PHYS 212 Final Review

Final Exam

Queue

Exam 1 Overview

1) Coulomb's Law

2) Electric Field

3) Electric Flux

4) Gauss's Law

5) Electric Potential

6) Capacitance

Exam 2 Overview

9/10) Simple Circuits and Kirchhoff's Laws

11) RC Circuits

12) Magnetic Force

13) Forces and Magnetic Dipoles

14) Biot-Savart Law

15) Ampere’s Law

16) Motional EMF

Exam 3 Overview

17) Faraday's Law

18) RL Circuits

19) LC Circuits

20) AC Circuits

21) AC Power and Resonance

22) Maxwell's Displacement Current

23) EM Waves

24) Polarization

25) Reflection and Refraction

Final Exam Overview

26) Lenses

27) Mirrors

1-25) Everything else

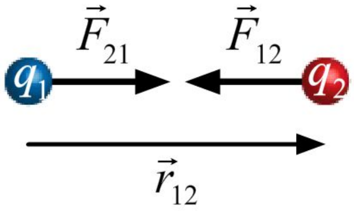
Coulomb's Law

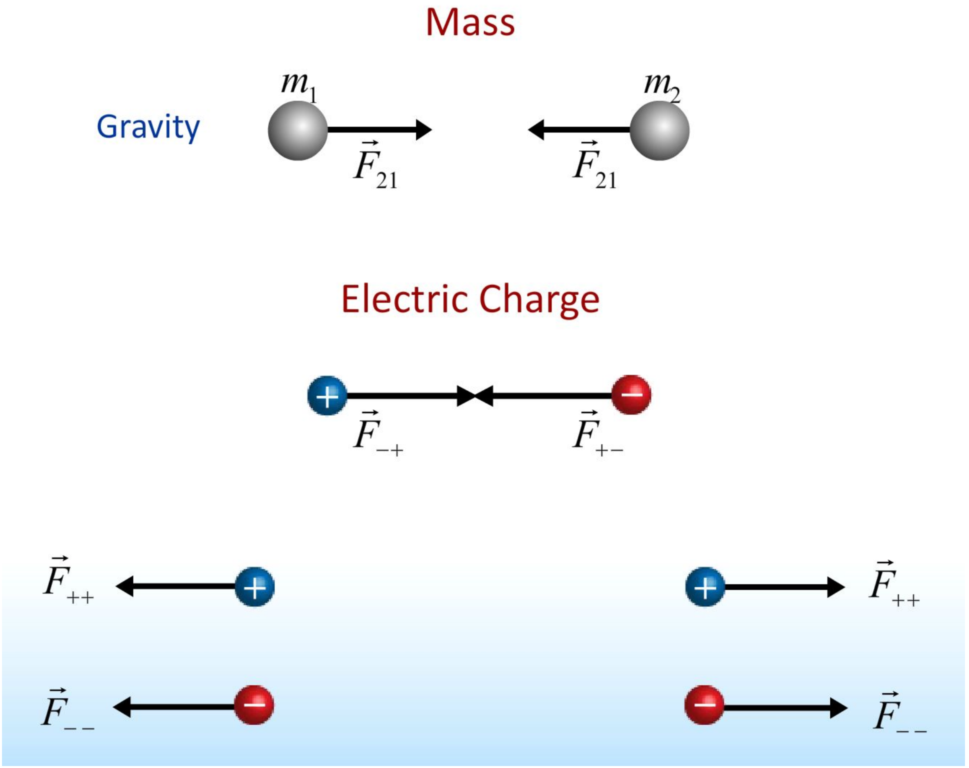
Electrostatic force between 2 charges

Newton's Third Law: $F_1 = - F_2$

Coulomb's Law (1785)

$$\vec{F}_{12} = k \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$





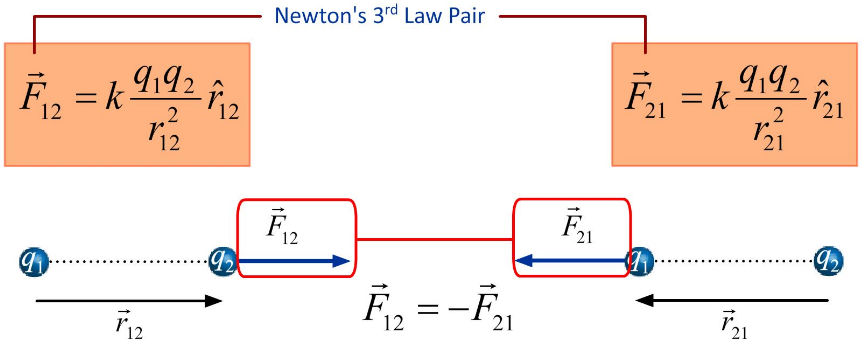
Superposition

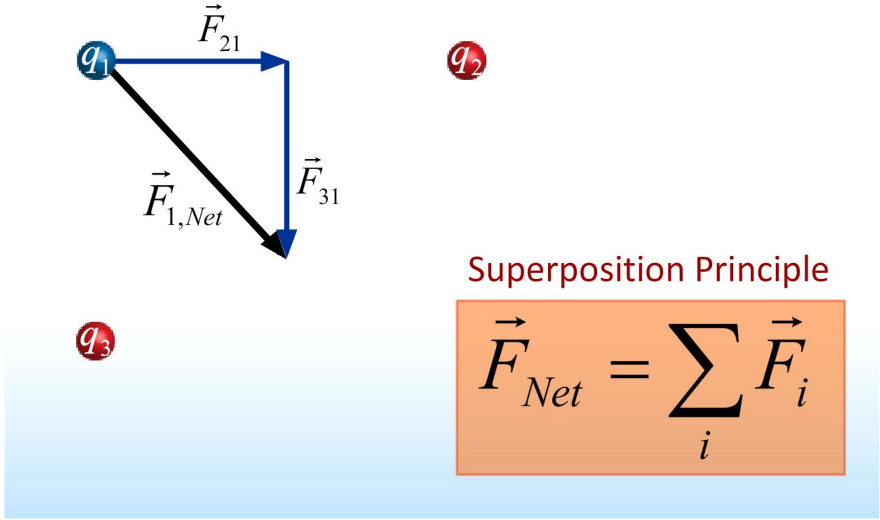
The total electric force on a charge

is the **sum of all the forces** exerted

by "n" charges on that one charge

$$\vec{F}_{1,Net} = \vec{F}_{21} + \vec{F}_{31}$$

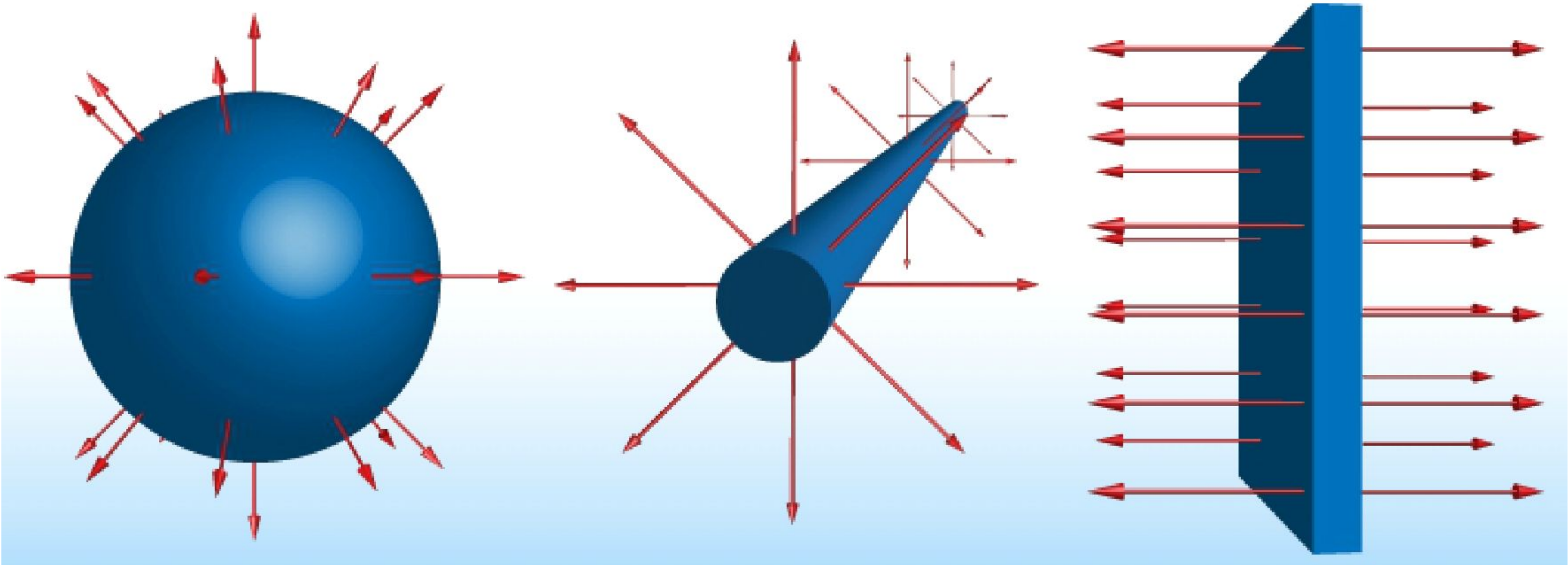




Electric Fields

3 main sources of electric fields:

