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Transportation Infrastructure Precast Innovation Center (TRANS-IPIC)

University Transportation Center (UTC)

Evaluating Prestressed Concrete Beams with Cracks using Machine Learning
UB-23-RP-01

FINAL REPORT

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Executive Summary

Bridge owners face difficult decisions on whether a bridge should be posted, repaired or replaced when prestressed concrete (PC) members have shear related cracks due to overloading. The decisions are currently made based on engineering judgment, costly load-testing or time consuming and complex modeling. Guidance is needed to interpret cracks and their impact on shear capacity to avoid overly conservative load ratings and to keep bridges operational, without compromising safety and economy. This project developed a reliable and efficient tool through machine learning (ML) to relate cracking to load history of bridge members. Shear strength was also predicted through ML as a reference point for load history.

A database of 806 shear test results for PC beams is compiled and filtered from the literature to predict shear strength. A second database with 79 beams, which is a subset of the larger database, is compiled for load history predictions. The beams in this second database have 373 data points with shear crack width measurements and corresponding loads (load history). The predictive features for shear strength were the product of prestressing steel area and its effective prestress ($A_{ps}\, f_{se}$), product of longitudinal tensile reinforcement area and its yield strength ($A_{sl}\, f_{yl}$), bottom flange width (b_{fb}), prestressing depth (d_p), ratio of the product of the shear reinforcement area and its yield strength to shear reinforcement spacing ($A_{sv}\, f_{yv}$ /s), shear span to prestressing depth ratio (a/d_p), web width (b_w), square root of the compressive strength of concrete ($\sqrt{f'_c}$), product of draped area of prestressing steel and sine angle of draping (A_{draped} sinα). Load history predictions had crack width (W_{cr}) as an additional predictive feature. Target features were shear strength or shear load history normalized by shear strength.

Three algorithms have been trained using these datasets: Ordinary Linear Regression, Support Vector Regression, and Gaussian Process Regression (GPR). The GRP algorithm is selected after a k-fold cross-validation showed that the error in shear strength predictions was consistently smaller (15% with nine predictive features) with GPR compared to the errors with the other algorithms (19% to 25% with nine predictive features). The most influential predictive features were d_p, $A_{ps}\, f_{se}$, $A_{sv}\, f_{yv}$ / s, a/d_p, and $\sqrt{f'_c}$ for shear strength predictions. These features were W_{cr}, and $A_{sl}\, f_{yl}$ for load history predictions.

Shear strength was also predicted using ACI 318-19 and the 9th edition of the AASHTO LRFD Bridge Design Specifications (BDS) as a comparison. The mean absolute percentage error of the ACI 318-19 and AASHTO LRFD BDS was 27%-40% and 39%, respectively, as compared to the 15% error for GPR. The higher errors in ACI 318-19 and AASHTO LRFD BDS predictions are partially explained by the intentional conservatism of design specifications.

The application of GRP for evaluating beams was demonstrated on four beams that were excluded from the training dataset. The prediction error ranged between 2% and 35% for shear strength. The normalized load history prediction error ranged between 3% and 24% for these beams. The error was generally larger for beams for which the training dataset size was smaller, indicating that predictions can be improved with additional data. Finite element modeling was used to demonstrate how analyses can supplement ML predictions by providing information such as reinforcement strains that correspond to a given load. A web application was created for evaluating beams for given geometric properties, material properties, reinforcement details and crack width.

TRANS-IPIC Final Report:

1. Statement of the Problem

Bridge owners face difficult decisions on whether a bridge should be posted, repaired or replaced when prestressed concrete (PC) members have shear related cracks due to overloading. The decisions are currently made based on engineering judgment, costly load-testing or time consuming and complex modeling. Guidance is needed to interpret cracks and their impact on shear capacity to avoid overly conservative load ratings and to keep bridges operational, without compromising safety and economy. This project developed a reliable and efficient tool through machine learning (ML) to relate cracking to load history of bridge members.

2. Research Plan

The project is composed of the following tasks:

<u>Task 1. Compile and filter test data:</u> Existing data in the literature on the shear behavior of PC beams is compiled and curated to create a comprehensive dataset. Existing databases are reviewed to obtain crack and design information. Any gaps in data are documented to plan for additional tests as needed.

<u>Task 2. Investigate ML algorithms:</u> ML is used to train a supervised learning model. The model determines relationships between structural design parameters and shear capacity from historical data presented in a training dataset. Linear and non-linear ML models are explored.

<u>Task 3. Predict load history and capacity:</u> Using a suitable ML algorithm, shear capacity and loading that corresponds to given crack widths are predicted. The input is geometric properties, material properties, reinforcement details and crack widths under increasing loading for PC beams for which test data is available. The models are fine-tuned using a cross-validation analysis.

<u>Task 4. Verify predictions:</u> The predictions of ML are tested on four beams that will be selected from the existing databases but are not part of the ML training. Shear load history and capacity of the beams predicted with ML are compared to the ones obtained from testing. Two of these beams are also modeled using finite element analysis to further examine the condition of the beams at a given load.

<u>Task 5. Develop software for implementation:</u> To facilitate the use of the evaluation method, a web application with a simple user interface is developed.

<u>Task 6: Write a final report</u>: A report that documents project goals, methods and results is prepared.

