Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

PHYS 212 – University Physics: Electricity and Magnetism

Mid-semester Review Worksheet Solutions

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1:

Can't make it to a session? Here's our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

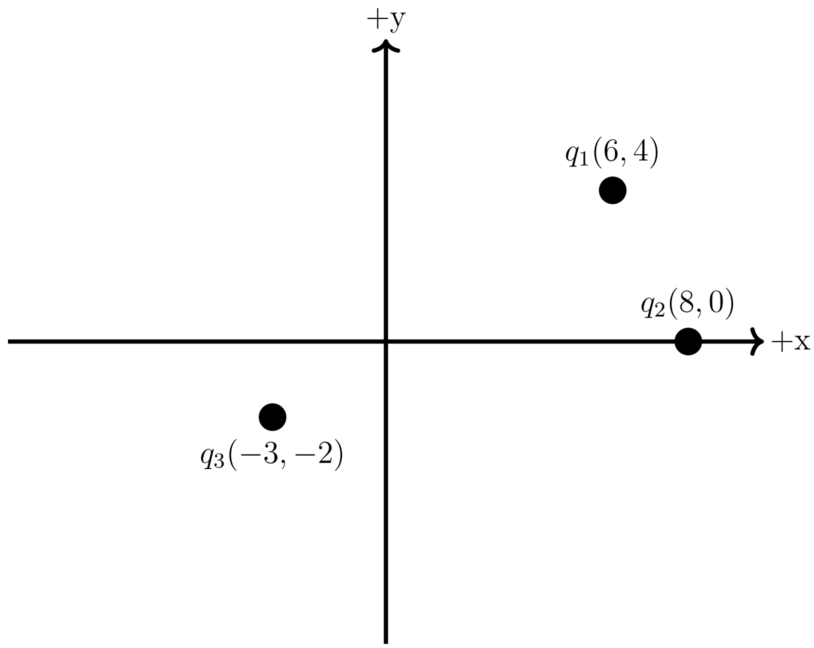
Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/848
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

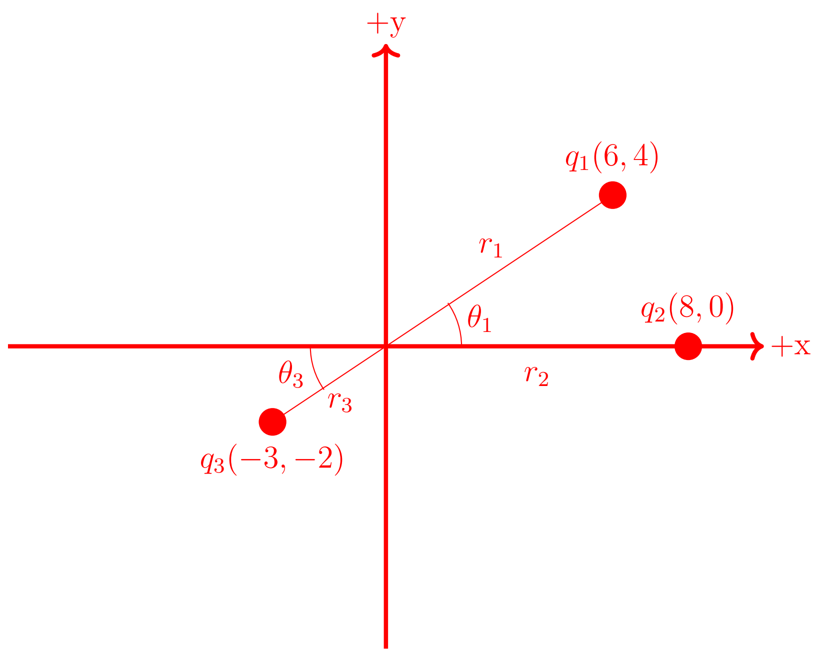
1. Three charges with charges $q_1 = 10\ \mu C$, $q_2 = -20\ \mu C$, and $q_3 = -30\ \mu C$ exist in the x-y plane. Their positions are as shown below. The coordinate is in units of meters.



(i) Find the electric field at the origin. Express the answer as a vector.

(ii) If a negative charge of $q = -2C$ is placed at the origin, what is the force on the negative charge? Express the answer as a vector.

(i) To find the electric field at the origin, we have to find the electric field generated by the three charges and add them through vector addition. But since vector addition can be sometimes confusing, we are going to break the three electric fields into their respective x and y components and add them together.