Point Charges, Infinite Lines of Charge, and Infinite Sheets of Charge



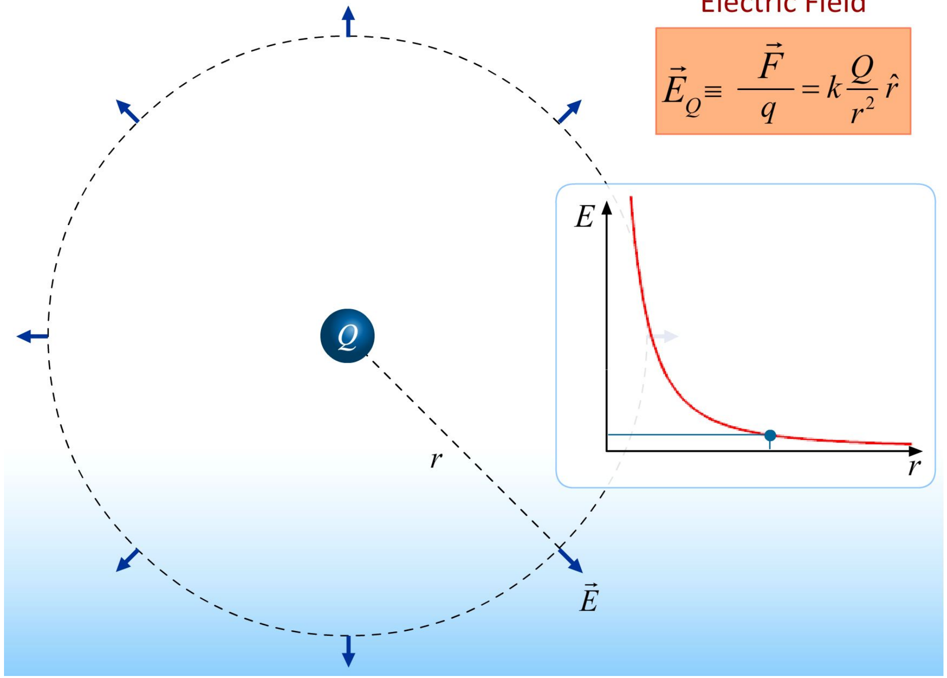
Point Charge

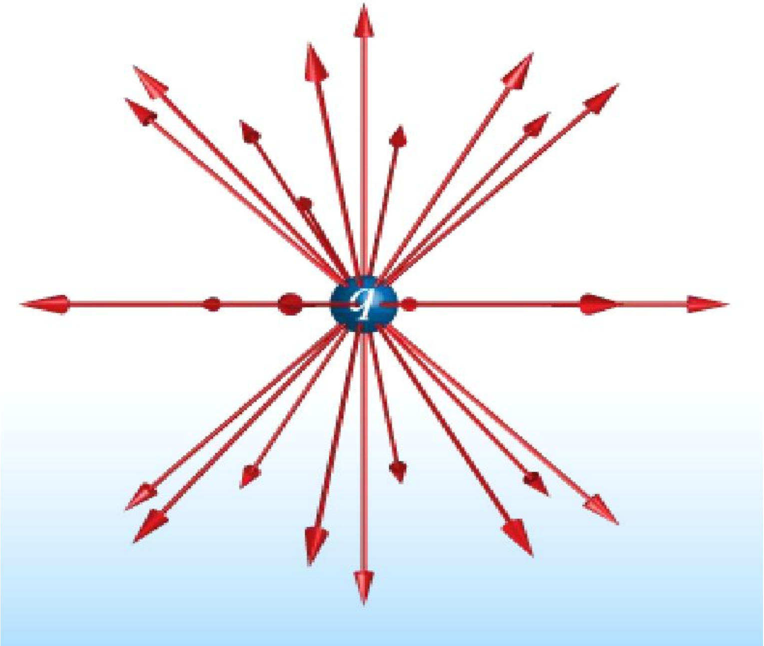
3D symmetry - **magnitude depends on r^2**

$$E = k\frac{q}{r^2}$$

Electric Field

$$\vec{E}_Q \equiv \frac{\vec{F}}{q} = k\frac{Q}{r^2}\hat{r}$$





Infinite Line of Charge

$$E = 2k\frac{\lambda}{r}$$

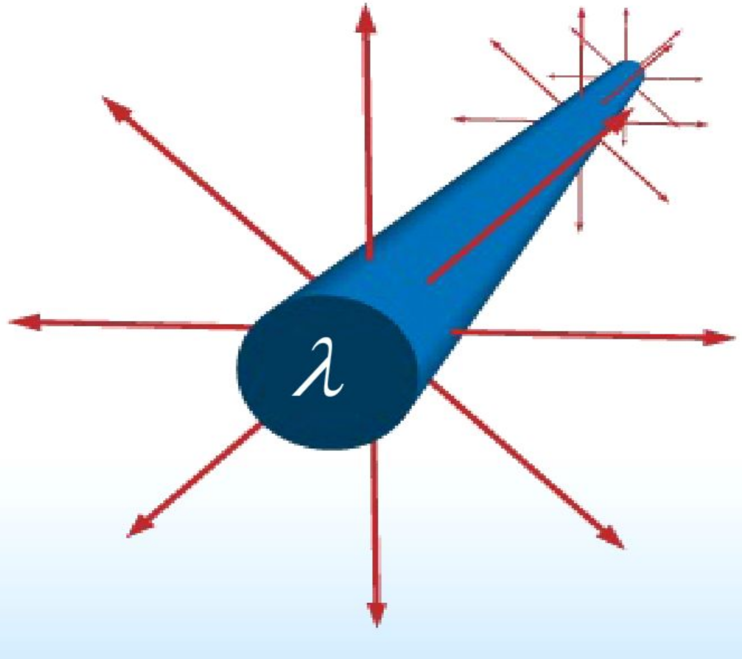
2D symmetry - **magnitude depends on r**

charge density - **λ = Q/L** (units: C/m)

Integral Setup Questions:

- Bounds are the **length** of the line of charge
- Inside the integral is of form **k(q/r^2)**
- **dQ = λdx**

$$E_y = \int_{x=-\infty}^{x=\infty} dE_y \qquad E_y = \int_{x=-\infty}^{x=\infty} k\frac{dq}{s^2}\cos\theta = \int_{x=-\infty}^{x=\infty} k\frac{\lambda\, dx}{s^2}\cos\theta$$

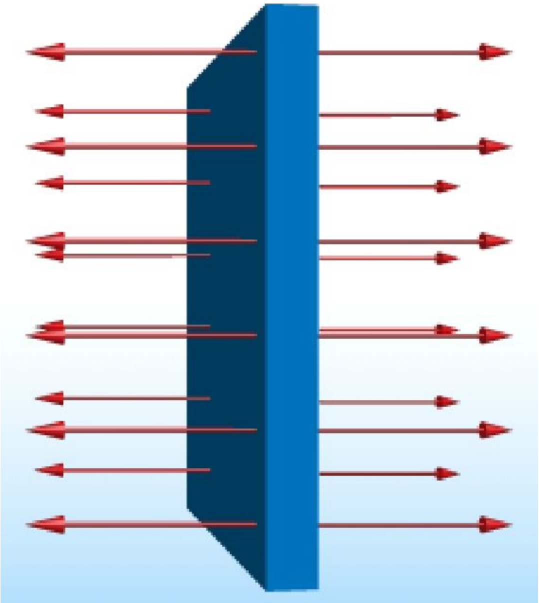


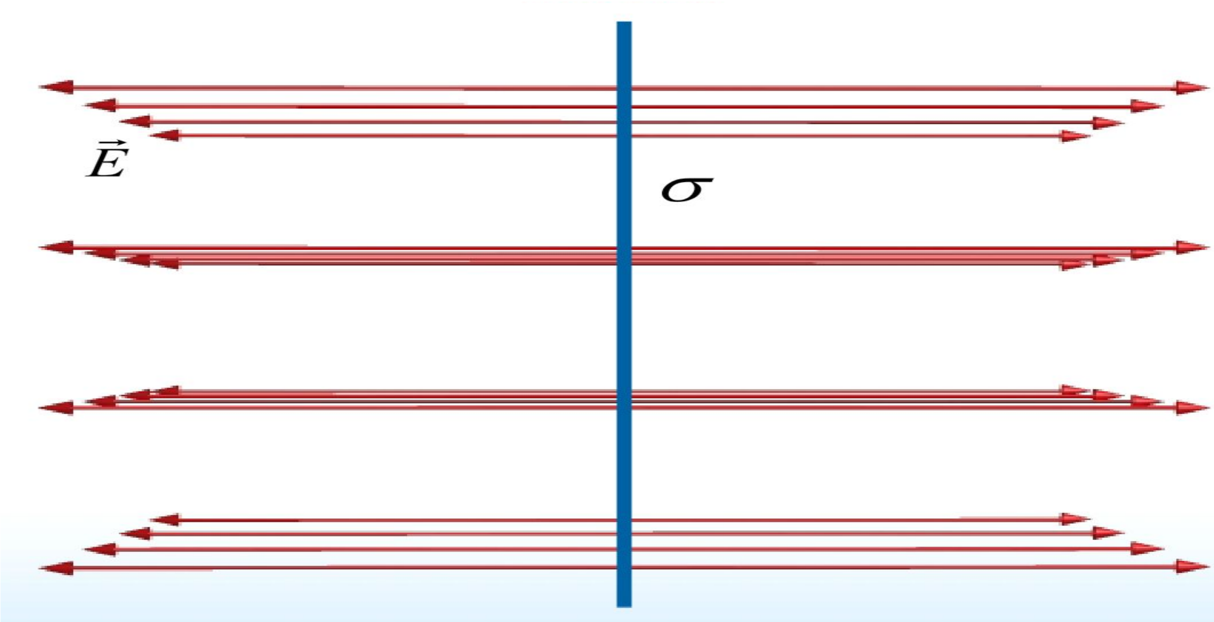
Infinite Sheet of Charge

1D symmetry - **magnitude has no dependance on r**

charge density - **σ = Q/A** (units: C/m^2)

$$E = \frac{\sigma}{2\varepsilon_o}$$





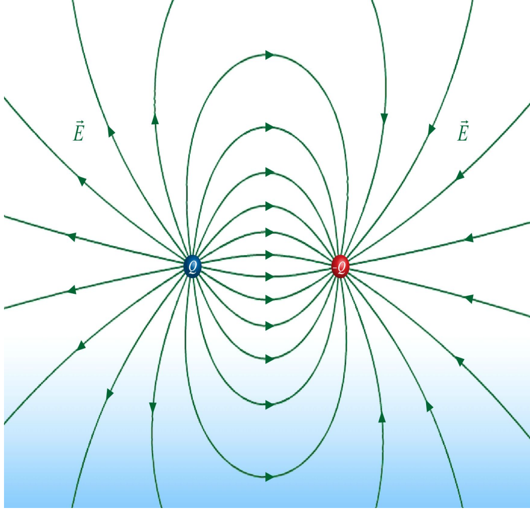
Electric Field Lines and Flux

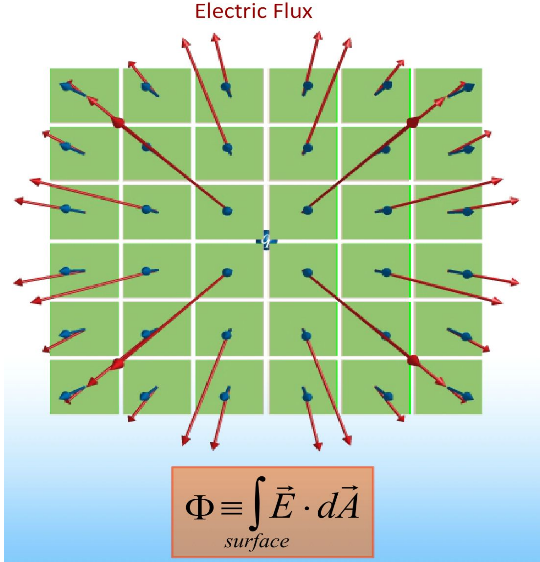
Density of field lines indicates electric field strength

- More dense lines => stronger electric field
- Less dense lines => weaker electric field
- **# of field lines is proportional to charge's magnitude**

Flux is the number of field lines that pass through a surface

- Positive flux points outwards
- Negative flux points inwards
- **Pay close attention** to Φ_{net} vs Φ_{left} or Φ_{right}





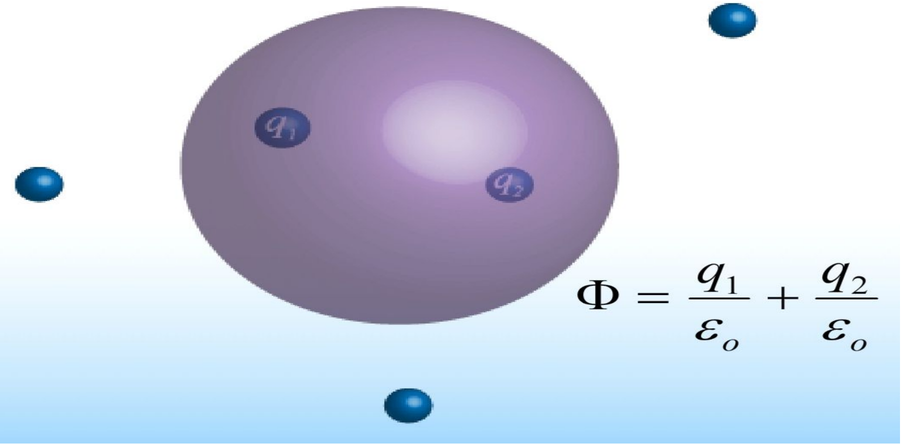
Gauss's Law

$$\Phi_{Net} = \oint_{surface} \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

3 shapes have enough symmetry for easy

integration, so that we can get **E · A = Q**$_{\mathbf{enc}}$

- Sphere (Point Charge)
- Cylinder (Infinite Line of Charge)
- Plane (Infinite Sheet of Charge)



Generally, a cylinder will be used but any symmetrical object would suffice (cube, sphere, etc.)

