# ILLINOIS

# Center for Academic Resources in Engineering (CARE)
# Peer Exam Review Session

**Phys 212 − University Physics: Electricity and Magnetism**

**Midterm 3 Worksheet Solutions**

*The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.*

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Thursday, May 1st, 6:30 - 7:50 PM CIF 2036 Ray, Aaron

Session 2: Friday, May 2nd, 4:00 - 5:20 PM CIF 2036 Jacob, Ray

Can't make it to a session? Here's our schedule by course:

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/848

2. Click "New Question"

3. Add your NetID and Name

4. Press "Add to Queue"

**Please be sure to follow the above steps to add yourself to the Queue.**

Good luck with your exam!

1. A circuit is composed of a battery with voltage $V = 10$ V, one resistor $R = 75\ \Omega$, a capacitor $C = 20$ pF, an inductor $L = 20$ mH and a switch S. The switch has been open for a long time; at $t = 0$, it is closed.
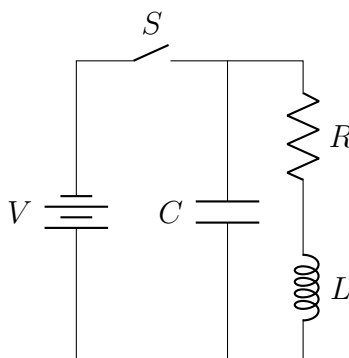


(i) What is the voltage across the capacitor after the switch has been closed for a long time?

    a) 10 V

    b) 0 V

    c) 15 V

    d) 7.5 V

(ii) What is the charge across the capacitor at this time?

    a) 20 pC

    b) 0 C

    c) 0.2 nC

    d) 2 nC

(iii) What is the voltage across $R$ after the switch has been closed for a long time?

    a) 7.5 V

    b) 10 V

    c) 0 V

    d) 5 V

(iv) If the switch is now reopened after a long period of time, what will the initial current going through $R$ be?

    a) 0.133 A

    b) 0 A

    c) 0.5 A

    d) 0.167 A

(i) The voltage source and capacitor are in parallel, and after a long time, the capacitor charge has approached its maximum. So voltage across capacitor equals the battery voltage $V$. The answer is **(a)**.

(ii)

$$\text{Using } Q = CV$$

$$C = 20 \text{ and } V = 10$$

$$Q = (20) * (10) = \boxed{200 \text{ pC or } 0.2 \text{ nC}}$$

(iii) After a long time, the inductor acts as a normal wire, so there is no voltage drop across it. Essentially, this means that the resistor is in parallel with both the battery and capacitor, both of which have a voltage drop of $\boxed{10V}$.

(iv) When the switch is opened, the inductor prevents in an instantaneous change in current. Therefore, the current right before and right after must be the same. Using Ohm's law for the current before the open switch:

$$I = \frac{V}{R}$$

$$\boxed{I = 0.133 \text{ A}}$$

2. The electric field for a plane electromagnetic wave in a vacuum is given by

$$\vec{E}(y, t) = 2100 \sin(\omega t + 0.8y) \, \hat{x}$$

(i) What is the frequency of the wave?

(ii) What is the magnitude of the magnetic field?

(iii) What is the direction of the Poynting vector?

     a) $\hat{y}$
     b) $-\hat{y}$
     c) $-\hat{z}$
     d) $\hat{z}$

(iv) What is the direction of $\vec{B}$?

     a) $\hat{y}$
     b) $-\hat{y}$
     c) $-\hat{z}$
     d) $\hat{z}$

(v) What is the average intensity of the electromagnetic wave?

(i) In a vacuum the speed of light is $c$. We can use the following dispersion relation to find $\omega$.

$$c = \frac{\omega}{k}$$

where $k = 0.8$ from the equation given. Rearrange the equation to solve for $\omega$. This can then be related to the normal frequency by $\omega = 2\pi f$. Solving for $f$:

$$\boxed{f = 38 \text{ MHz}}$$

(ii) The relationship between the magnitude of the electric and magnetic fields is

$$E = cB$$

We can solve for $B$ using $E = 2100$ from the equation

$$|B| = \frac{2100}{c} = \boxed{7 \times 10^{-6} \text{ T}}$$

(iii) The Poynting vector points in the direction of the wave's motion. In this case, that's along the $y$-axis because $E$ is a function of $y$. To determine the sign, we must examine the signs of the function's argument. Since both $\omega t$ and $ky$ have the same sign, the wave moves along $\boxed{-\hat{y}}$.