The strength of the electric field from a given charge can be calculated using

$$E = \frac{kQ}{r^2}$$

Therefore, the strength of the three electric fields from the three charges are

$$E_1 = \frac{kq_1}{r_1^2} = \frac{k \times 10 \times 10^{-6}}{6^2 + 4^2} = 1731\ N/C$$

$$E_2 = \frac{kq_2}{r_2^2} = \frac{k \times -20 \times 10^{-6}}{8^2 + 0^2} = -2812\ N/C$$

$$E_3 = \frac{kq_3}{r_3^2} = \frac{k \times -30 \times 10^{-6}}{(-3)^2 + (-2)^2} = -20769\ N/C$$

Now that we have the strength of the electric fields, we can break them into their respective x and y components to add them together. Note that the negative sign for the strength implies that the electric field is going into the charge.

$$\Sigma\vec{E}_x = \vec{E}_{1,x} + \vec{E}_{2,x} + \vec{E}_{3,x} = -|E_1|cos(\theta_1)\hat{x} + |E_2|cos(0)\hat{x} - |E_3|cos(\theta_3)\hat{x} = -15909\hat{x}\ N/C$$

$$\Sigma\vec{E}_y = \vec{E}_{1,y} + \vec{E}_{2,y} + \vec{E}_{3,y} = -|E_1|sin(\theta_1)\hat{y} + |E_2|sin(0)\hat{y} - |E_3|sin(\theta_3)\hat{y} = -12481\hat{y}\ N/C$$

$$\vec{E}_{origin} = \Sigma\vec{E}_x + \Sigma\vec{E}_y = \boxed{-15909\hat{x} - 12481\hat{y}\ N/C}$$

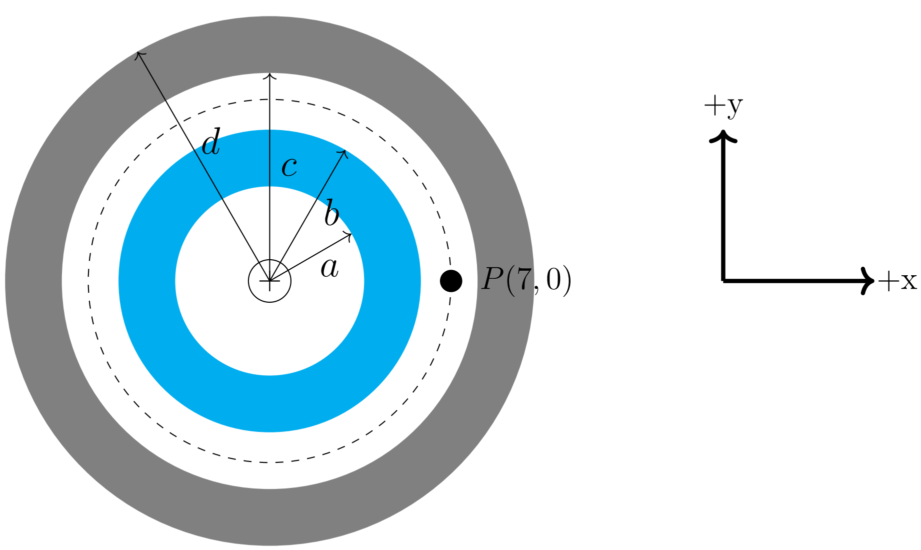
(ii) The force on a charge inside a given electric field can be computed using

$$\vec{F} = q\vec{E}$$

Since we have already computed the electric field, the force on the given negatively charged particle would be:

$$\vec{F}_{particle} = q_{particle}\vec{E}_{origin} = \boxed{31818\hat{x} + 24962\hat{y}\ N}$$

2. A positively charged particle with charge $Q_0 = 5\ \mu C$ is surrounded by two shells. The inner shell is made out of a conductor and the outer shell is made out of an insulator. The conductor shell is electrically neutral and has an inner radius of $a = 3\ m$ and an outer radius of $b = 5\ m$. The insulator shell carries a charge $Q_I = -10\ \mu C$ spread uniformly throughout and has an inner radius of $c = 9\ m$ and an outer radius of $d = 13\ m$.



(i) Find the strength of the electric field at point P.

(ii) Find Q_{inner}, the charge on the inner surface of the conductor shell.