Gauss's Law says the number of field lines out of a surface is directly related to the charge(s) enclosed

Gauss's Law cont.

- A is the surface area of the **chosen Gaussian surface** (sphere, cylinder, cube, etc.)
- Charge denstitions (λ, σ, ρ) come from the **given physical object** we are working with

- We can use charge densities to find q_{enc}
  - $\boldsymbol{\lambda = q_{enc} / L}$ (L is length - m)
  - $\boldsymbol{\sigma = q_{enc} / A}$ (A is area - m^2)
  - $\boldsymbol{\rho = q_{enc} / V}$ (V is volume - m^3)

$$\Phi_{Net} = \oint_{surface} \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

Conductors

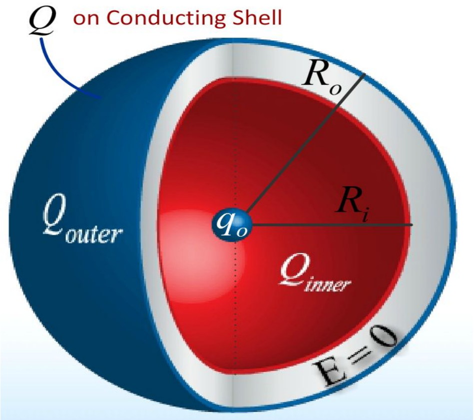
Electric field inside a conductor is **ALWAYS 0,** since all the charge goes the surface

For charges inside a conducting shell:

- **Q_{inner} = opposite value of the center charge**
- **Q_{outer} = value of the charge on the surface + value of the center charge**

$$Q_{inner} = -q_o$$

$$Q_{outer} = Q + q_o$$



Insulators

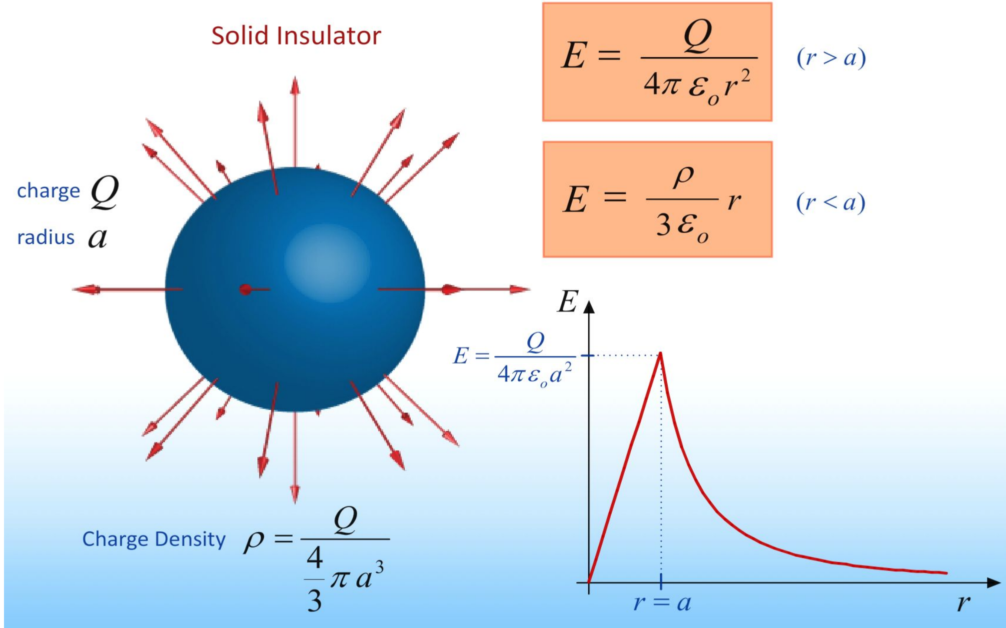
Charge is **uniformly (equally) distributed** throughout the entire insulator

The net charge inside an insulator behaves differently than outside the insulator

Outside - behaves like a point charge

Inside - behaves linearly

- **Memorize second equation**
- **Saves you time from deriving it**



Electric Potential Energy (Units: J)

Solving Systems of Particle Problems

1. $U_1 = 0$, for whatever particle you chose first
2. $U_2 = kq_2q_1 / (d_{21})$
3. $U_3 = kq_3q_1 / (d_{31}) + kq_3q_2 / (d_{32})$
4. Repeat process for all additional charge pairs and sum them up ($U_1 + U_2 + U_3 + \dots U_n$) to get U_{sys}
5. **Remember that W = - U**

$$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

Electric Potential (Voltage - Units: V=J/C)

Energy required to move a positive test charge through a constant electric field

- **$V_{\text{point charge}}$ = U / q (where little q is the test charge)**

Electric Potential Difference

$$\Delta V_{A \to B} = -\int_{A}^{B} \vec{E} \cdot d\vec{l}$$

Equipotential Lines:

- Perpendicular to electric field lines
- Electric field lines always point from higher to lower electric potential
- **More dense lines => Stronger electric potential**
- **Equal electric potential along on the same equipotential lines**

Capacitance (Units: Farads - F)

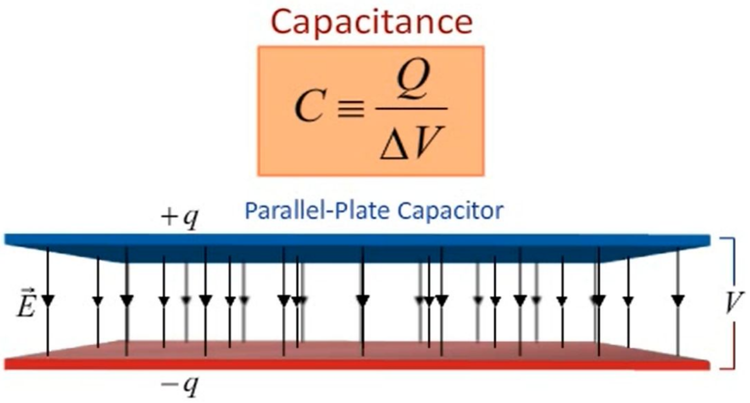
Capacitance primarily depends on the **geometry**

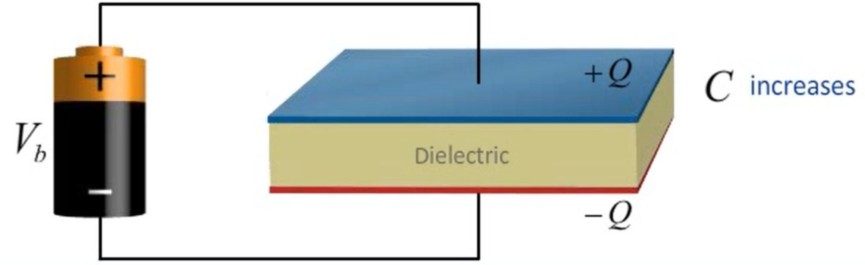
$$C = \frac{\kappa \epsilon_0 A}{d}$$

Units - Farads (F)

Energy of a capacitor: U = 0.5CV²

Dielectric - adding a dielectric to a capacitor increases its capacitance





Capacitors in Series/Parallel

Series - $1/C_1 + 1/C_2 + 1/C_3 + \ldots 1/C_n = 1/C_{total}$

***Shortcut (Product over Sum)**: only works with **2 capacitors at a time**, repeat process for all capacitors until C_{total}

$(C_1 \times C_2) / (C_1 + C_2) = C_{1,2}$ ==> **Multiply C_1 and C_2 (product) and divide by their sum**

Parallel - just add them up

$C_1 + C_2 + C_3 + \ldots C_n = C_{total}$

Current and KCL

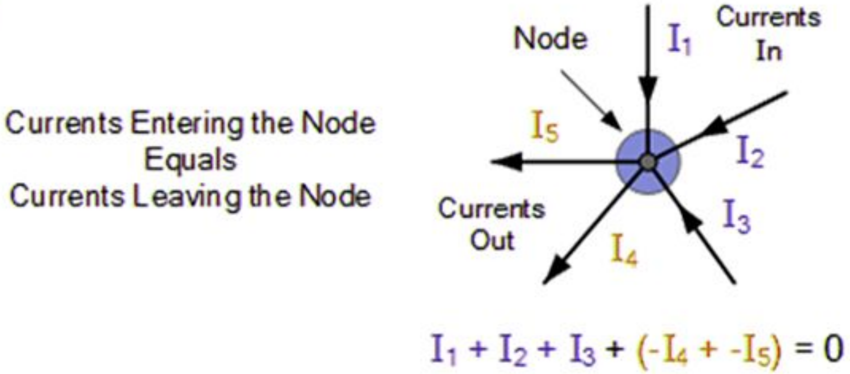
Current (I) is the flow of charge per second

Units: Amperes (A) - Coulombs/second (C/s)

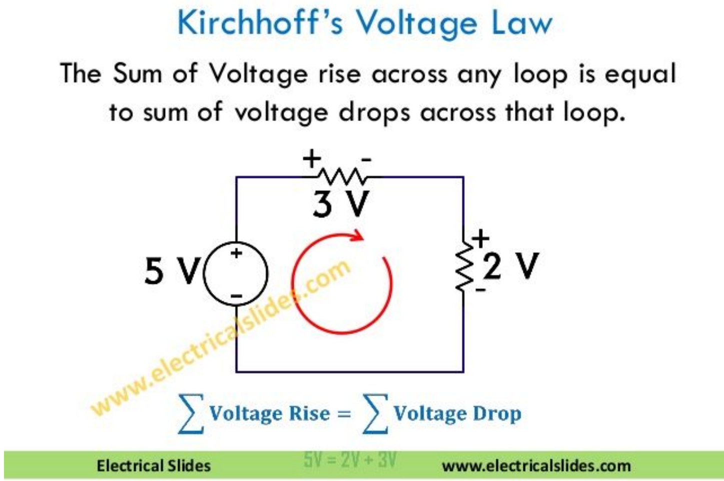
Kirchhoff's Current Law - KCL

- **The amount of current going in is equal to the amount of current coming out**

$$I_{in} = I_{out}$$



Voltage and KVL



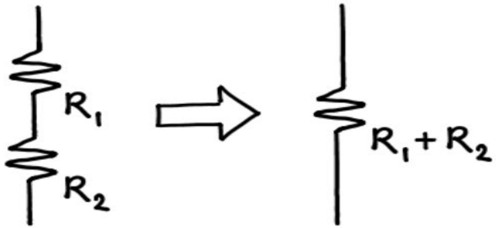
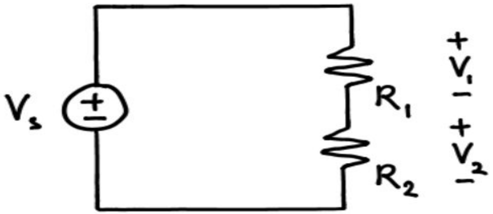
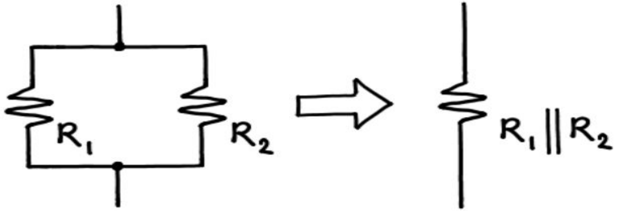
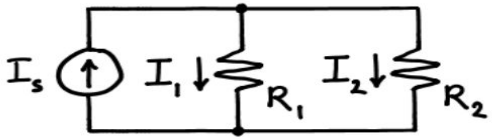
Voltage (V) is the amount of energy per unit charge