(iv) The equation for the Poynting vector is

$$\vec{S} \propto \vec{E} \times \vec{B}$$

Since we know the direction of $\vec{S}$ ($-\hat{y}$) and the direction of $\vec{E}$ we can determine the direction of $\vec{B}$ using the right hand rule. We must determine the unit vector $\hat{A}$ that makes $-\hat{y} = \hat{x} \times \hat{A}$ true.
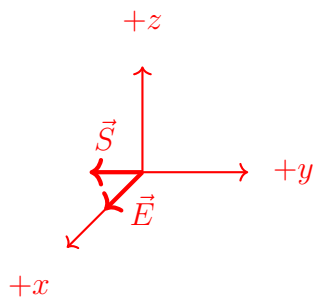


From the diagram, we can see that the only unit vector that works is $\boxed{+\hat{z}}$.

(v) The intensity of an electromagnetic wave is the magnitude of its Poynting vector.

$$I = \left| \frac{\vec{E} \times \vec{B}}{\mu_0} \right| = \frac{E \cdot B}{\mu_0} = \frac{E^2}{c \cdot \mu_0}$$

Since we want the average intensity, the expression then becomes:

$$I_{avg} = \frac{E_{avg}^2}{c \cdot \mu_0} = \frac{(E_{max}/\sqrt{2})^2}{c \cdot \mu_0} = \frac{E_{max}^2}{2c \cdot \mu_0} = \boxed{5849 \text{ } W/m^2}$$

3. An ideal transformer has $N_1 = 100$ turns in the primary coil and $N_2 = 10$ turns in the secondary coil. An RMS voltage of $V = 120$ V and 60Hz AC voltage is connected to the primary coil. A resistor with resistance $R = 20 \text{ } \Omega$ is connected to the secondary coil as shown in the figure.
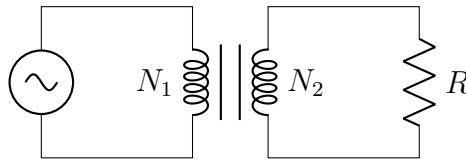
(i) What is the average voltage across the resistor?

    a) 120 V

    b) 1200 V

    c) 12 V

    d) 24 V

(ii) What is the average power in the resistor?

    a) 0 W

    b) 14.4 W

    c) 0.6 W

    d) 7.2 W

(i)

$$\text{Using } \frac{V_1}{N_1} = \frac{V_2}{N_2}$$
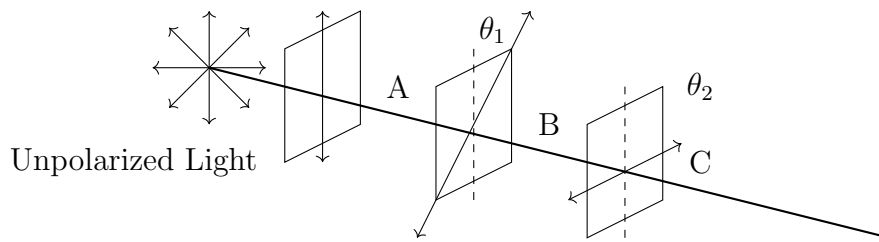
where $N_1 = 100$, $N_2 = 10$, and $V_1 = 120$ V to get

$$\boxed{V_2 = 12 \text{ V}}$$

(ii) Using the average voltage found in the previous question

$$P = \frac{V^2}{R} = \frac{12^2}{20} = \boxed{7.2 \text{ W}}$$

4. A beam of unpolarized light of intensity $I_0$ passes through a series of ideal polarizing filters with their transmission axis turned to various angles, as shown in the figure ($\theta_1 = 75°$ and $\theta_2 = 90°$, both relative to the vertical)



(i) What is the light intensity (in terms of $I_0$) in regions A, B and C?

(ii) If we remove the middle filter, what will be the intensity at point C?

(iii) If the second filter was rotated 15° clockwise, what would the light intensity be at point C?

(iv) How far should the first filter be rotated in order to maximize the intensity of light at point C?

(i) Unpolarized light through a linear polarizer cuts intensity in half. Then use

$$I = I_0 \cos^2(\theta)$$

to solve for intensities going through the other two filters

$$I_A = \boxed{\frac{I_0}{2}}$$
$$I_B = I_A \cos^2(\theta_1) = \boxed{0.0335 I_0}$$
$$I_C = I_B \cos^2(\theta_2 - \theta_1) = \boxed{0.03125 I_0}$$

(ii) If the middle filter is removed, the angle between the first and third filters is 90°. Vertically polarized light cannot enter a horizontal polarizer.

$$\boxed{I_C = 0}$$

(iii) If the second filter is rotated 15° clockwise, $\theta$ between the first and second filter would be 90°, so $I_B = 0$ and $I_C = 0$
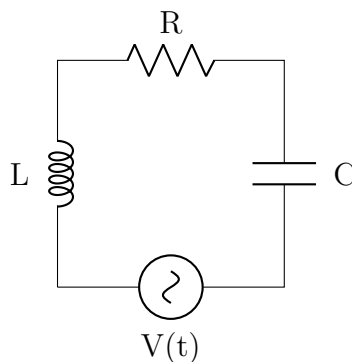
$$\boxed{I_C = 0}$$

(iv)

$$\boxed{75° \text{ clockwise}}$$

To maximize the intensity of light at point C, rotate the first filter 75° to make $\theta$ between the first and second filter 0. This allows $I_1 = I_2$ so there's no decrease in intensity through the second filter.