3. Research Progress and Results

<u>Task 1 [100% completed]:</u>

A database of 806 shear test results for PC beams is compiled as summarized in Table 1. The majority of the data comes from the shear dataset collected by Nakamura, Avendaño et al. (2013). Additional filtering is applied to Nakamura, Avendaño et al. (2013) such that the filtered dataset only includes slender, normal-weight concrete, simply supported beams prestressed with bonded strands. Slender PC beams are *defined as those with an $a/d_p \geq 2$, where a is the shear span, and d_p is the distance between the extreme compression fiber and centroid of the strands at midspan. This filtered dataset is further supplemented with recent experimental studies published since the compilation of the Nakamura, Avendaño et al. (2013) dataset as listed in Table 1. It should be noted that most of the shear test results published after 2013 focus on unconventional materials (e.g., concrete with steel or synthetic fibers) and are not included in our study.*

Table 1. Experimental dataset of PC beams collected to train ML algorithms.

Reference	Number of tests
Shear dataset by Nakamura, Avendaño et al. (2013) with additional filtering	796
De Wilder, Lava et al. (2015)	7
Joshi, Thammishetti et al. (2018)	1
Perumalla, Yogeendra et al. (2022)	2
Total	806

Each experimental sample consists of up to 9 predictive features that relate to geometry, material properties, reinforcement details and one target feature (experimental shear strength). These predictive features are: product of prestressing steel area and its effective prestress ($A_{ps} f_{se}$), product of longitudinal tensile reinforcement area and its yield strength ($A_{sl} f_{yl}$), bottom flange width (b_{fb}), prestressing depth (d_p), ratio of the product of the shear reinforcement area and its yield strength to shear reinforcement spacing ($A_{sv} f_{yv} /s$), shear span to prestressing depth ratio (a/d_p), web width (b_w), square root of the compressive strength of concrete ($\sqrt{f'_c}$), product of draped area of prestressing steel and sine angle of draping ($A_{draped} \sin\alpha$). The experimental shear strength reported by the studies is assumed to include the self-weight of the beam unless stated otherwise by the studies.

Figure 1 presents the relative frequency distributions for different parameters, and includes the minimum, mean, maximum, and standard deviation for each parameter. Relative frequency distribution provides a visual representation of the data's spread and statistics and is also an indicator of the applicability range of the ML algorithms. For example, most beams in the dataset have d_p and b_w less than 20 inches and 6 inches, respectively. These values are small compared to bridge beams. Although this may hinder the accuracy of the ML algorithms to make predictions for larger members, the accuracy can be improved by retraining ML algorithms when new test data becomes available.