- Units: Volts (V) = Joules/Coulomb (J/C)

Kirchhoff's Voltage Law - KVL

- **The total voltage in a loop is the sum of all the voltage drops and rises**
  - Voltage drop - "+" to "-"
  - Voltage rise - "-" to "+"

You can solve all the circuit problems you will see in this course by applying KCL and KVL

Name	Diagram	Formulas
Series Resistors		$\text{Equivalent resistance} = R_1 + R_2$
Voltage Divider		$V_1 = \dfrac{R_1}{R_1 + R_2} V_s \qquad V_2 = \dfrac{R_2}{R_1 + R_2} V_s$
Parallel Resistors		$\text{Equivalent resistance} = R_1 \| R_2 = \dfrac{R_1 R_2}{R_1 + R_2}$
Current Divider		$I_1 = \dfrac{R_2}{R_1 + R_2} I_s \qquad I_2 = \dfrac{R_1}{R_1 + R_2} I_s$

Power

Power is the amount of energy per second being delivered/absorbed

- Units: Watts (W) = Joules/second (J / s) ==> amount of energy per second
- **$P_{resistor}$ = IV = V^2/R = I^2R (These last 2 equations are for resistors ONLY)**

The sign ("+" or "-") is very important when it comes to power **(Not on your test)**

- Negative power means that circuit element is delivering energy to the circuit (sources, capacitors, inductors)
- Positive power means that the circuit element is absorbing energy from the circuit (resistors, capacitors, inductors)

RC Circuits

Time Constant

$$\tau = RC$$

τ - tau is the time constant which affects the rate of growth/decay

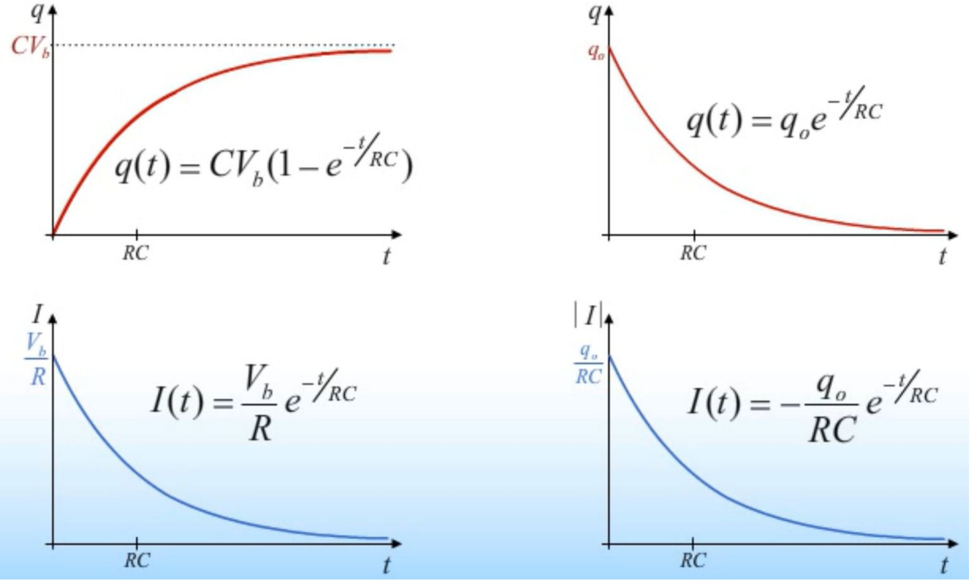
Charging and Discharging Equations

$$Q(t) = Q(\infty)\left(1 - e^{-t/\tau}\right)$$

$$Q(t) = Q(0)e^{-t/\tau}$$

Charging

Discharging



RC Circuits cont.

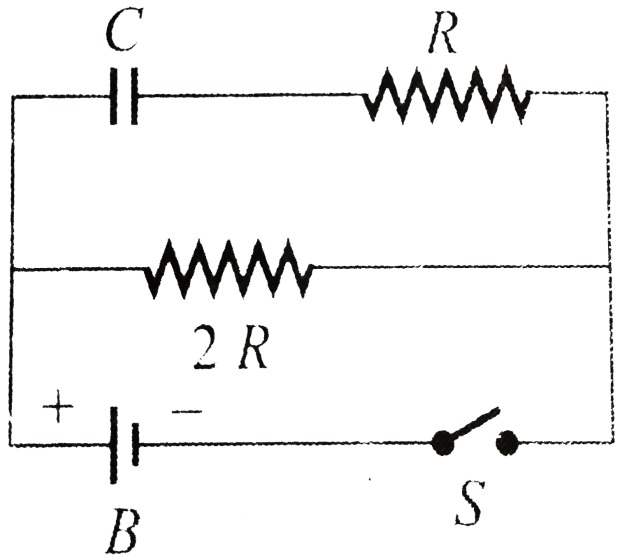
Charging

t = 0 → capacitor acts like a wire (short circuit)

- V = 0 V, but there is a current

t = ∞ → no current thru capacitor (open circuit)

- I = 0 A, but there is a voltage

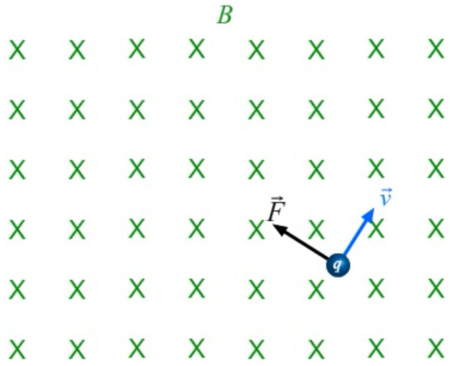


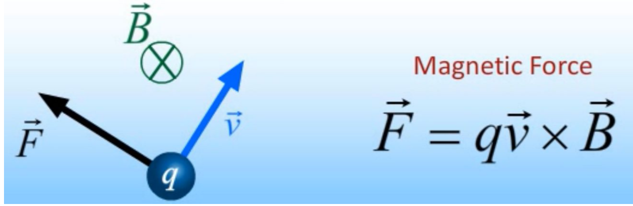
Discharging

t = 0 → capacitor acts like a battery (C = Q/V where V is found when charging up)

t = ∞ → capacitor acts like a wire (all the charge is dissipated aka gone)

Magnetic Force on Charges





- F_m = qv X B
  - we know that **F = ma**
  - and for these problems **$a = a_c = v^2/r$**
  - If we substitute in for F we get **mv^2/r = qv X B**
  - We use this to solve for any missing variable

Right-Hand Rule (1st RHR)

- Point fingers or hand along the direction of v
- Curl fingers in the direction of B
- Thumb points in the direction of the force*

***This works for positive charges, flip your thumb 180° for a negative charge**

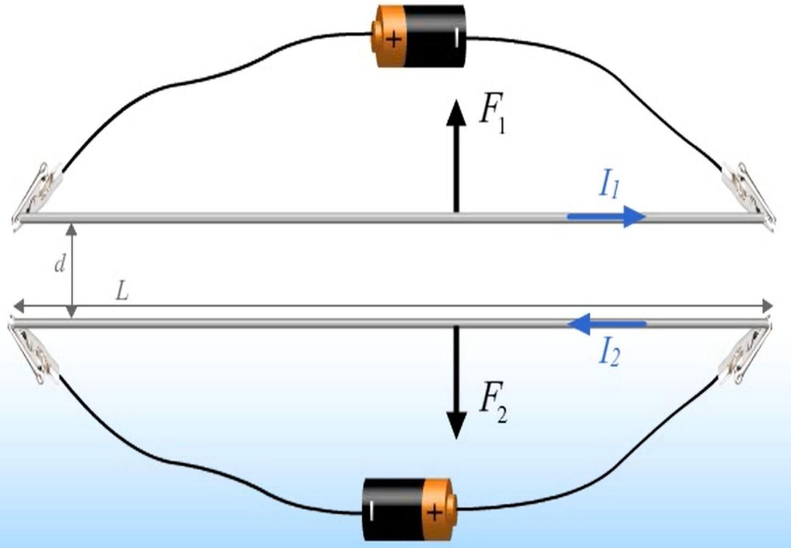
Forces on Current Wires and Loops

$F_{wire} = I\,L \times B$ (1st RHR)

- The force around an entire loop of current is always zero (assuming B is constant) but be careful because it may not be zero at a segment of the loop

Currents traveling in the same direction - attract

Currents traveling in opposite directions - repel



Torques and Energy on Current Loops

Remember sin(θ) goes with cross products and cos(θ) goes with dot products

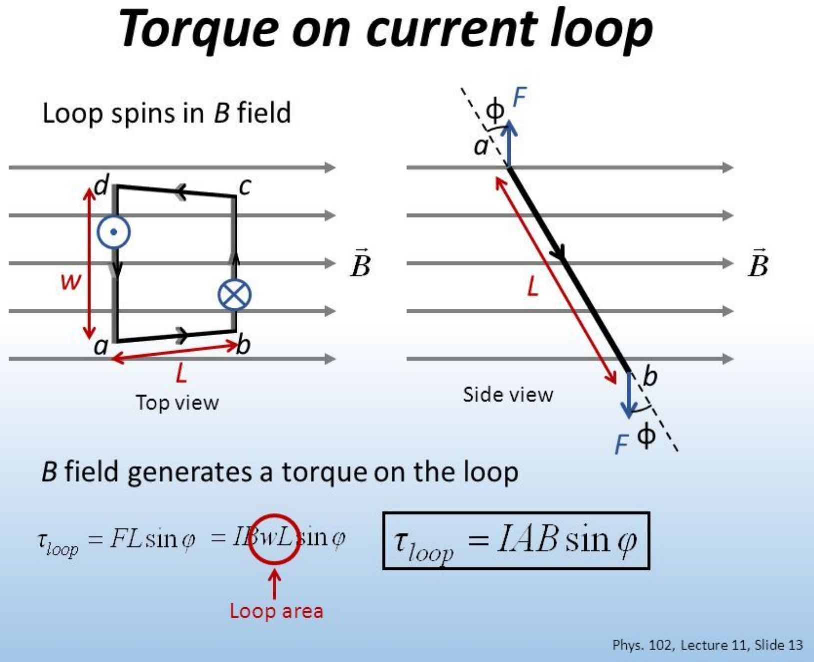
Magnetic Dipole: **µ = n * I * A (2nd RHR)**

- **n = # of turns**
- **I = current through loop**
- **A = area of the loop**

Torque: **τ = µ x B = |µ||B|sin(θ) (1st RHR)**

Potential Energy: **U = µ · B = |µ||B|cos(θ)**

Work: **W = -U**



Torques and Energy Cont.

