ILLINOIS

Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

Math 241 – Calculus III

Midterm 2 Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Mar. 9, 5-7 pm Meredith, Rick, Pallab

Session 2: Mar. 25, 5-7 pm Lydia, Gabe, Johail

Can't make it to a session? Here's our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/845
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. Calculate the following derivatives:

(a) Find $\frac{\mathrm{d}f}{\mathrm{d}t}$ for $f(x, y) = xe^{xy}$, $x(t) = t^2$, $y(t) = \frac{1}{t}$

(b) Find f_t for $f(x, y) = 2xy$, $x(s, t) = st$, $y(s, t) = s^2t^2$

2. Show that $f(x, y) = y^2e^{xy}$ is differentiable at (0, 2) and use linear approximation to find the value of $f(0.1, 2.1)$.

3. A tiny spaceship is orbiting a path given by $x^2 + y^2 = 4$. The solar radiation at a point (x, y) in the plane of the orbit is $f(x, y) = xy + 2y$.

   Use the method of *Lagrange multipliers* to find the maximum value and minimum value of solar radiation experienced by the tiny spaceship in its orbit.

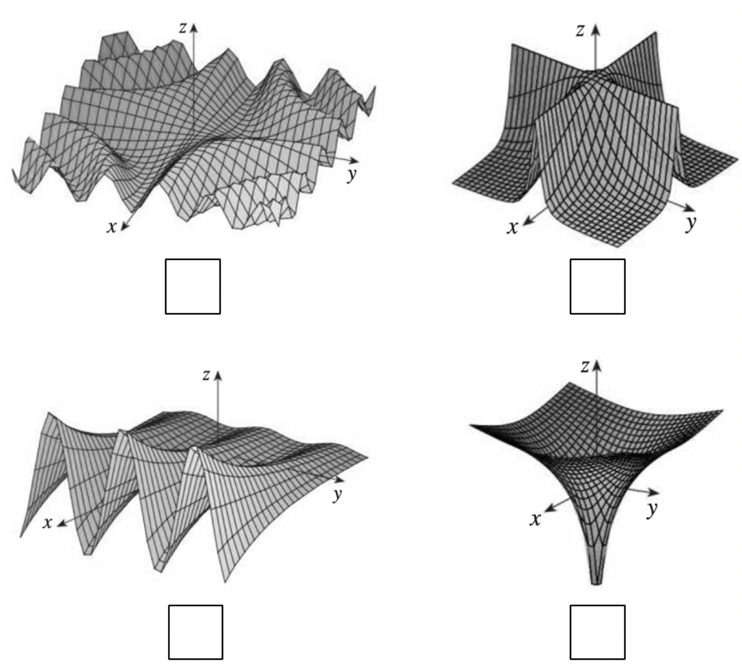
4. For each equation below, match it with the corresponding graph.

(A) $z = ln(x^2 + y^2)$

(B) $z = sin(xy)$

(C) $z = e^x cos(y)$

(D) $z = \frac{1}{1+x^2y^2}$



5. Let $f(x, y)$ be a differentiable function on the disk $\{D : x^2 + y^2 \leq 400\}$, where:

(I) $f(x, y) = 19$ for every point on the boundary of the disk $x^2 + y^2 = 400$

(II) $f(0, 0) = 7$

(III) $f(x, y)$ has only one critical point which is at $(-1, 2)$

Decide which statement is true:

A) $f(-1, 2) > 7$

B) $f(-1, 2) < 7$

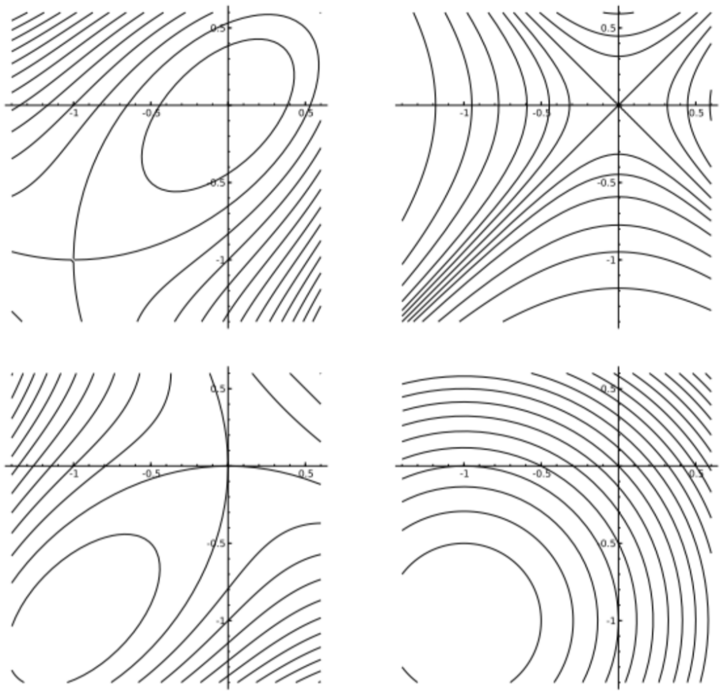
C) $f(-1, 2) = 7$

D) Not enough information is given

6. Consider the function $f(x, y) = x^3 + y^3 + 3xy$

(a) The critical points of f are $(0, 0)$ and $(-1, -1)$. Classify them into local minima, local maxima and/or saddle points

(b) Based on your answer in (a), identify the correct contour diagram of f



7. What is the partial derivative of $f(x, y, z) = e^x \sin(yz)z^3 \ln(y)$ with respect to x.

8. Consider a function $f(t) = f(x(t), y(t), z(t)) = xyz - z^2$, where $x(t)$, $y(t)$, $z(t)$ are defined as followed:

$$x(t) = 2t^2 + 1 \qquad y(t) = 3 - \frac{1}{t} \qquad z(t) = 3$$

Find the following values:

(a) $f_z(3, 1, 2)$ (b) $\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{(t=0)}$ (c) $\left.\frac{df}{dt}\right|_{(t=1)}$

9. Consider the limit

$$\lim_{(x,y)\to(0,0)} \frac{y^4 \cos^2 x}{x^4 + y^4}$$

Does this limit exist? If so, what is its value? Justify your answer.

10. If $f(x, y, z) = xye^z$, find the gradient of f and the directional derivative at (2,5,0) in the direction of $\vec{v} = 2\hat{i} - \hat{j} + \hat{k}$.

11. Evaluate the following functions with $\lim_{(x,y)\to(0,0)}$:

(a) $f(x,y) = \dfrac{3xy - x^2y}{x^2 + y^2 + xy}$ (b) $f(x,y) = \dfrac{y\sin(x) + y^2e^x}{y}$ (c) $f(x,y) = \dfrac{\left(x^2 + y^2\right)^5}{x^{10} + y^4}$

12. Find min/max of f(x,y,z) = $3x^2 + 8y^2 + z^2 - 2z$ defined on the domain $x^2 + 4y^2 + 2z \leq 8$ and $z \geq 0$

(a) The domain is (select all that apply)
   I) open
   II) closed
   III) bounded
   IV) unbounded

(b) Where are the critical points inside the domain? Evaluate the function value on these points.

(c) What is the minimum and maximum on $x^2 + 4y^2 + 2z = 8$?

(d) What is the minimum and maximum on $z = 0$?

(e) What is the global minimum and maximum of the whole domain?

13. Compute the double integral over the indicated rectangle. Confirm your answer by switching the order of integration and recomputing.

$$\iint_R 2x - 4y^3 \; dA \qquad R = [-5, 4] \times [0, 3]$$

14. Making an appropriate change of variables, compute the following double integral over the region bound by a circle of radius 2 and a circle of radius 5.

$$\iint_D e^{x^2+y^2} \; dA$$