(iii) Find the location(s) on the positive x-axis where the electric field is 0. Assume the positive charge sits at the origin.

(i) To find the electric field at point P, we have to use Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

In this case, we construct a spherical surface centered at the origin with a radius r = 2.5, as shown in the dashed circle in the initial figure. The electric field is the same everywhere on this sphere because of the spherical symmetry of the setup. Therefore, we can simplify the integral and get:

$$\oint \vec{E} \cdot d\vec{A} = E \oint dA = 4\pi r^2 E = \frac{q_{enclosed}}{\epsilon_0} \Rightarrow E = \frac{q_{enclosed}}{4\pi\epsilon_0 r^2}$$

In this case, the $q_{enclosed}$ is simply $Q_0 = 5\ \mu C$, so the electric field at point P is

$$E_P = \frac{Q_0}{4\pi\epsilon_0 r^2} = \boxed{918.4\ N/C}$$

(ii) We can construct a spherical Gaussian surface with $a < r < b$ to determine the charge on the inner surface of the conductor shell. We know that the electric field inside a conductor has to be 0. Therefore, according to Gauss's Law, the charge enclosed by the Gaussian surface also has to be zero. Since the Gaussian surface is only enclosing the charge Q_0 and the inner conductor surface, we have

$$q_{enclosed} = Q_0 + Q_{inner} = 0$$
$$Q_{inner} = -Q_0 = \boxed{-5\ \mu C}$$

(iii) We know that the electric field has to be zero inside a conductor. Therefore, $a < x < b$ is part of our answer. In this conductor-insulator setup, we can see that the charge enclosed by a spherical Gaussian surface when $0 < x < c$ is always Q_0 since the conductor is not charged. However, when $x > d$, the charge enclosed is $Q_0 + Q_I = -5\ \mu C$ which is negative. This implies that in the interval $c < x < d$ there must exist a point where $Q_{enclosed} = 0$. At that point, the electric field must also be zero to satisfy Gauss's Law. In the region of $c < x < d$, the charge enclosed can be expressed as

$$Q_{enclosed} = Q_0 + Q_I'$$

where Q_I' is the amount of charge from the insulator that is enclosed by the Gaussian surface. Q_I' can be calculated using the enclosed volume of the insulator $V_{I,enclosed}$ and the total volume of the insulator V_I:

$$Q_I' = Q_I \times \frac{V_{I,enclosed}}{V_I}$$

With this information, we can then calculate the point where $Q_{enclosed} = 0$.

$$Q_{enclosed} = Q_0 + Q_I' = Q_0 + Q_I \times \frac{V_{I,enclosed}}{V_I} = Q_0 + Q_I \times \frac{\frac{4}{3}\pi x^3 - \frac{4}{3}\pi c^3}{\frac{4}{3}\pi d^3 - \frac{4}{3}\pi c^3} = Q_0 + Q_I \times \frac{x^3 - c^3}{d^3 - c^3} = 0$$

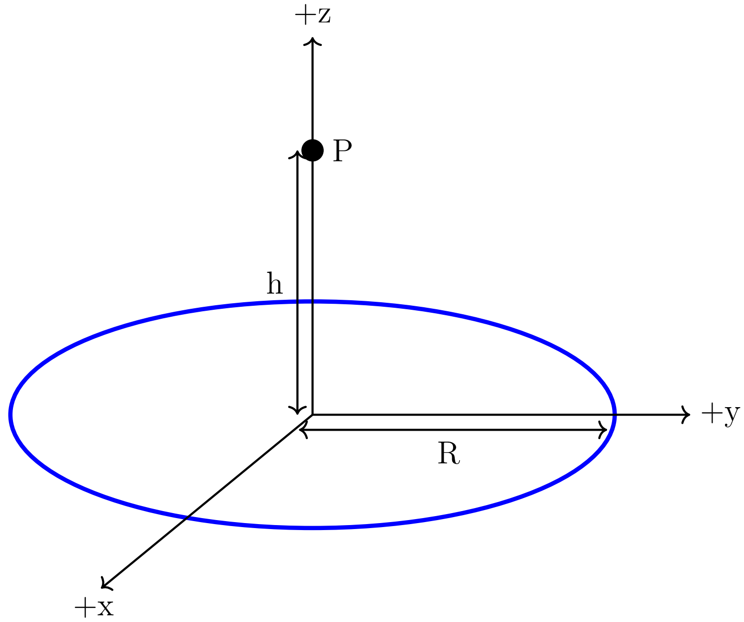
$$\frac{x^3 - c^3}{d^3 - c^3} = \frac{-Q_0}{Q_I}$$