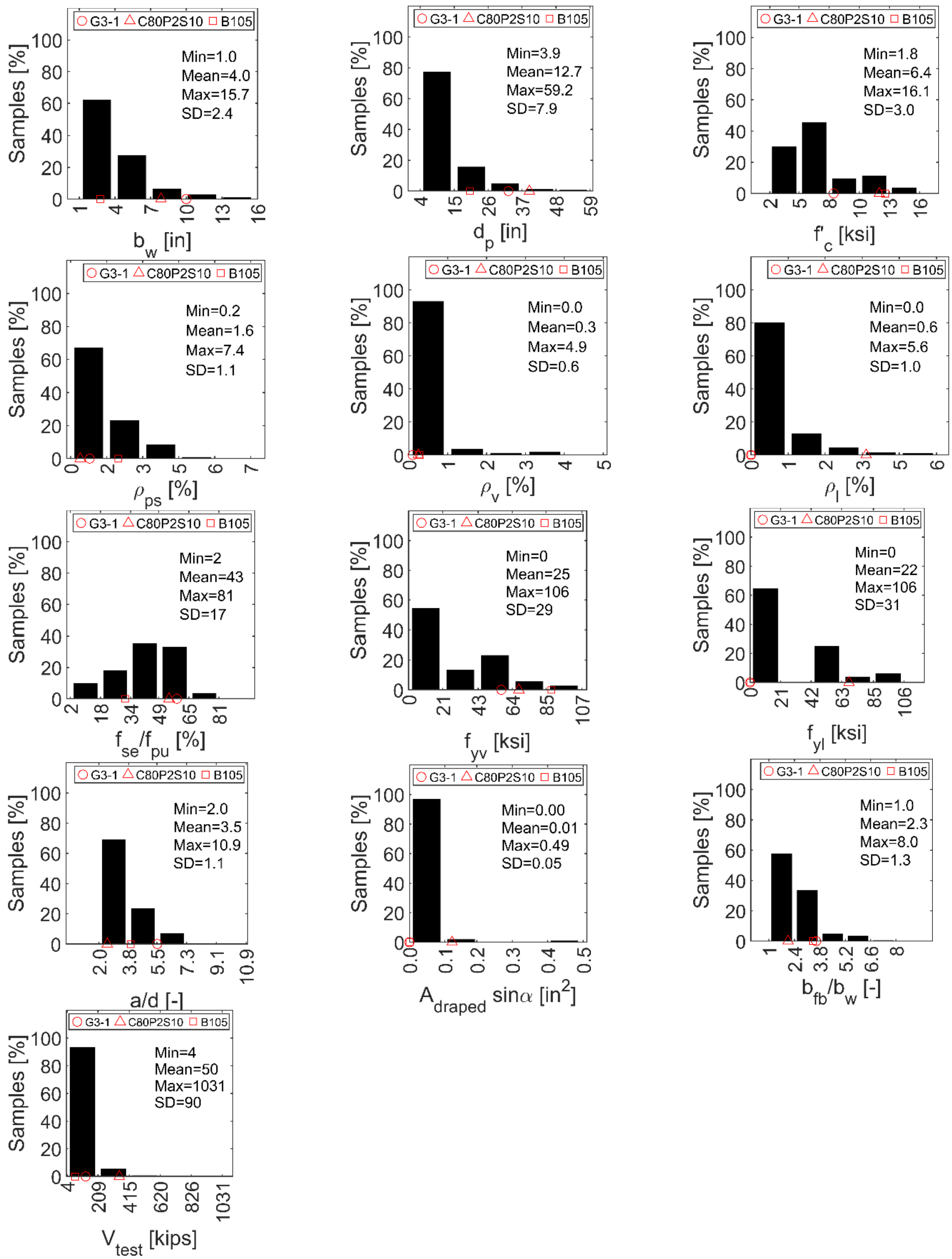


Figure 1. Relative frequency distribution of parameters for shear strength database.

A second database with 79 beams, which is a subset of the larger database, is compiled for load history predictions. The beams in this second database have crack width *measurements (373 data points) and corresponding shear load, in addition to information on geometric properties, material properties, and reinforcement details. Each experimental sample consists of up to 10 predictive features and one target feature (measured shear load corresponding to a crack width normalized by the measured shear strength). These predictive features are $A_{ps} f_{se}$, $A_{sl} f_{yl}$, b_{fb}, d_p, $A_{sv} f_{yv} /s$, a/d_p, b_w, $\sqrt{f'_c}$, A_{draped} sinα, and crack width (W_{cr}). Figure 2 presents the relative frequency distributions for different parameters of this smaller database.*

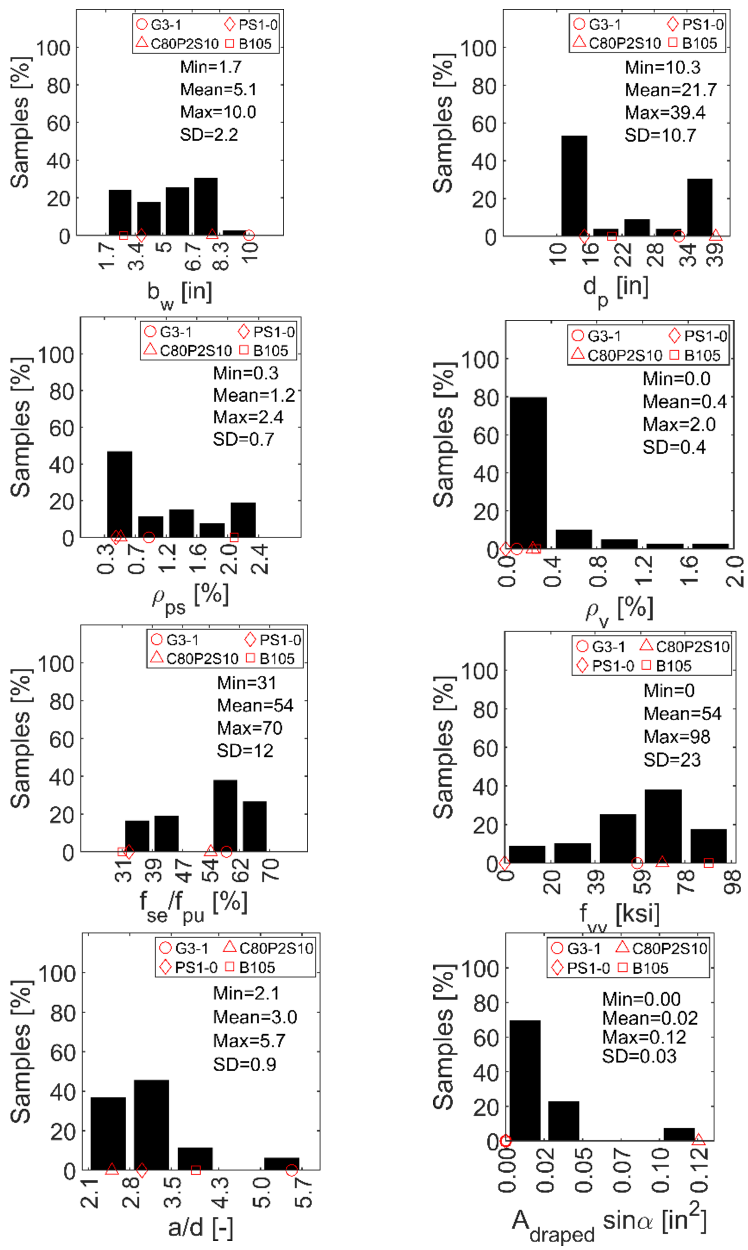


Figure 2. Relative frequency distribution of parameters for load history database.

The crack width (W_{cr}) measurements with their corresponding shear that is normalized by the measured shear capacity ($V_{applied}/V_{max}$) are visualized in Figure 3. Although several trends could potentially fit this data, the results exhibit significant scatter, *indicating that crack width alone is insufficient as an indicator of the normalized shear. Therefore, beam material and geometric properties are considered as predictive features in addition to crack width to improve the prediction of normalized shear.*

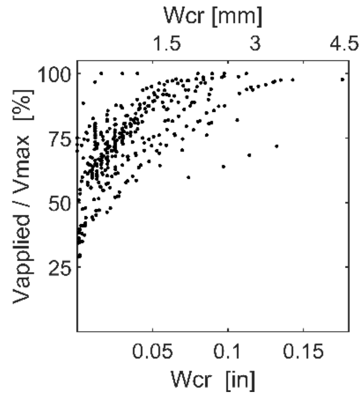


Figure 3. Experimental crack width versus normalized shear in the collected database.