Remember sin(θ) goes with cross products and cos(θ) goes with dot products

Torque: τ = **μ x B** = **|μ||B|sin(θ)**

Max when sin(θ) = 1 → θ = 90 → when μ and B are perpendicular

Potential Energy: **U = μ · B = |μ||B|cos(θ)**

Max when cos(θ) = 1 → θ = 0° → when μ and B are parallel in the same direction

Min when cos(θ) = -1 → θ = 180° → μ and B are parallel in opposite directions

Work: **W = -U**

Biot-Savart Law

By using the Biot-Savart Law, we were able to derive the equation for the **magnetic field produced by a current carrying wire (in orange)**

Direction of B is always tangent to the circle (3rd RHR)

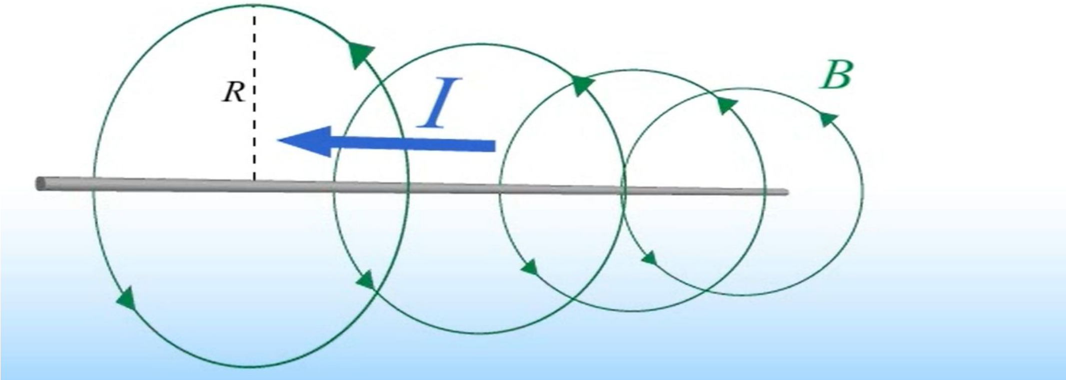
(Not used often, painful to integrate)

$$B = \frac{\mu_o I}{2\pi R}$$

Right Hand Rule

1. Place thumb in direction of I
2. Fingers curl in direction of B

$$B_z = \frac{\mu_0 I R^2}{2(z^2 + R^2)^{3/2}}$$



Right-Hand Rules (3 Total)

1st RHR - Cross Products

- Place your fingers along the first vector, curl your fingers in the direction of the second vector, your thumb gives you the direction of the force, torque, etc.

2nd RHR - Magnetic Dipole

- Curl your fingers along the direction in which the current is flowing, your thumb gives you the direction of the magnetic dipole

3rd RHR - Magnetic Fields

- Place your thumb along the direction of current, curl your fingers to give you the direction of the “circular path”, B is tangent to the “circular path”

Ampere's Law

Think of it as the 2D version of Gauss's Law, but for magnetic fields now

By convention for line integrals, **traversing a closed loop counter-clockwise (CCW) is positive and traversing it clockwise (CW) is negative**

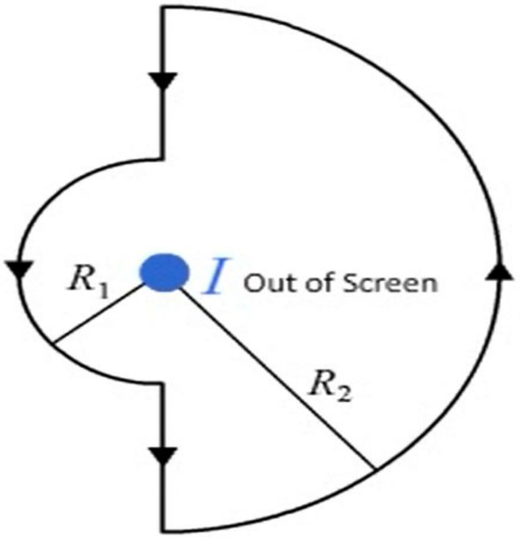
Current density: J = I / A

Units: (A/m^2)

I - Current

A - Area

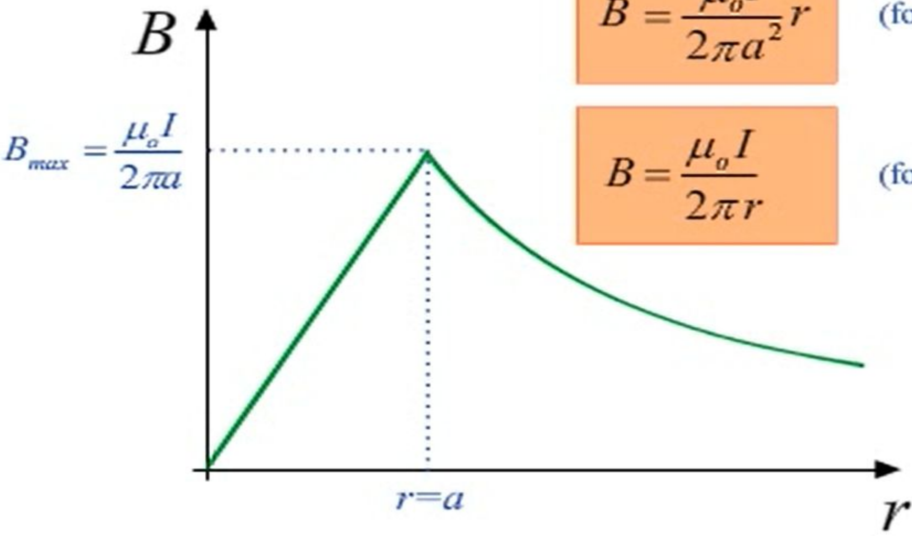
$$\oint_{loop} \vec{B} \cdot d\vec{l} = \mu_o I_{enclosed}$$



Ampere's Law Cont.

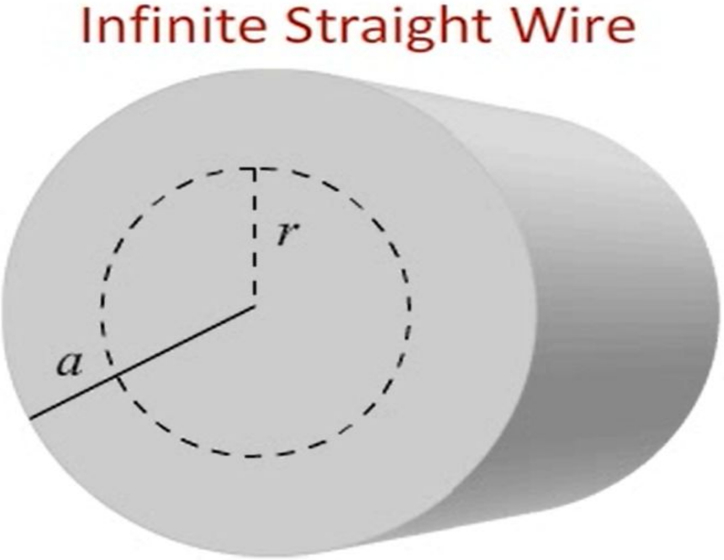
Magnetic field equations inside and outside a current-carrying wire

Memorize inside equation (#1), it will save you time from deriving it on the exam



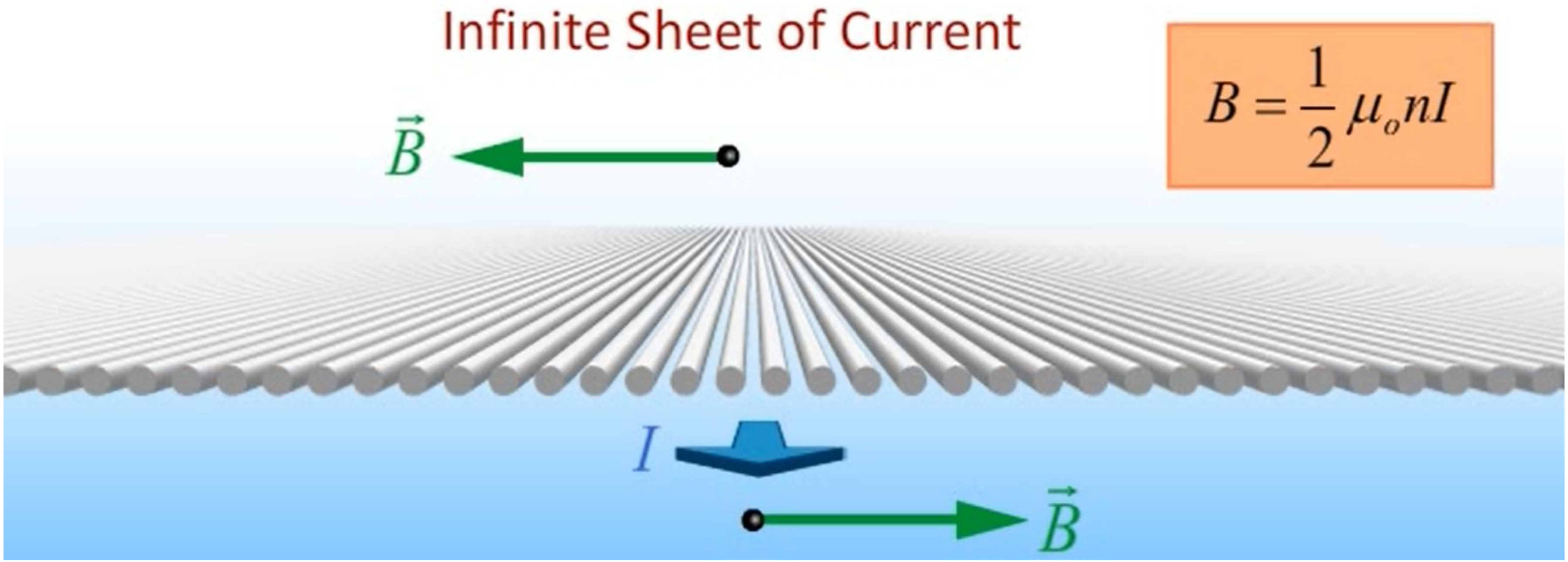
$$B = \frac{\mu_o I}{2\pi a^2} r \quad \text{(for } r < a\text{)}$$

$$B = \frac{\mu_o I}{2\pi r} \quad \text{(for } r > a\text{)}$$



Ampere's Law Cont.

