Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

Math 231 – Calculus II

Midterm 3 Worksheet Solutions

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: April 7, 7:00-8:30pm Bella, Sofi, and Hriday

Session 2: April 8, 7:00-8:30pm Grace, Addy, Anush

Can't make it to a session? Here's our schedule by course:

`https://care.grainger.illinois.edu/tutoring/schedule-by-subject`

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/844
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. Determine whether the series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{n^2+3}{n^3+3}$$

$\frac{n^2+3}{n^3+3}$ can be compared to $\frac{n^2}{n^3}$ by the comparison test.

$\frac{n^2}{n^3} = \frac{1}{n}$ which diverges by the p-test (p =1).

Because $\frac{n^2+3}{n^3+3} > \frac{n^2}{n^3}$, the series $\boxed{\text{diverges}}$.

2. Determine whether the series converges or diverges. Note that $\lim_{n\to\infty}(1+\frac{1}{n})^n = e$.

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

Use the divergence test: $a_n = \frac{n!}{n^n}$ and $a_{n+1} = \frac{(n+1)!}{(n+1)^{n+1}}$

$\frac{a_{n+1}}{a_n} = [\frac{(n+1)!}{(n+1)^{n+1}}][\frac{n^n}{n!}] = \frac{n^n}{(n+1)^n} = (\frac{n}{n+1})^n$

$\lim_{n\to\infty}(\frac{n}{n+1})^n = \lim_{n\to\infty}(\frac{1}{1+\frac{1}{n}})^n = \frac{1}{e} < 1$

So the series $\boxed{\text{converges}}$.

3. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{5^n}{n+6^n}$$

$\frac{5^n}{n+6^n} < \frac{5^n}{6^n} = (\frac{5}{6})^n$ which converges. Thus, $\frac{5^n}{n+6^n}$ $\boxed{\text{converges}}$.

4. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} (\frac{n^2}{2n^2+5})^n$$

Use the root test:

$(\frac{n^2}{2n^2+5})^n \rightarrow [a_n]^{\frac{1}{n}} \rightarrow [(\frac{n^2}{2n^2+5})^n]^{\frac{1}{n}}$

$= \frac{n^2}{2n^2+5}$

$\lim_{n\rightarrow\infty} \frac{n^2}{2n^2+5} = \frac{1}{2} < 1 \rightarrow \boxed{\text{Series converges}}$.

5. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{17n^2}{3n^4-1}$$

Use the limit comparison test:

$a_n = \frac{17n^2}{3n^4-1}$ and $b_n = \frac{1}{n^2}$

$\frac{a_n}{b_n} = (\frac{17n^2}{3n^4-1})(\frac{n^2}{1}) = \frac{17}{3} \rightarrow$ both do the same thing.

$\frac{1}{n^2}$ converges by the p-test $\rightarrow \boxed{\text{both converge}}$.

6. Determine if the following series converges absolutely, converges conditionally, or diverges.

$$\sum_{n=1}^{\infty} \frac{(2n+1)(-2)^n}{n!}$$

$$a_n = \frac{(2n+1)(-2)^n}{n!}, \qquad a_{n+1} = \frac{(2n+3)(-2)^{n+1}}{(n+1)!}$$

$$\begin{aligned}
\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| &= \lim_{n\to\infty} \left|\frac{-2(2n+3)n!}{(n+1)!(2n+1)}\right| \\
&= \lim_{n\to\infty} \left|\frac{-2(2n+3)}{(n+1)(2n+1)}\right| \\
&= \lim_{n\to\infty} \left|\frac{-4n-6}{2n^2+...}\right| \\
&= \lim_{n\to\infty} = 0 < 1
\end{aligned}$$

Therefore the series $\boxed{\text{converges absolutely.}}$

7. Determine whether the series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

Using the integral test:

$a_n = \frac{1}{n \ln n}$

$\int_2^{\infty} \frac{1}{x \ln x} dx$; Using u-substitution to solve the integral: $u = \ln x, du = \frac{1}{x} dx$

$\int_2^{\infty} \frac{1}{x \ln x} dx = \int_2^{\infty} \frac{1}{u} du = \lim_{t \to \infty} \ln(\ln t) - \ln(\ln 2) = \infty$

$\frac{1}{n \ln n}$ diverges by the integral test $\rightarrow$ $\boxed{\text{The series diverges}}$.