<u>Task 2 [100% completed]:</u>

The literature is reviewed for ML algorithms suitable for the objectives of this project. Ordinary linear regression (OLR), support vector regression (SVR) and Gaussian process regression (GPR) algorithms are selected to be investigated because they can be applied to relatively small datasets such as the ones available for PC beams and do not require high computational power to be trained.

A k-fold cross-validation is performed, which shuffles the samples randomly, and divides the dataset into k subgroups with one subgroup selected for validation and the remaining subgroups for training. This process is repeated k times so that each subgroup is selected for validation. The mean absolute percentage error (MAPE), defined in Equation (1), is selected as the performance metric to evaluate the prediction accuracy. MAPEs obtained from the three algorithms predicting the shear strength were compared to determine the most suitable algorithm.

$$MAPE = \frac{\sum \left| \frac{Predicted\ value - true\ value}{true\ value} \right|}{number\ of\ samples} \tag{1}$$

The algorithms are trained to predict the shear strength of PC beams considering all possible combinations of the following features: $A_{ps} f_{se}$, $A_{sl} f_{yl}$, b_{fb}, d_p, $A_{sv} f_{yv} /s$, a/d_p, b_w, $\sqrt{f'_c}$, and A_{draped} sinα. Figure 4 compares the shear strength prediction errors of the three algorithms for a given number of features included in a prediction. The reported errors are from the specific feature(s) that led to the lowest prediction errors. This error is calculated as the average of the errors from k-fold cross-validations for these specific features. The lowest MAPE of OLR, SVR, and GPR are 35%, 19%, and 15%, respectively, when 9 features are considered. Except for the prediction with only 1 predictive feature (which is not realistic), GPR consistently had smaller MAPEs compared to the other algorithms. Therefore, GPR is selected as the most suitable algorithm for further analysis.

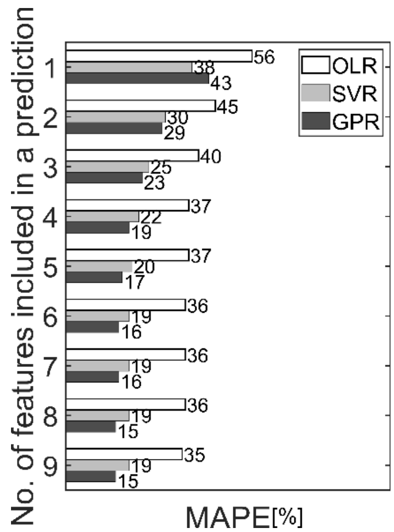


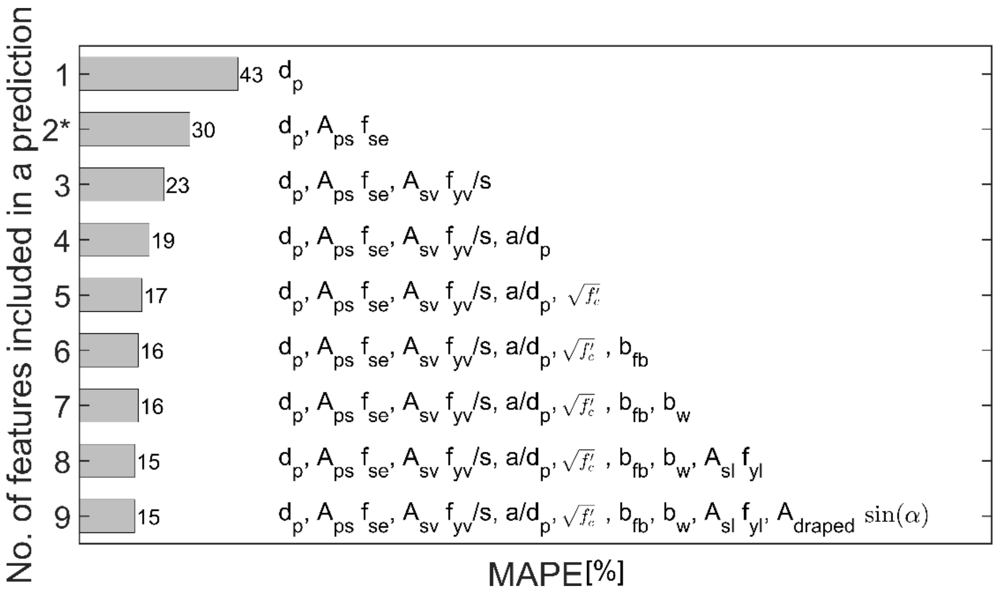
Figure 4. Comparison of shear strength prediction errors for the three algorithms considering specific number of features.

Task 3 [100% completed]:

The GPR algorithm is used to predict the shear strength given beam properties, and then to predict the normalized shear given beam properties and crack widths. MAPE is calculated for all possible combinations of features to identify the most influential features on shear strength of normalized shear.

