Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

PHYS 212 – University Physics: Electricity and Magnetism

Mid-semester Review Worksheet Solutions

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1:

Can't make it to a session? Here's our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

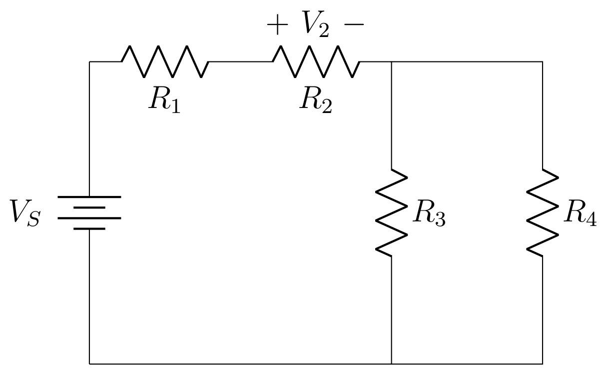
Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/848
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

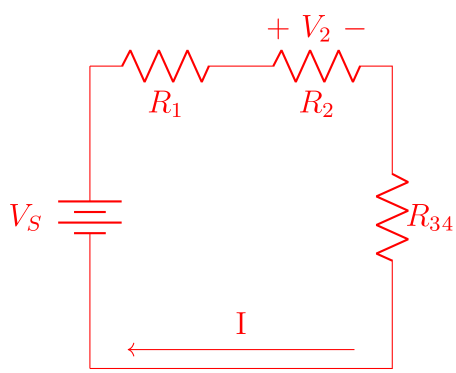
1. Consider the circuit below, where $V_S = 10$ V, $R_1 = 5\ \Omega$, and $R_3 = R_4 = 4\ \Omega$.



(i) If V_2 is 3 V, what is the resistance of R_2?

(ii) The cross-sectional area of R_3 is doubled and the length of R_1 is halved. What is the new value of V_2?

(i) To solve for R_2, we would have to use Kirchhoff's Laws. However, KVL would be easier if R_3 and R_4 is combined since the simplified circuit would only contain one current. This produces the following circuit with current I:



In this case, the combined resistance of R_{34} would be

$$R_{34} = (\frac{1}{R_3} + \frac{1}{R_4})^{-1} = (\frac{1}{4} + \frac{1}{4})^{-1} = 2\ \Omega$$

Using KVL and summing up all the voltages, we obtain

$$-V_S + IR_1 + V_2 + IR_{34} = 0$$

$$I(R_1 + R_{34}) = V_S - V_2$$

$$I = \frac{V_S - V_2}{R_1 + R_{34}} = \frac{10 - 3}{5 + 2} = 1\ A$$

Using Ohm's Law, we can then calculate the resistance of R_2 to be

$$R_2 = \frac{V_2}{I} = \frac{3}{1} = \boxed{3\ \Omega}$$

(ii) We know the resistance of the resistor is related to its geometry through

$$R = \frac{\rho L}{A}$$

Therefore, by adjusting the cross-sectional area of R_3 and the length of R_1, we obtain new resistance R_3' and R_1':

$$R_3' = \frac{\rho L}{2A} = \frac{R_3}{2} = 2\ \Omega$$

$$R_1' = \frac{\rho L/2}{A} = \frac{R_1}{2} = 2.5\ \Omega$$

Since R_3 has changed, the combined resistance R_{34} will also change to R_{34}'

$$R_{34}' = (\frac{1}{R_3'} + \frac{1}{R_4})^{-1} = (\frac{1}{2} + \frac{1}{4})^{-1} = 1.333\ \Omega$$

To obtain the current I, we further combine R_1', R_2, and R_{34}' together to get R_{1234}'