Magnetic field equation for an infinite sheet of current



Motional EMF

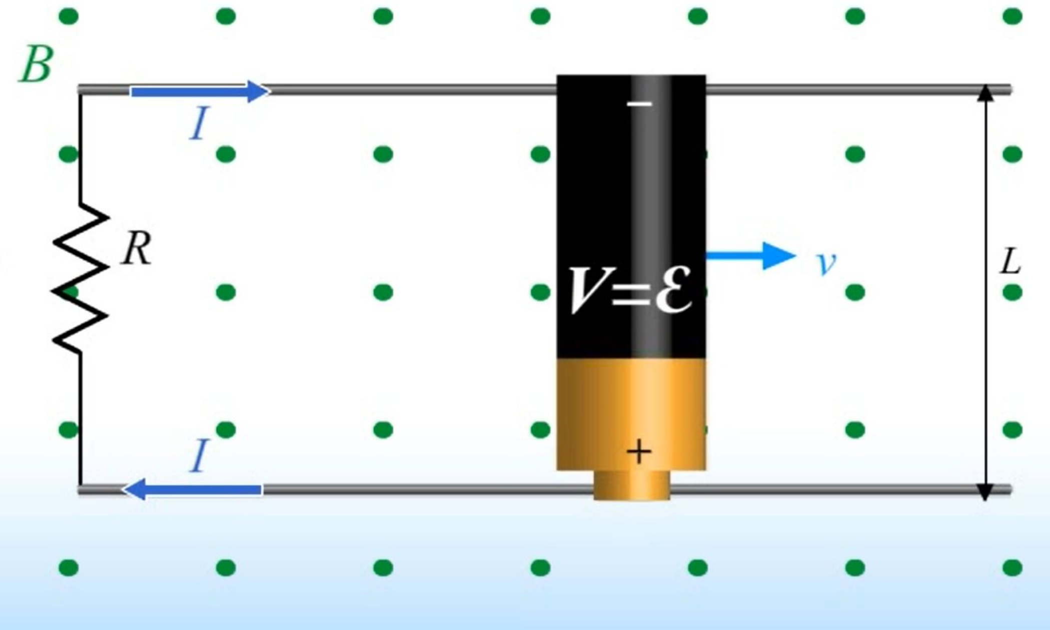
Potential difference = Voltage = Electromagnetic Force (EMF)

$$\mathcal{E} = vBL$$

v - velocity

B - magnetic field

L - length of the loop



To find direction of current: 1st RHR

- RHR wrt the magnet: **F = qv x B**
- **Your thumb gives you the direction of the current**

Faraday's Law

$$\mathcal{E}_{induced} = -\frac{d\Phi_B}{dt}$$

Main Idea: A changing magnetic flux creates an electric field

The induced EMF (voltage) always opposes the change in magnetic flux

The induced EMF gets **multiplied by N turns** if the loop has N turns in it

3 ways to change the magnetic flux

- **Making the area of the loop smaller or larger**
- **Moving the loop around in a constant magnetic field**
- **Having a time-varying magnetic field (i.e. B is not constant with time)**

Faraday's Law cont.

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\mathcal{E}_{induced} = -\frac{d\Phi_B}{dt}$$

Steps for solving Faraday's Law problems (2 types)

Type 1: (Usually given B as a function of time or on a graph)

1) Find the magnetic flux **(B · A)**
2) Solve for the induced EMF by take the negative derivative of the magnetic flux with respect to time **(-d/dt of the magnetic flux)**

Type 2: (Usually a picture with one or "N" conducting loops)

1) **Determine the change in magnetic flux, $B_{induced}$ will always point in the opposite direction** to the change in magnetic flux
2) **Use the 3rd RHR:** Point your fingers in the direction of $B_{induced}$ and curl your fingers to give you the direction of the induced current

RL Circuits

Inductors behave "oppositely" to capacitors (i.e. at t=0 and t=∞ when charging up)

Inductors in circuits add in series and in parallel like resistors

$$L \equiv \frac{\Phi_B}{I}$$

Inductance: L = magnetic flux / current

Time constant: τ = **L / R**

$$\tau = \frac{L}{R} \qquad V = L\frac{dI}{dt}$$

Charging and Discharging Equations

$$I(t) = I(\infty)\left(1 - e^{-t/\tau}\right) \qquad I(t) = I(0)e^{-t/\tau}$$

RL Circuits cont.

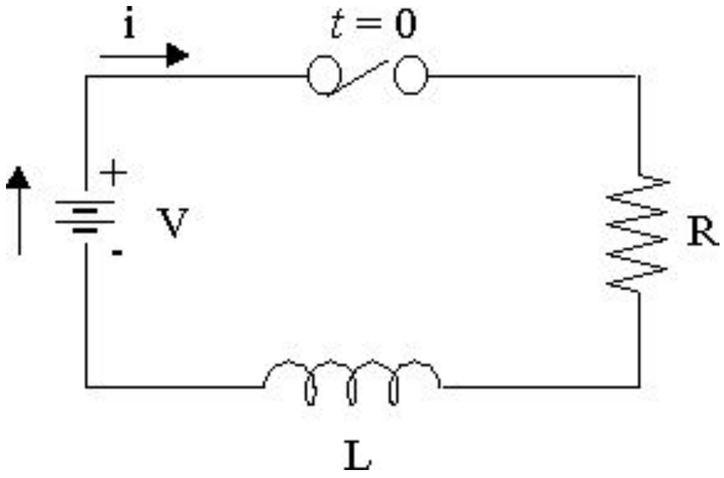
Charging

t = 0 → inductor acts like an open circuit

- I = 0 A, but there is a voltage

t = ∞ → inductor acts like a wire (short circuit)

- V = 0 V, but there is a current



Discharging

t = 0 → inductor acts like a current source (I at t = 0 is the same as I at t = ∞ found when charging up)

t = ∞ → inductor acts like a wire (no more current in the circuit)

LC Circuits

Inductors and capacitors are storage devices so their energies are constantly oscillating between one another (given an initial voltage/current)

Total Potential Energy: $U_{total} = U_{inductor} + U_{capacitor} = 0.5LI^2 + 0.5CV^2$

Resonance only occurs at the natural frequency: ω_0

Natural Frequency

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$U = \frac{1}{2}LI^2 \qquad U = \frac{1}{2}CV^2$$

AC Circuits (RLC)

Resistor is in phase with the current

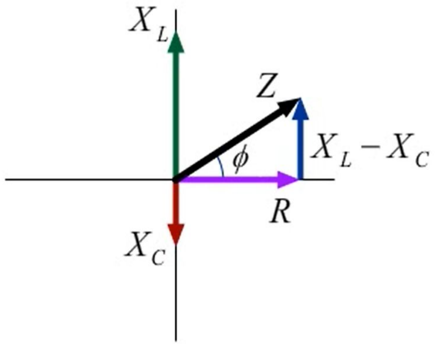
Inductor leads current by 90 degrees

Capacitor lags current by 90 degrees

Steps for AC Circuit Problems:

1) Find the reactances first (X_L and X_C)
2) Then find impedance (Z)
3) Now you can solve for I_m
4) Solve for phase of the generator
    a) **If phase is positive → generator voltage leads current**
    b) **If phase is negative → generator voltage lags current**

Impedance Phasor Diagram



$$\tan\phi = \frac{X_L - X_C}{R}$$ Phase

$$I_m = \frac{\mathcal{E}_m}{Z}$$ Maximum Current

Inductor Reactance $X_L = \omega L$

Capacitor Reactance $X_C = \dfrac{1}{\omega C}$

Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Average Power and Resonance

Resonance occurs when $\boldsymbol{\omega = \omega_o}$

This makes $X_L = X_C$ thus $Z = R$ => this is when $\mathbf{I_m}$ **is at its maximum value**

$$\langle P_{Generator} \rangle = \mathcal{E}_{rms} I_{rms} \cos\phi$$

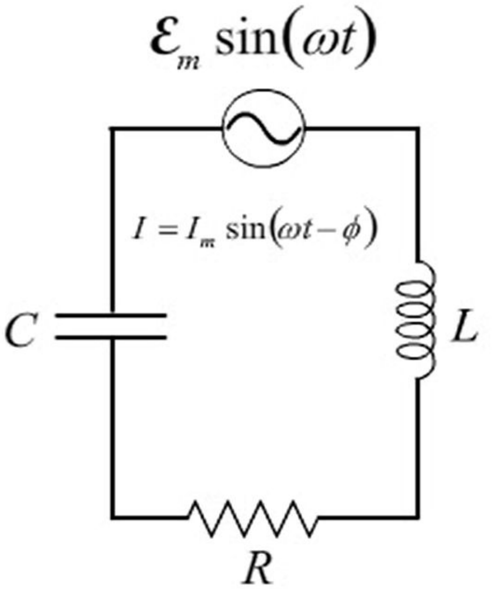
Root Mean Square (rms)

$$\mathcal{E}_{rms} = \frac{\mathcal{E}_m}{\sqrt{2}}$$ Voltage

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$ Current

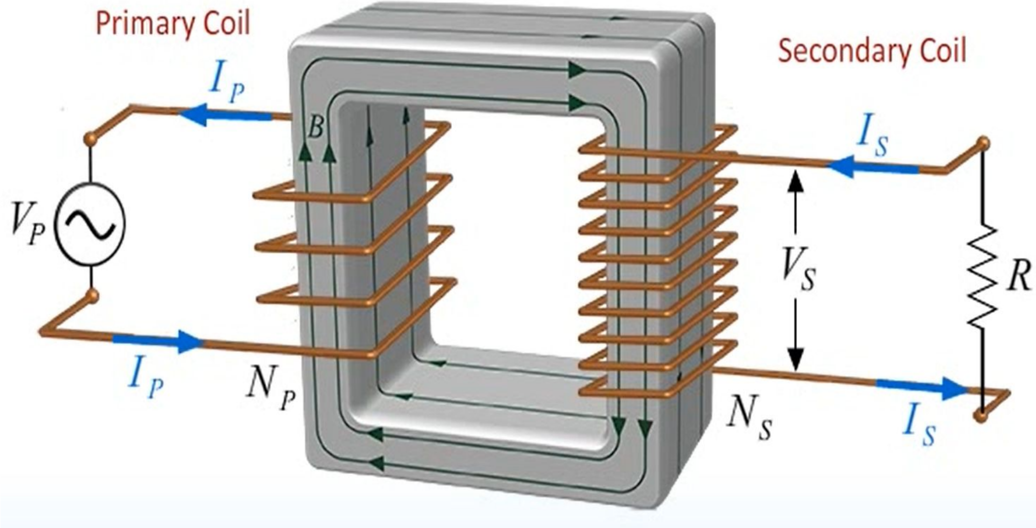
Natural Frequency

$$\omega_o = \frac{1}{\sqrt{LC}}$$



Transformers

Transformers are used to convert from high voltages to low voltages and vice versa



Voltage Relation

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

Current Relation

$$\frac{I_P}{I_S} = \frac{N_S}{N_P}$$

EM Wave Properties

$$E_x = E_o \cos(kz - \omega t)$$

E and B have the same waveform: If E is sin(kz-ωt) then B is also sin(kz-ωt)

Magnitude of B is smaller: $B_0 = E_0$ / c where c is the speed of light (3 x 10^8 m/s)

The “x, y, or z” variable inside the argument tells you the direction of propagation
cos(kz - ωt) travels in +z-direction, cos(kz + ωt) travels in -z-direction

Wave parameters: $\omega = 2\pi f$, $v = \lambda f = \omega / k$ (v = c for EM waves in free-space)

Poynting vector (S) points in the same direction the wave is traveling

$S = (E \times B) / \mu_0$

Power = S x A (units: W) , **Intensity = Power / Area = S** (units: W/m^2)