Features influential for shear strength predictions

Figure 5 shows the feature(s) that yield the lowest shear strength error considering a specific number of features. The lowest MAPE of the shear strength prediction is also shown in the figure. For example, when the prediction is based on a single feature, d_p yields the lowest error, 43%, compared to other combinations with a single feature, indicating that d_p is the most influential feature. Figure 5 shows that MAPE does not decrease considerably (≤1%) when five or more features are included in the predictions. This indicates that adding features beyond d_p, $A_{ps}\,f_{se}$, $A_{sv}\,f_{yv}\,/\,s$, a/d_p, and $\sqrt{f'_c}$ provides no significant benefit to the predictions and the remaining features (b_{fb}, b_w, $A_{sl}\,f_{yl}$, and $A_{draped}\,\sin\alpha$) have minimal influence on reducing the error. The low predictive power of $A_{sl}\,f_{yl}$ and $A_{draped}\,\sin\alpha$ may be due to the fact that over 50% of the beams in the training dataset had no longitudinal reinforcement or draped strands.



* For no. of features between 2-6, there were other combinations of features that led to virtually identical error to the ones presented here. For example, when 2 features were considered, *d_p and b_w led to nearly the same MAPE as d_p and A_{ps} f_{se}.*

Figure 5. Features with the least shear strength prediction error using the GPR algorithm for a specific number of features.

Even though the results presented in this section show that some features may have insignificant predictive power, training the GPR algorithm using all features does not require high computational power. Therefore, all nine features are considered for training the GPR algorithm.

Features influential for load history predictions

Figure 6 shows the feature(s) that yield the lowest normalized MAPE considering a specific number of features. Figure 6 shows that MAPE does not decrease considerably (≤1%) when two or more features are used in the predictions. This indicates that adding features beyond W_{cr}, and A_{sl} f_{yl} does not improve predictive performance. The remaining features (A_{ps} f_{se}, b_{fb}, d_p, A_{sv} f_{yv} / s, b_w, $\sqrt{f'_c}$, a/d_p, and A_{draped} sinα) do not have a significant influence on the error. This can be explained due to the limited number of data points for beams with crack width measurements, which inadvertently allows the GPR algorithm to fit the small dataset effectively with a few predictive features. Since GPR predictions are prone to overfitting, all ten features are considered for training the GPR algorithm.

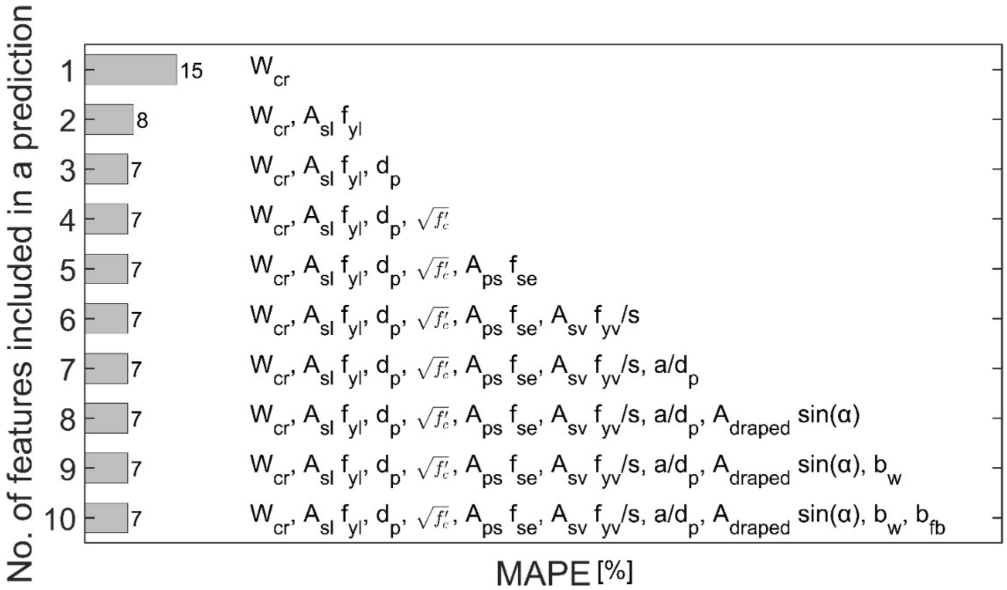


Figure 6. Features with the least normalized shear error using the GPR algorithm for a specific number of features.

<u>*Comparison of ML and design code predictions for shear strength*</u>
The shear strength of the beams in the dataset is also calculated per ACI 318-19 (ACI, 2019) and AASHTO LRFD Bridge Design Specifications (BDS) (AASHTO, 2020) to compare the prediction error of the specifications to the ones obtained from the GPR algorithm. For ACI 318-19, the shear strength is calculated at a distance equal to (a – d) and (h/2) from the nearest support for beams under concentrated and distributed loads, respectively, where a is the shear span, d is the distance of reinforcement from the compression face, and h is the beam height. For AASHTO LRFD BDS 2020, the shear strength is calculated at a distance equal to ($a - d_v$) and (d_v) from the nearest support for beams under concentrated and distributed loads, respectively, where d_v is the effective shear depth as defined by AASHTO LRFD BDS.

The MAPE for the shear strength predictions by ACI 318-19's simplified approach, ACI 318-19's detailed approach, and AASHTO LRFD BDS 2020 are 40%, 27%, and 39%, respectively. As previously shown in Figure 5 MAPE for the GRP algorithm is 15%. Although the GPR algorithm has a smaller error compared to the design codes, the higher error of the design specifications is partially due to their intentional conservatism to serve design purposes.

