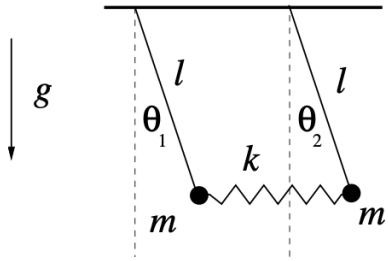
Q1: Two identical plane pendulums, each consisting of a massless rod of length l to which a mass m is attached, are connected by a massless spring with spring constant k. The spring is unstretched when $\theta_1 = \theta_2 = 0$. Gravity is acting downwards. The pendulums move only in the plane, so you can use the angles θ_1 and θ_2 to describe the motion. These angles can be assumed small enough that $\sin\theta_1 \approx \theta_1$, *etc.*



a) Find expressions for the potential and kinetic energies of the system in terms of m, l, k, and θ_1, θ_2 and their time derivatives. Hence write down the Lagrangian for this system.

b) Use your Lagrangian to write down the equations of motion for the coupled pendulums.

c) Find the characteristic frequencies and eigenvectors of the normal modes of the system.

d) Describe (in words) the motion of the pendulums in each of the normal modes.

e) Both pendulums are initially stationary, but held so that $\theta_1 = 0$ while $\theta_2 = \Theta > 0$. At time $t = 0$ the pendulums are simultaneously released, and so free to move. Write down the formulae giving $\theta_1(t)$ and $\theta_2(t)$.