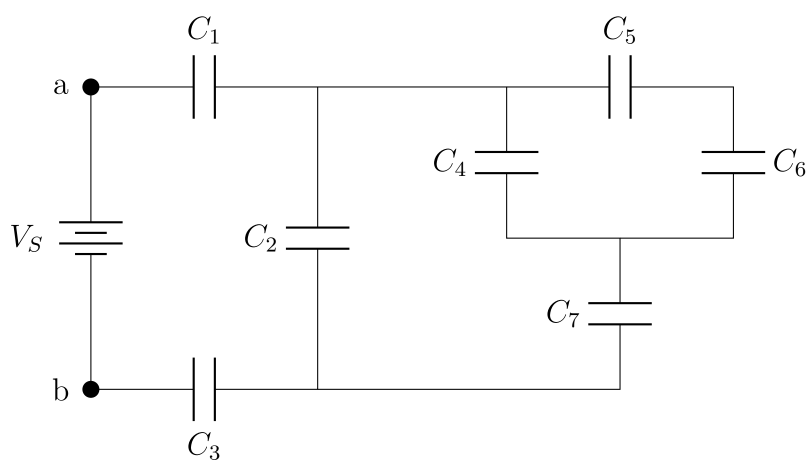
$$R_{1234}' = R_1' + R_2 + R_{34}' = 2.5 + 3 + 1.333 = 6.833\ \Omega$$

$$I = \frac{V_S}{R_{1234}'} = \frac{10}{6.833} = 1.463\ A$$

Finally, to calculate V_2, we again apply Ohm's Law

$$V_2 = IR_2 = \boxed{4.39\ V}$$

2. Question 2 refers to the circuit shown below where $C_1 = C_5 = C_6 = C_7 = 40\ \mu F$ and $C_2 = C_3 = C_4 = 20\ \mu F$:



(i) Find the equivalent capacitance as seen by the voltage source.

(ii) An electron travels from node a to node b. What is its change in electric potential energy if V_S = 5 V?

(iii) What is the energy stored by the equivalent capacitance?

(iv) How does the energy stored in the equivalent capacitor change if we double the distance between each capacitor's plates?

(i) To combine capacitors, we use the following equations:

$$C_{parallel} = C_A + C_B$$

$$C_{series} = (\frac{1}{C_A} + \frac{1}{C_B})^{-1}$$

It is also important to combine everything in a systematic way to simplify the process. Therefore, we are going to combine the capacitors starting from the top right corner and make our way to the left.

$$C_{56} = (\frac{1}{C_5} + \frac{1}{C_6})^{-1} = 20\ \mu F$$

$$C_{456} = C_4 + C_{56} = 40\ \mu F$$

$$C_{4567} = (\frac{1}{C_{456}} + \frac{1}{C_7})^{-1} = 20\ \mu F$$

$$C_{24567} = C_2 + C_{4567} = 40\ \mu F$$

$$C_{1234567} = (\frac{1}{C_1} + \frac{1}{C_{24567}} + \frac{1}{C_3})^{-1} = \boxed{10\ \mu F}$$

(ii) Going from point a to point b, we enter the battery from the positive end and exit from the negative end. This results in a drop of voltage $\Delta V = -V_S =$ -5 V. Since we know the change in electric potential energy can be expressed as

$$\Delta U = q\Delta V$$

we can calculate the electron's change in electric potential energy to be:

$$\Delta U_{electron} = q_{electron}\Delta V = -1.6 \times 10^{-19} \times -5 = \boxed{8 \times 10^{-19} J}$$

(iii) The energy stored in a capacitor can be calculated using

$$U = \frac{1}{2}CV^2$$

Since the equivalent capacitor is in parallel with the voltage source, the equivalent capacitor has the same voltage as the voltage source, V_S. This means the energy stored in the equivalent capacitor is

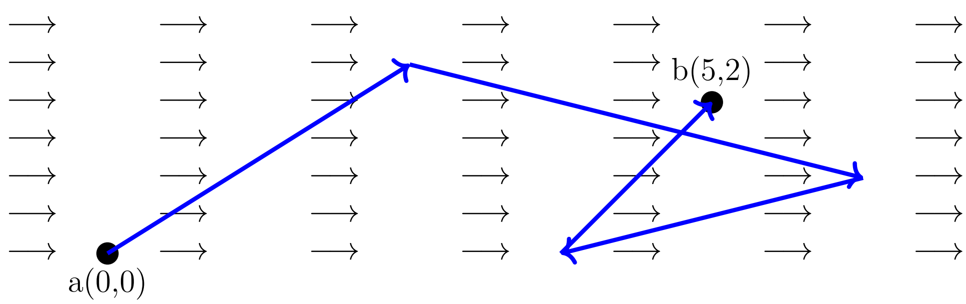
$$U_{Ceq} = \frac{1}{2} \times 10\ \mu F \times 5^2 = \boxed{125\ \mu J}$$

(iv) The capacitance of a capacitor is related to its geometry through

$$C = \frac{\epsilon_0 A}{d}$$

Thus, by doubling the distance (d) between each capacitor, we cut the capacitance of the original capacitors by half. Since all the capacitors are halved, the equivalent capacitance is also halved. However, the equivalent capacitor is still in parallel with the voltage source. Therefore, its voltage does not change, although its capacitance has changed. Since the energy stored in the equivalent capacitor is proportional to its capacitance, cutting its capacitance in half results in the stored energy also being halved.