<u>Task 4 [100% completion]:</u>
Four beams were excluded from the training dataset for additional validation of the GPR algorithm. ML algorithms are used to predict the shear strength and normalized shear for these beams to demonstrate error for beams that were not seen by the algorithms and the use of the ML algorithms. Some of these beams were also modeled using finite element analysis to obtain supplementary information that ML predictions cannot provide (e.g., reinforcement strains).

<u>*ML predictions for unseen beams*</u>
The four beams were selected from the studies of De Wilder, Lava et al. (2015), Hanson and Hulsbos (1971), Lee, Cho et al. (2010), Maruyama and Rizkalla (1988) denoted as G3-1, PS1-0, C80P2S10, and B105, respectively. Of these four beams, two had shear reinforcement less than the minimum required by ACI 318-19. These types of beams

composed a very small part of the training dataset. The other two beams had more than the minimum required shear reinforcement. These types of beams composed a larger portion of the training dataset. Therefore, beams with well or poorly represented features could be identified. Of these two beams with more than the minimum required shear reinforcement, one had features well represented by the training dataset and the other had features poorly represented by the training dataset. A well-represented beam is defined as a beam that has half or more of its relevant features within the 25^{th} and 75^{th} percentiles of the dataset distribution. Values of the features for these beams are marked on Figure 1 and 2.

Shear strength and normalized shear were predicted for these four beams using GPR. The results are summarized in Figure 7. It should be noted that Beam PS1-0 failed due to loss of bond between strands and concrete. Therefore, the measured shear strength of this beam was considered unavailable, and the shear strength was not predicted by ML. Overall, the ML had 3% to 35% error in shear strength predictions. The error was higher for beams with less than the minimum required shear reinforcement, possibly due to the much smaller number of beams in this group.

ML predicted normalized shear corresponding to crack widths with an average error between 2% and 24%. The predictions for three out of four beams are within 11% and follow the measured data. The largest error (24%) is observed for a beam with less than the minimum required shear reinforcement, likely due to the limited number of similar samples in the dataset. For this beam, the ML captured the trend reasonably well but did not capture the magnitudes of normalized shear accurately.

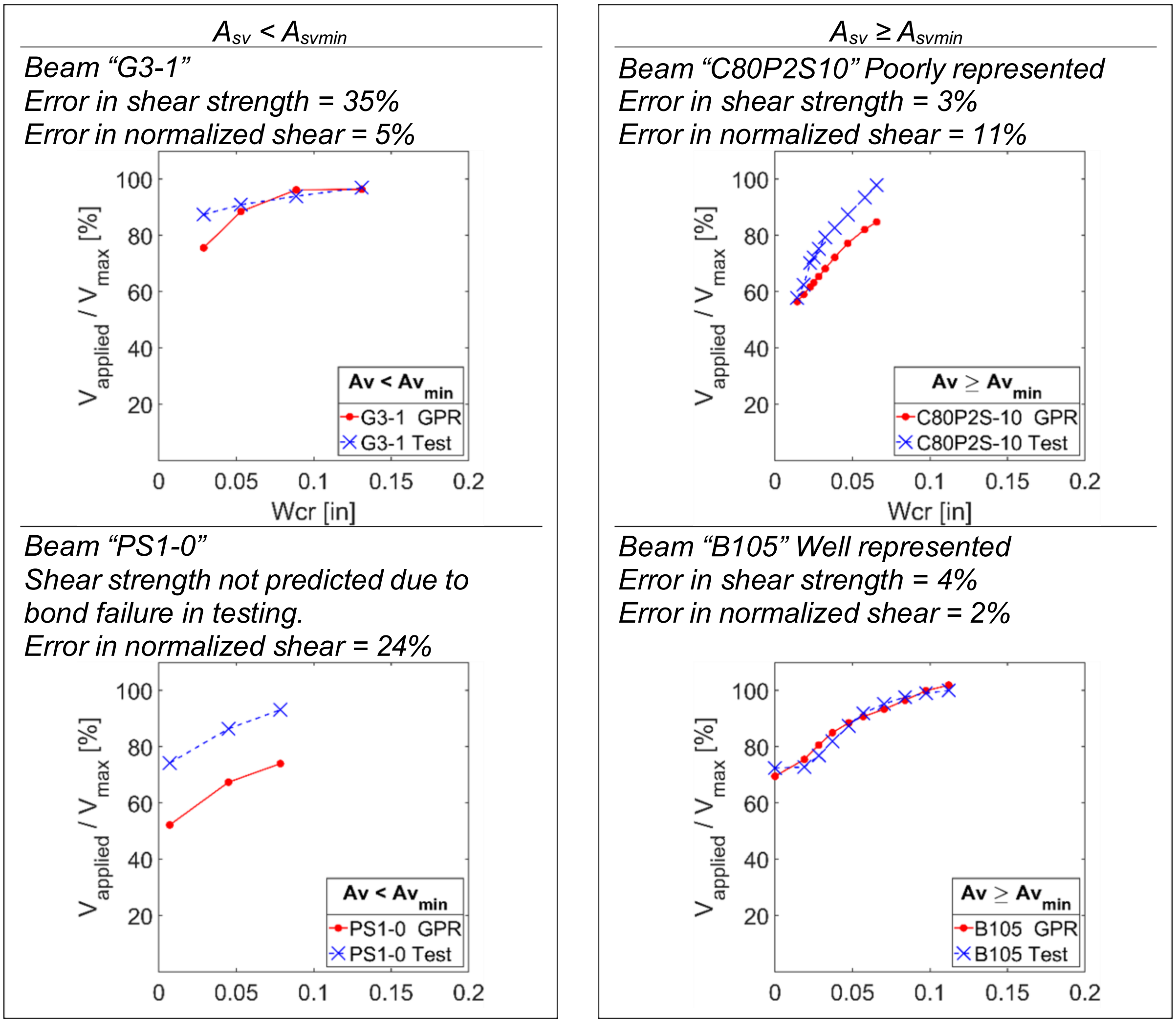


Figure 7. Shear strength and normalized shear error for four sample beams excluded from the training dataset.