Doppler Shift

$$f' = f\sqrt{\frac{1 \pm \beta}{1 \mp \beta}} \xrightarrow{\beta \ll 1} f' \approx f(1 \pm \beta)$$

where $\beta \equiv \frac{v}{c}$

Decreasing Separation

$$f' = f\sqrt{\frac{1+\beta}{1-\beta}}$$

$(f' > f)$

Increasing Separation

$$f' = f\sqrt{\frac{1-\beta}{1+\beta}}$$

$(f' < f)$

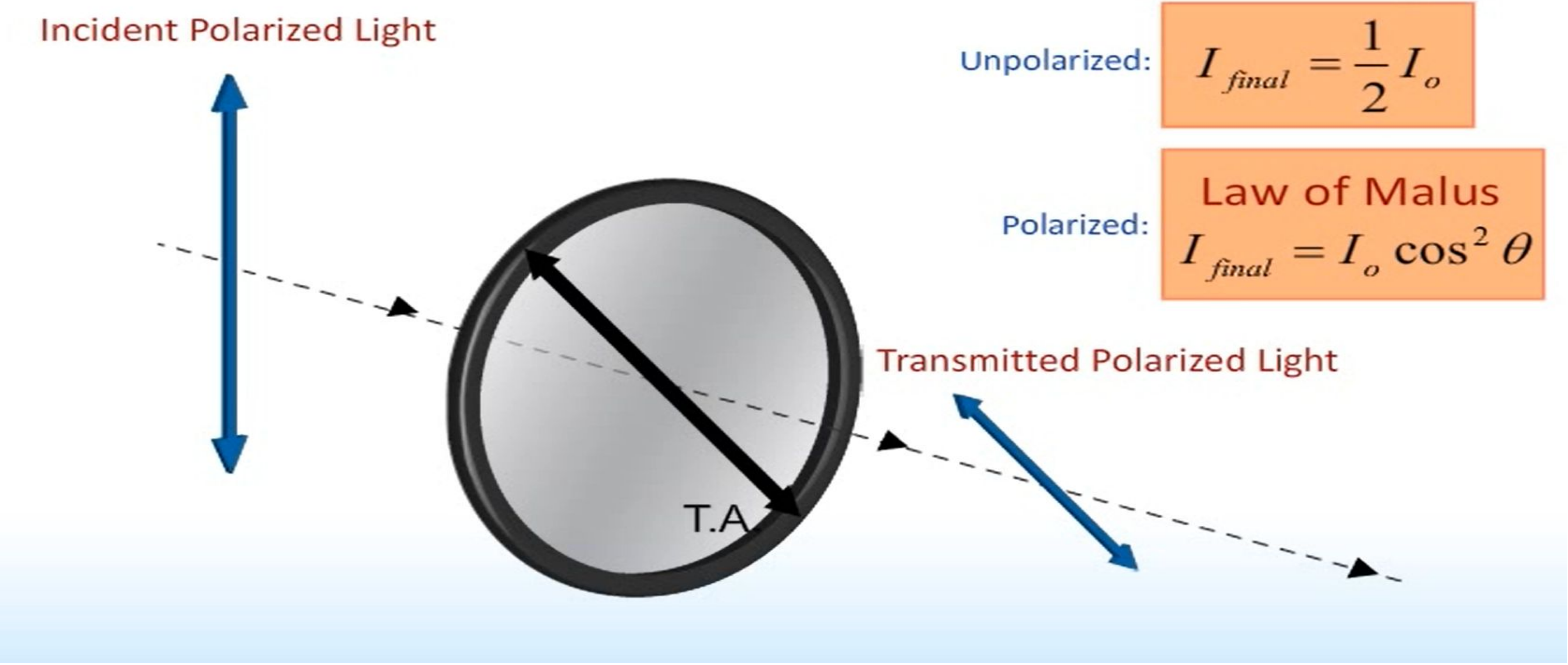
Linear Polarization

Incident Light

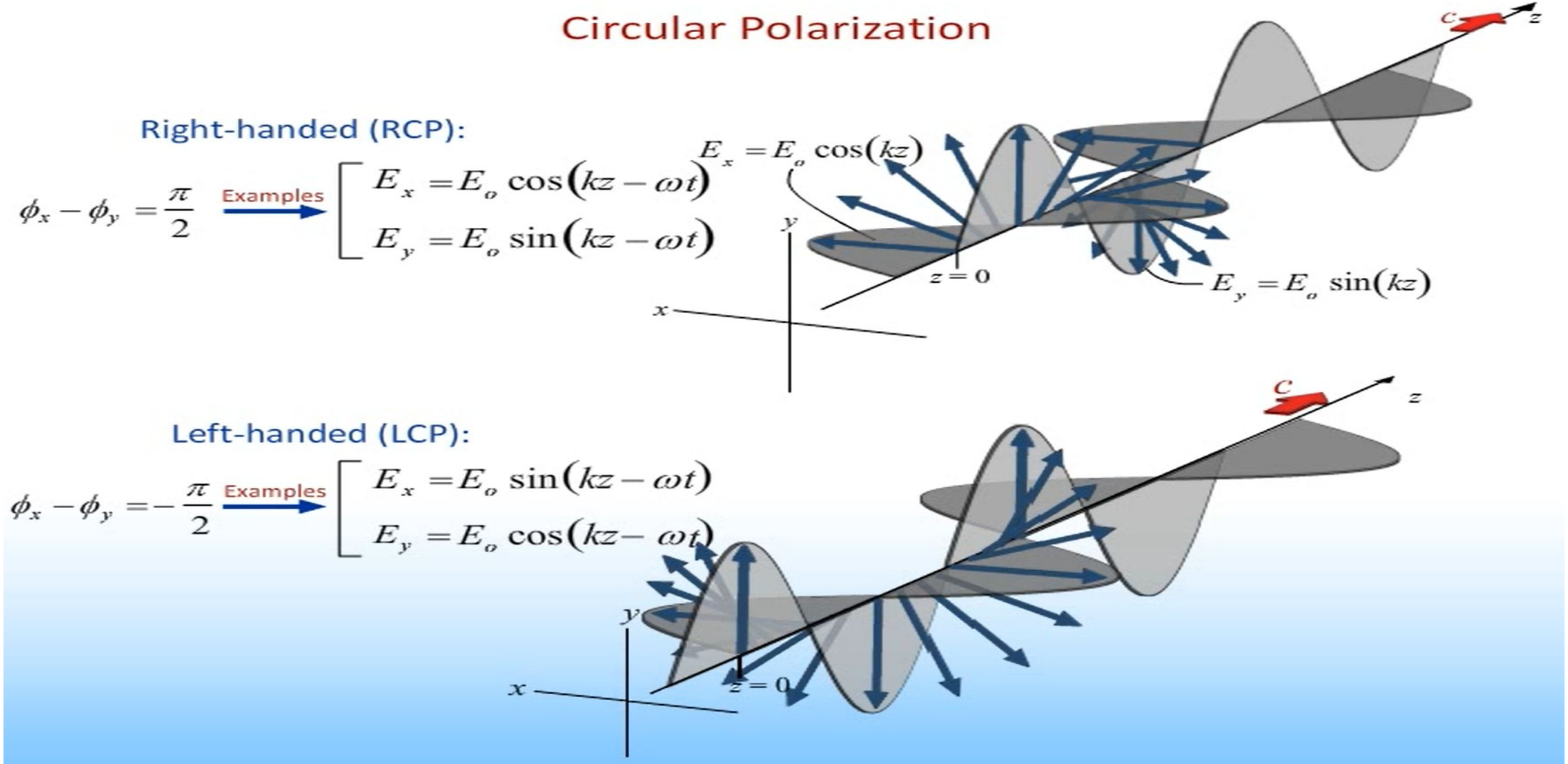
Unpolarized: $$I_{final} = \frac{1}{2} I_o$$

