Midterm 3
Welcome to Math 231 Exam Review!

CARE

QR Code to Queue and Worksheet:

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

Sequences

- A list of numbers in a certain order
  - $a_1, a_2, a_3, a_4, a_5, \ldots, a_n, \ldots$
- Taking the Limit of Sequences:
  - Limit as n approaches infinity
    - If it exists, the sequence is convergent
    - DNE, sequence is divergent
    - If the limit is infinity, a_n diverges to infinity

Ex: $1/5$, $-2/25$, $3/125$, $-4/625$, … , $\mathbf{(-1)^{(n-1)} n/5^n}$

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

Series

- Similar to sequences, but now add the terms together!
  - $a_1 + a_2 + a_3 + a_4 + a_5 + \ldots + a_n + \ldots$
- Convergent if $a_1 + a_2 + \ldots + a_n + \ldots = s$
  - Divergent if the sum diverges
- Geometric Series:

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}; |r| < 1$$

What about |r| >= 1? It diverges!

- Test for Divergence
  - If the limit of an does not exists or does not equal 0
    - The series is divergent

Integral Test

- Suppose $a_n = f(n)$

  If converges $\int_1^\infty f(x)dx$ then $\sum_{n=1}^\infty a_n$ converges

  If diverges $\int_1^\infty f(x)dx$ then $\sum_{n=1}^\infty a_n$ diverges

- P-Test

$\sum_{n=1}^\infty \frac{1}{n^p}$

When does this series converge?

When p > 1!

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

Comparison Test

- If given two series and know the convergence or divergence of one:

If $\sum_{n=1}^{\infty} b_n$ is convergent and $b_n \geq a_n$ then $\sum_{n=1}^{\infty} a_n$ converges

If $\sum_{n=1}^{\infty} b_n$ is divergent and $b_n \leq a_n$ then $\sum_{n=1}^{\infty} a_n$ diverges

- Limit Comparison Test:

If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ and c is finite and > 0:

Either both series converge or both series diverge

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

Alternating Series

- The series alternates between positive and negative!
- Convergence:
  - Terms are decreasing, $b_n >= b_{n+1}$
  - $\lim_{n\to\infty} b_n = 0$
- Absolute and Conditional Convergence
  - Absolute Convergence: The absolute value of a series is convergent
  - Conditional Convergence: The absolute value of a series is convergent, but the original series diverges
  - If a series is absolutely convergent, then it is convergent

Ex: $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ Conditional or Absolute convergence?

Conditionally Convergent!

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

Ratio Test

- Take the limit of absolute value of the ratio of the n^{th} term and the $n^{th}+1$ term

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L$$

- If $L < 1$: Absolutely Convergent
- If $L > 1$ or $L = \infty$: Divergent
- If $L = 1$: Inconclusive

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

Root Test

- Take the limit of n^{th} root of the absolute value of the n^{th} term

$$\lim_{n \to \infty} \sqrt[n]{|a_n|} = L$$

- If L < 1: Absolutely Convergent
- If L > 1 or L = ∞: Divergent
- If L = 1: Inconclusive

Putting It All Together

A general order for how you might want to go by solving problems:

1. Test for Divergence
2. p-Series Test
3. Geometric Series Test
4. Comparison Test
5. Alternating Series Test
6. Ratio Test
7. Root Test
8. Integral Test

Putting It All Together

1. **Check divergence with limit**
2. Look for easy P-Test/Geometric
3. Inspection

TEST	SERIES	CONVERGES IF…	DIVERGES IF…	COMMENTS
*n*th Term Test for Divergence	$\sum_{n=1}^{\infty} a_n$	n/a	$\lim_{n\to\infty} \neq 0$	should be first test used. Inconclusive if limit = 0.
Geometric Series Test	$\sum_{n=1}^{\infty} a_n r^{n-1}$	$\lvert r\rvert < 1$	$\lvert r\rvert \geq 1$	use if there is a "common ratio" $S_n = \frac{a}{1-r}$
P-Series Test	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	harmonic series when p=1. Useful for comparison tests.
Integral Test	$\sum_{n=1}^{\infty} a_n$ $a_n = f(x)$	$\int_1^{\infty} f(x)\,dx$ converges	$\int_1^{\infty} f(x)\,dx$ diverges	$f(x)$ must be continuous, positive, and decreasing
Direct Comparison Test	$\sum_{n=1}^{\infty} a_n$	$0 \leq a_n \leq b_n$, $\sum_{n=1}^{\infty} b_n$ converges	$0 \leq b_n \leq a_n$, $\sum_{n=1}^{\infty} b_n$ diverges	to show convergence, find a larger series. to show divergence, find a smaller series.
Limit Comparison Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{a_n}{b_n} > 0$, $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n\to\infty} \frac{a_n}{b_n} > 0$, $\sum_{n=1}^{\infty} b_n$ diverges	apply l'hospital's rule if necessary; inconclusive if limit equals 0 or ∞
Alternating Series Test	$\sum_{n=1}^{\infty} (-1)^{n+1} a_n$	$a_{n+1} \leq a_n$, $\lim_{n\to\infty} a_n = 0$	$\lim_{n\to\infty} a_n \neq 0$	must prove that the limit equals 0 and that terms are decreasing
Ratio Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert < 1$	$\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert > 1$	test fails if : $\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert = 1$
Root Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} < 1$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} > 1$	test fails if: $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = 1$