<u>Finite element analysis</u>

Once ML algorithms predict the normalized shear for a given crack width, finite element analysis (FEA) can be used to evaluate the condition of the beam (e.g., stresses and strains in steel and concrete) under the corresponding load. To illustrate the concept, beam "B105" was modeled using VecTor2 (Wong, Vecchio et al., 2013). VecTor2 utilizes modified compression field theory for nonlinear FEA of concrete membrane structures.

Figure 8 shows the results of the FEA model for load-displacement response compared with the experimental results. The model is considered validated as the shear strength, the load at the onset of flexural cracking, and the load at the onset of web shear cracking are predicted within 5%, 6%, and 2% of the test results, respectively. In addition, similar crack patterns at the ultimate load from FEA and testing provided additional qualitative validation as also shown in Figure 8.

The validated model is used to obtain additional information on beam condition. Figure 8 shows the load at which the onset of stirrup yielding occurs and the extent of stirrup yielding at the ultimate load. For example, at a crack width of 0.7 mm, the normalized shear predicted using the GPR algorithm for beam "B105" is 80%. This corresponds to

190 kN of shear obtained from FEA. At this shear level, the FEA results show that stirrups do not yield. Therefore, the beam is considered repairable.

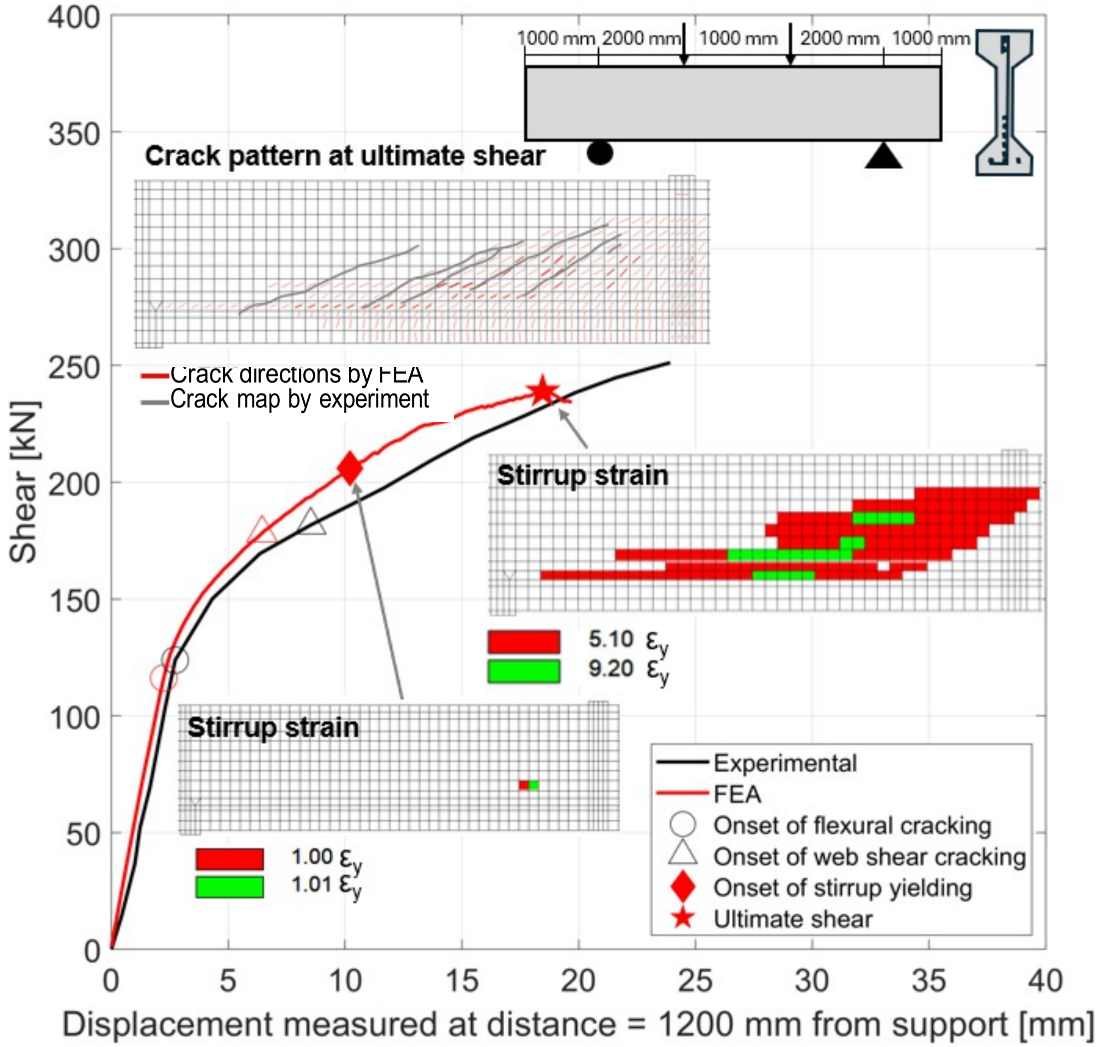


Figure 8. Comparison of crack patterns and shear-displacement relationship from observations and FEA.