Law of Malus

Polarized: $$I_{final} = I_o \cos^2 \theta$$

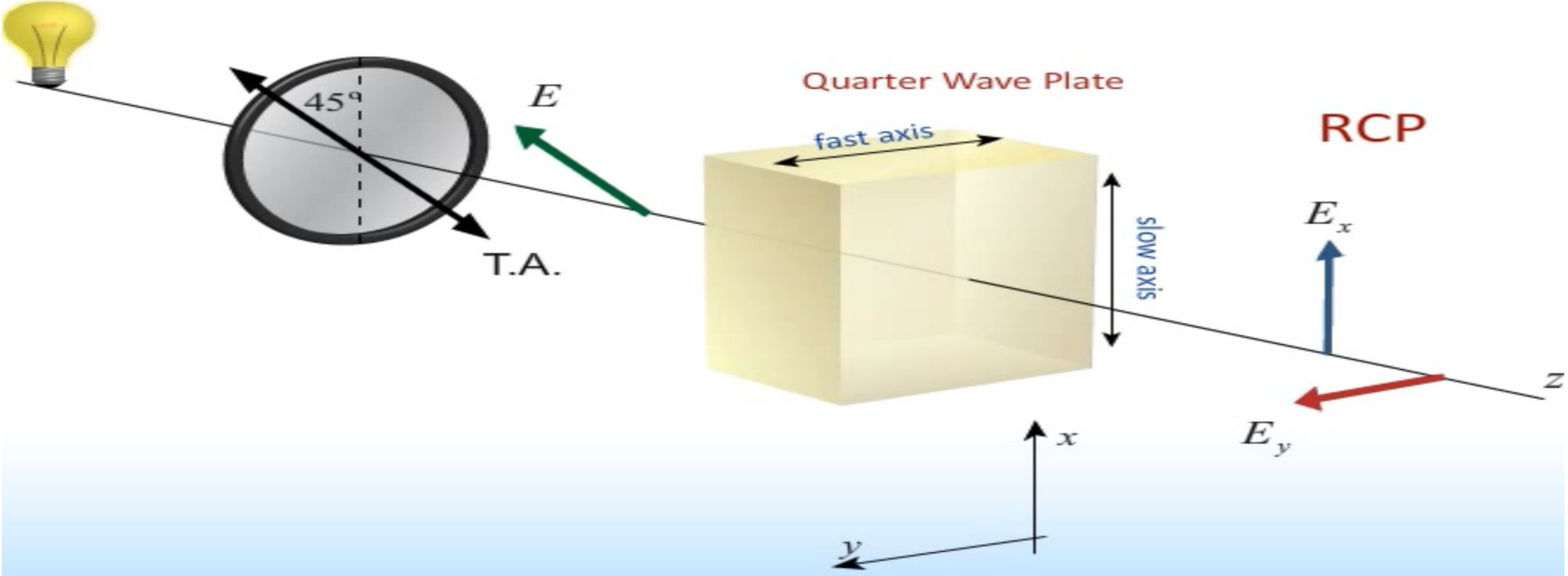


Circular Polarization



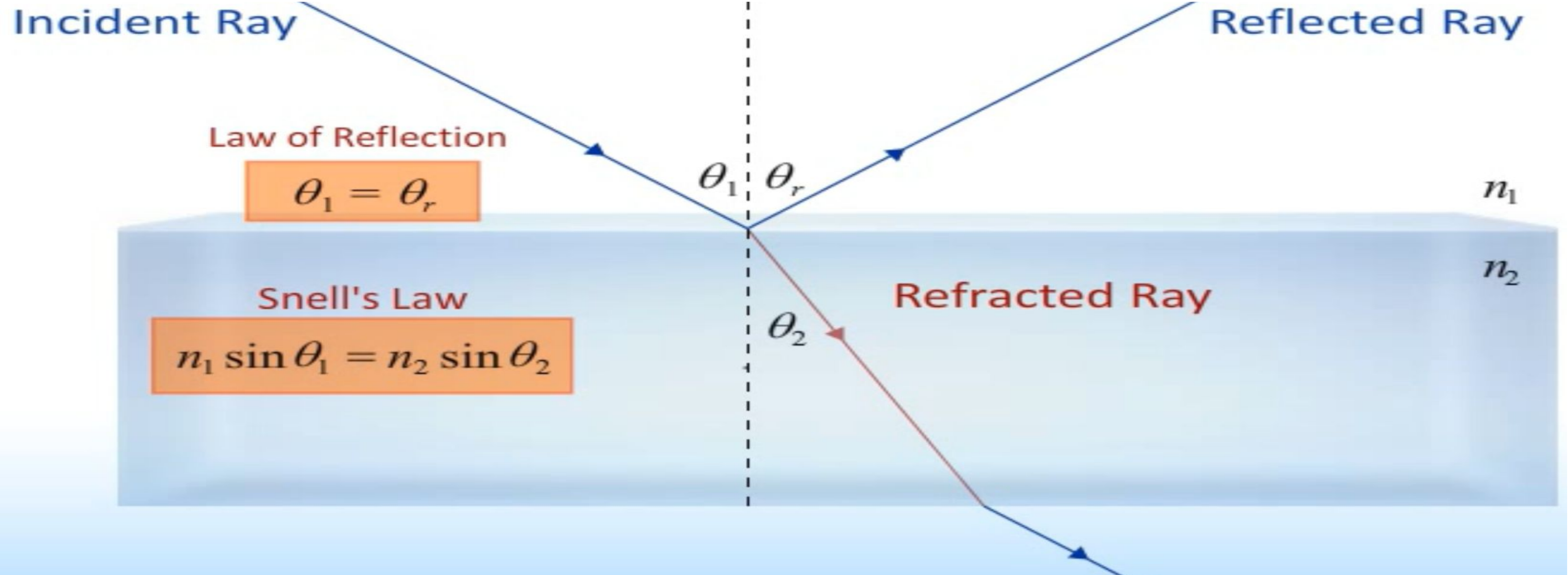
Circular Polarization cont.

- Produced by passing linear polarized light through a quarter wave plate **(only if the light isn't 100% vertically or horizontally linearly polarized beforehand)**
- If **Slow-Axis X Fast-Axis = Direction of Wave → RCP , otherwise LCP**



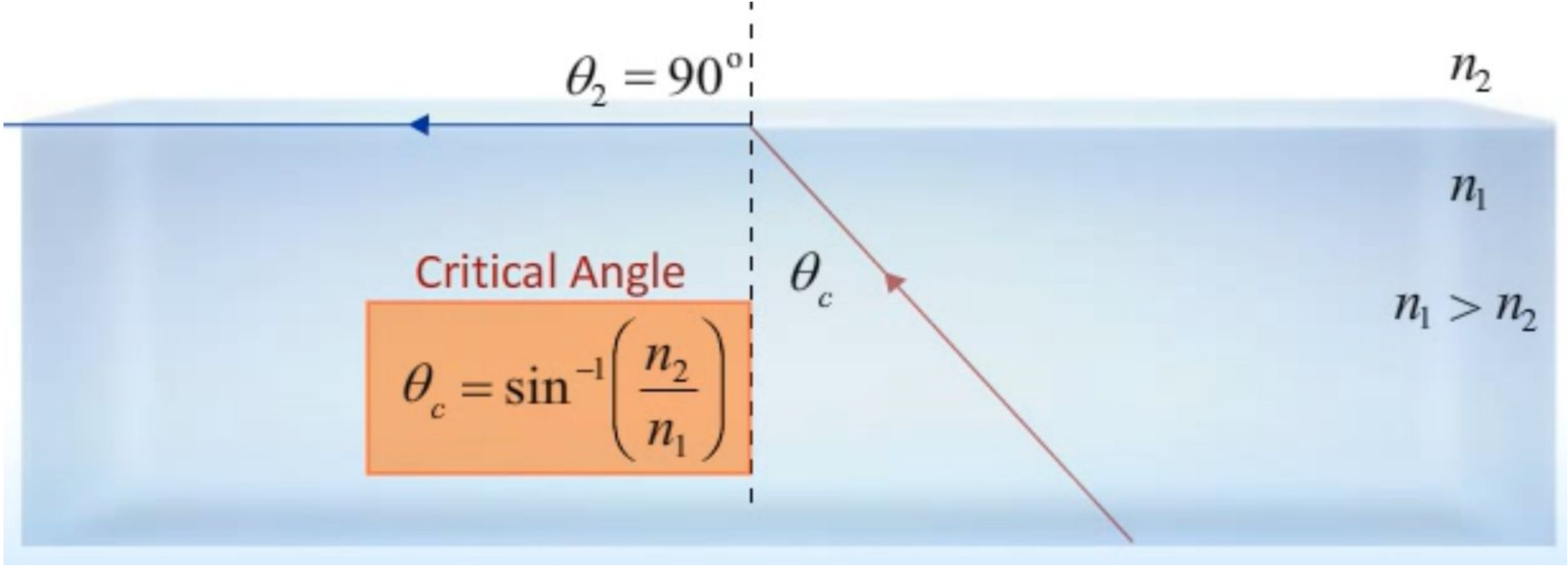
Reflection and Refraction

- **Law of Reflection** - the incident angle is equal to the reflected angle wrt the normal
- **Index of Refraction** - material specific: for air n = 1 and for glass n = 1.5 **(v = c/n)**
- **Snell's Law** - used to find the angle of the refracted ray wrt the normal

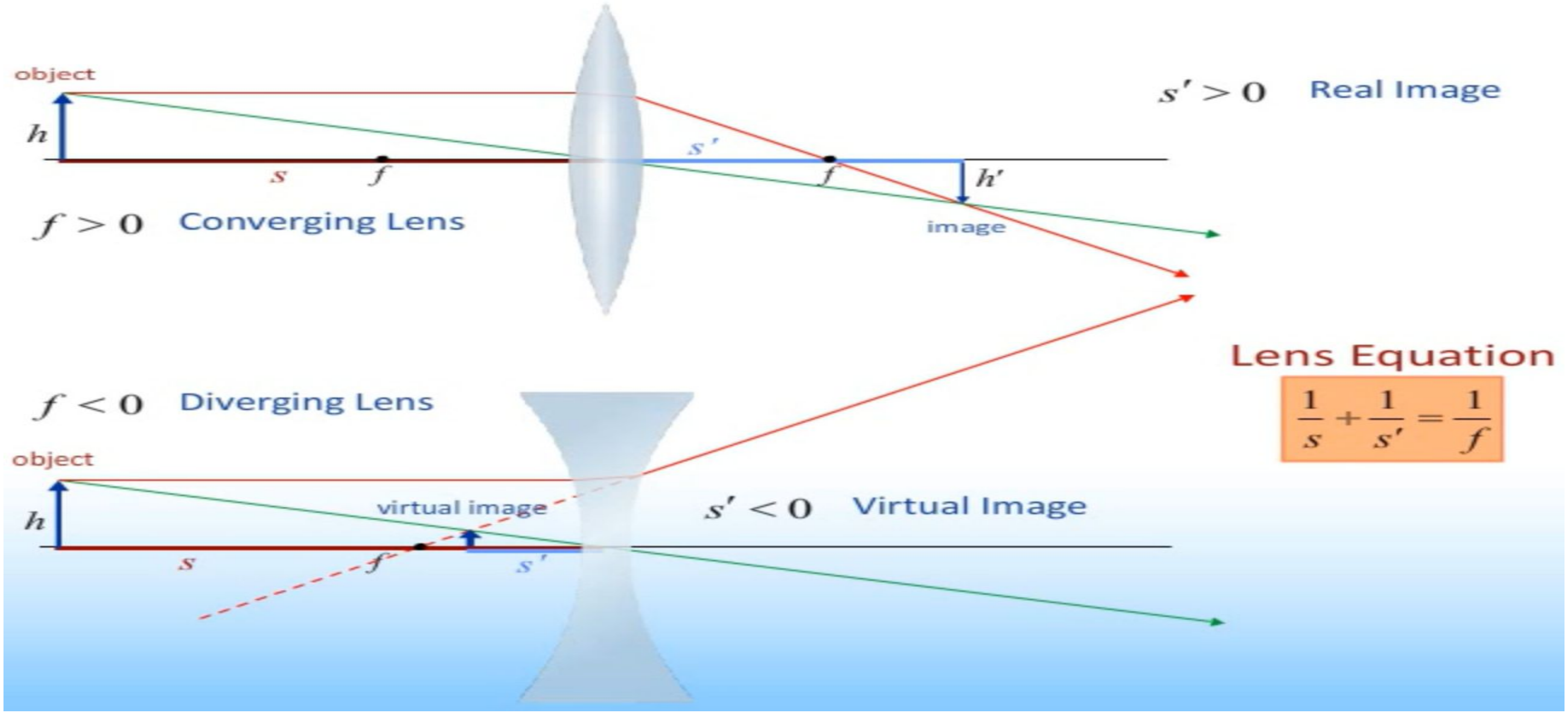


Reflection and Refraction cont.

Total Internal Reflection - only happens when rays are at the critical angle or at angles larger than the critical angle



Lenses



object
h
s
f
s'
f
h'
image
$s' > 0$ Real Image
$f > 0$ Converging Lens
Lens Equation
$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$
$f < 0$ Diverging Lens
object
h
virtual image
s
f
s'
$s' < 0$ Virtual Image

Lenses cont.

Len's Equation

- Converging (f > 0) vs Diverging Lenses (f < 0)
- Real Image (S' > 0) vs Virtual Image (S' < 0)

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Magnification

$$M \equiv \frac{h'}{h} \rightarrow M = \frac{-f}{s-f}$$

- Upright Image (M > 0) vs Inverted Image (M < 0)
- Real Images are always inverted and Virtual Images are always upright

General Lensmaker's Formula

$$\frac{1}{f} = (n-1)\frac{1}{R}$$

Mirrors

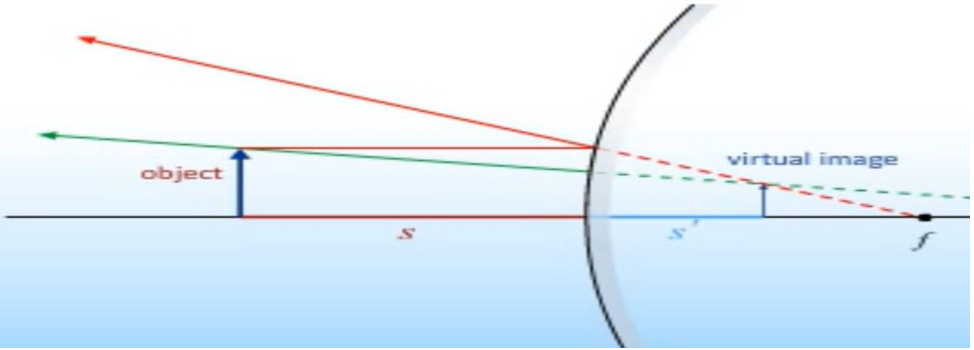
Lens Equations and Mirror Equations are the same

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$M = \frac{h'}{h} = -\frac{s'}{s}$$

Sign Conventions

$f > 0$	Concave Mirrors
$f < 0$	Convex Mirrors
$s' > 0$	Real Image
$s' < 0$	Virtual Image



Sign into queue for worksheet!