5. An AC circuit is shown below. The generator follows $V(t) = 1500 cos(50t)$. The Resistor has resistance R = 60 $\Omega$. The inductance L is 40 H. The capacitance C is 10 $\mu F$.



(i) Find the magnitude of the current through the circuit at t = 73 s. (Hint: what is the phase difference between the generator and the current?)

(ii) The frequency of the generator has been decreased to 40. Does the generator voltage now lags the current or leads the current?

(iii) The frequency of the generator is still set to 40. At a certain instant $t_0$, the generator voltage has its maximum. What is the magnitude of the current in the circuit at that instant?

(i) To calculate the current, we first have to find $I_{max}$ and the phase difference between the generator and the current. We know that the relationship between the peak voltage $V_{max}$ and the peak current $I_{max}$ is

$$V_{max} = I_{max}Z$$

$$Z = \sqrt{X_R^2 + (X_L - X_C)^2}$$

$$\phi = tan^{-1}(\frac{X_L - X_C}{X_R})$$

In this case, $X_R, X_L, X_C$ are calculated to be:

$$X_R = R = 60 \ \Omega$$

$$X_L = \omega L = 50 \times 40 = 2000 \ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{50 \times 10 \times 10^{-6}} = 2000 \ \Omega$$

Therefore, the phase difference between the generator voltage and the current is

$$\phi = tan^{-1}(\frac{2000 - 2000}{60}) = 0$$

This implies that the generator voltage and the current are in phase. Therefore the current can be expressed as

$$I(t) = I_{max}cos(50t) = \frac{V_{max}}{Z}cos(50t) = 25cos(50t)$$

Finally, the current in the circuit at t = 73 s is

$$I(t = 73s) = 25cos(50 \times 73) = \boxed{21.56 \ A}$$

(ii) We know that $X_L$ is proportional to the generator frequency and that $X_C$ is inversely proportional to the generator frequency. Therefore, by lowering the frequency, $X_L$ decreases and $X_C$ increases. This results in the generator voltage lagging the current.

(iii) We again start off by calculating the impedance for each component in order to know the phase relation between the generator and the current:
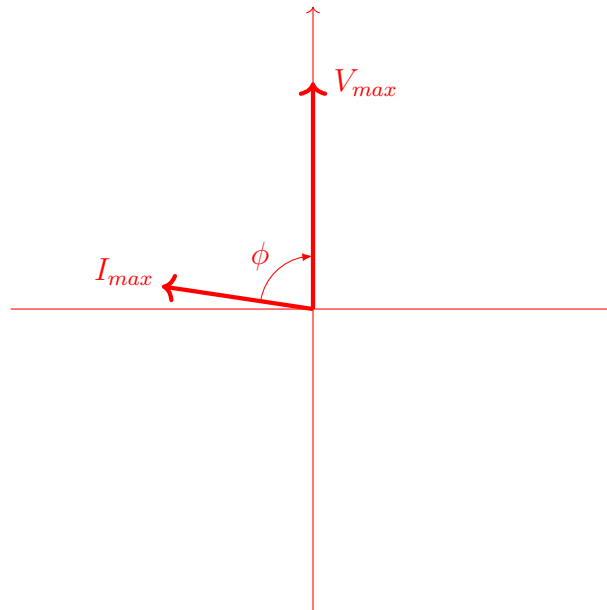
$$X_R = R = 60 \ \Omega$$

$$X_L = \omega L = 40 \times 40 = 1600 \ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{40 \times 10 \times 10^{-6}} = 2500 \ \Omega$$

$$\phi = tan^{-1}(\frac{1600 - 2500}{60}) = -86.19°$$

Therefore, we know that the generator lags the current by $-86.19°$. With this information, we can also construct the phasor diagram when the generator voltage is at its maximum:

We know that in order to get the current, we would need to project $I_{max}$ onto the y-axis. Since we know the angle $\phi$, we are able to calculate this projection:

$$I(t_0) = I_{max}cos(86.19°) = \frac{V_{max}}{Z}cos(86.19°) = \frac{1500}{\sqrt{60^2 + (1600 - 2500)^2}}cos(86.19°) = \boxed{0.11\ A}$$