Task 5 [100% completion]:

A MATLAB user interface tool that facilitates the use of the ML algorithm in shear capacity predictions has been developed. Considering that MATLAB is a licensed software, a second web application tool has been created using Python, an open-source programming software. This web application provides free access to the tool without the need for a license and enables users to predict shear strength and normalized shear for PC beams. The link to the developed web application is the following: https://hassan-lasheen.onrender.com/*. The research team will migrate this website to a buffalo.edu domain in the coming weeks before disseminating the link.*

Task 6 [100% completion]:

This report documents project goals, methods, and results.

4. Educational Outreach and Workforce Development

- *The research team hosted 30 high school students from across Western New York for Science Exploration Day on March 20, 2024 as shown in Figure 9a. Students were introduced to concrete bridges, potential uses of machine learning in bridge engineering and bridge evaluation during their visit.*

Figure 9. (a) Science Exploration Day, (b) An outreach event for high school students.

- *A presentation was delivered to the advisory board of the Institute of Bridge Engineering on April 30, 2024 virtually. There were 19 attendees at the meeting, which included engineers from transportation agencies, engineers from the industry, as well as faculty at the University at Buffalo.*
- *The project results were presented at the Northeastern Peer Exchange for Resilient and Sustainable Bridges that took place in Buffalo, NY on August 7, 2024. The event was attended by engineers from departments of transportation, industry, consultants and material producers, as well as academics and students.*
- *An outreach event with Women in Science and Technology was scheduled on August 27, demonstrations were prepared to introduce female freshmen level students to bridge*

engineering. Although no students chose to attend this event, the prepared demonstrations were used in other outreach activities.

- *A presentation was delivered September 24, 2024, in the graduate-level course "CIE580: Emerging Technologies in Bridge Engineering" at the Department of Civil, Structural and Environmental Engineering at UB. This course focuses on emerging technologies intended to enhance the analysis, design, construction, performance, and asset management of bridges and highway infrastructure.*
- *An outreach event for high school students was held on November 12, 2024 as shown in Figure 9b. Demonstrations were held to introduce the students to bridge engineering, condition assessment, machine learning, behavior of concrete structures, and shear failure.*
- *An undergraduate student was recruited and worked on the project during summer 2024. She was exposed to concepts related to machine learning, shear behavior of reinforced concrete and PC beams, data analysis, user interfaces for web applications.*

5. Technology Transfer

An online tool has been developed to facilitate technology transfer, as outlined in Task 5.

6. Papers

The following are the papers that acknowledge TRANS-IPIC in the acknowledgments section:

- *Hassan Lasheen, M., Okumus, P., Elhami-Khorasani, N. (2025). "Evaluation of structural cracking in reinforced and prestressed concrete bridges: A review and a machine learning-based framework." Transportation Research Board (TRB) Annual Meeting, January 5-9, Washington, DC.*

- *Two journal papers are under preparation:*
  Hassan Lasheen, M., Okumus, P., Elhami-Khorasani, N., Chandola, V. "Predicting shear strength of prestressed concrete beams using machine learning." In preparation.

  Hassan Lasheen, M., Okumus, P., Elhami-Khorasani, N., Chandola, V. "Data-driven prediction of shear crack-inducing loads for evaluating prestressed concrete beams" In preparation.

7. Presentations and Posters

- *Hassan Lasheen, M., Okumus, P., Elhami Khorasani, N. (2024). "Predicting shear strength of prestressed concrete beams using machine learning", poster presentation, TRB Annual Meeting, presented at the reception by Institute of Bridge Engineering, University at Buffalo, January 8.*
- *Hassan Lasheen, M., Okumus, P., Elhami Khorasani, N. (2024) "Evaluating prestressed concrete beams with cracks using machine learning.", presentation, TRANS-IPIC workshop, Rosemont, IL, April 22.*
- *Okumus, P., Elhami Khorasani, N., Hassan Lasheen, M. (2024) "Evaluating prestressed concrete beams with cracks using machine learning.", presentation, External Advisory Board Meeting of Institute of Bridge Engineering, University at Buffalo, the State University of New York, virtual, April 30.*
- *Okumus, P., Elhami Khorasani, N. (2024), "Bridge engineering research at University at Buffalo.", presentation, New York City DOT and University at Buffalo Meeting, New York City, NY, June 12.*

- *Hassan Lasheen, M., Okumus, P., Elhami Khorasani, N. (2024). "Machine learning for evaluating in-service concrete bridges.", presentation, Northeastern Peer Exchange Resilient and Sustainable Bridges, Buffalo, NY, August 7.*
- *Hassan Lasheen, M., Okumus, P., Elhami Khorasani, N. (2024) "Evaluating prestressed concrete beams with cracks using machine learning.", presentation, TRANS-IPIC monthly webinar, August 22.*

8. Other Events

Please see the list above in Sections 5 and 6.

9. References

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