3. Suppose you have a constant electric field of 2 V/m pointing to the right:



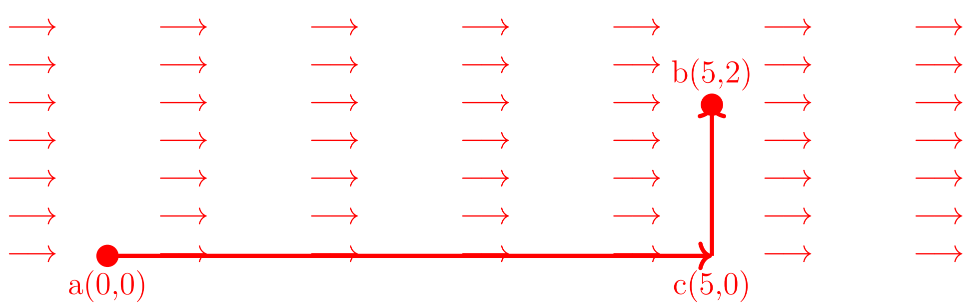
(i) What is the magnitude of the difference in electric potential between point a and point b? Assume the coordinate has a unit of meters.

(ii) If an electron is moved from point b to point a along the blue path labeled in the figure, what is its change in electric potential energy?

(i) The electric potential difference between two points can be calculated using

$$\Delta V = -\int_a^b \vec{E} \cdot d\vec{l}$$

But since the integral involves a dot product, it is easier to break the path from (0,0) to (5,2) into two parts: one parallel to the electric field, and one perpendicular to the electric field, as shown below:



Then, we can calculate the electric potential using its definition

$$\Delta V = -\int_a^b \vec{E} \cdot d\vec{l} = -\int_a^c \vec{E} \cdot d\vec{l} - \int_c^b \vec{E} \cdot d\vec{l}$$

In this case, the integral from c to b would simply be zero. This is because the path from c to b is perpendicular to the direction of the electric field, which makes the dot product inside the integral zero. As for the integral from a to c, since the path is in the same direction as the electric field, the dot product then becomes the product of the two vector magnitudes

$$\Delta V = -\int_a^b \vec{E} \cdot d\vec{l} = -\int_a^c \vec{E} \cdot d\vec{l} = -\int_a^c E \cdot dl = -E\int_a^c dl = -5E = -10V$$

Since the question is asking for the magnitude, we have

$$|\Delta V| = \boxed{10\ V}$$

(ii) The electric potential energy change of a charged particle can be calculated using $\Delta U = q\Delta V$. From the last part, we calculated that ΔV_{atob} is -10 V. Since the electron is traveling in the opposite direction from b to a, we then have

$$\Delta V_{b\to a} = -\Delta V_{a\to b} = 10V$$

$$\Delta U_{electron} = q_{electron}\Delta V_{b\to a} = \boxed{-1.6\times 10^{-18} J}$$

4. A 10 Ω cylindrical resistor of length 1 cm has a uniform electric field of 5 V/m inside it. What is the magnitude of the current through the resistor?

(i) To get the current, we have to use Ohm's Law: V = IR. But to get I, we first have to calculate V. From Question 3, we know that the calculation for the voltage difference ΔV can be simplified when the electric field and the direction of the path taken are in the same direction:

$$\Delta V = -\int_a^b \vec{E} \cdot d\vec{l} = -E\int_a^b dl = -E \times L$$

This is exactly the case we have, therefore the V for the resistor can be calculated to be

$$V = -5\ V/m \times 0.01\ m = -0.05\ V$$

The magnitude of the current through the resistor is then calculated to be

$$|I| = |V| \times R = 0.005\ A = \boxed{5\ mA}$$