$$x^3 = \frac{-Q_0}{Q_I}(d^3 - c^3) + c^3$$

$$x = (\frac{-Q_0}{Q_I}(d^3 - c^3) + c^3)^{1/3} = 11.35$$

Therefore our final answer would be $\boxed{3 < x < 5 \text{ or } x = 11.35}$

3. Consider a circular ring with radius R in the x-y plane centered at the origin carrying a charge of Q uniformly distributed in the ring. Point P lies on the z-axis and is a distance h from the x-y plane.



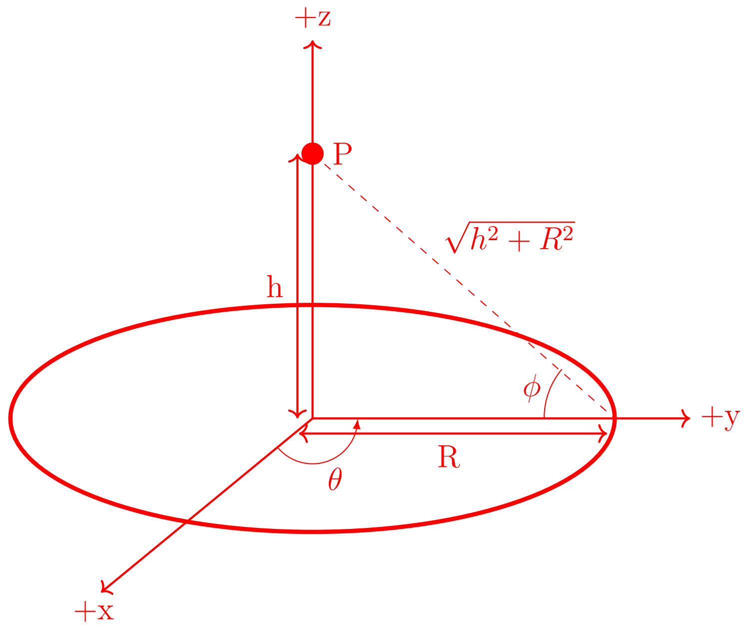
(i) Write out an integral that gives the z-component of the electric field at point P. The answer should be algebraic.

(ii) What is the electric field at point P? How is this electric field different from what you got in the first part?

(iii) Let $h >> R$, what does your answer in part (i) approximate to? Does the result resemble anything?

(i) Since the source of the electric field is not a point charge but a continuous distribution of charges, we have to integrate dE to get the final electric field.

$$E = \int dE = \int \frac{kdQ}{r^2}$$



Since the charge Q is uniformly dispersed on the ring, we can calculate the charge density of the ring and use it to calculate dE:

$$\lambda = \frac{Q}{2\pi R}$$

$$\int dE = \int \frac{kdQ}{r^2} = \int \frac{k\lambda dl}{r^2} = \int \frac{k\lambda R d\theta}{r^2}$$

In this case, dl is a very short arc length of the ring. Since the arc length of a circle is $l = R\theta$, $dl = Rd\theta$. However, since we only want the z-component of the electric field at point P, we have to multiply the electric field by $sin(\phi)$

$$E_Z = \int dE_Z = \int \frac{k\lambda R d\theta}{r^2} sin(\phi) = \int \frac{k\lambda R d\theta}{h^2+R^2}\frac{h}{\sqrt{h^2+R^2}} = \boxed{\int_0^{2\pi} \frac{k\lambda h R d\theta}{(h^2+R^2)^{3/2}}}$$

(ii) We have already calculated the electric field in the z-direction. So if we can figure out the electric field parallel to the x-y plane, we would know the total electric field at point P. However, if we only consider the component of the electric field parallel to the x-y plane, we can see that the electric field from any point (x_0, y_0) on the ring would be canceled out by the electric field from another point $(-x_0, -y_0)$ since both electric fields have the same magnitude but different direction. Therefore, the electric field we calculate in part (i) is exactly the total electric field at point P.

$$\boxed{E = E_Z}$$

(iii) If $h >> R$, then $h^2 + R^2 \approx h^2$ and we get

$$E = \int_0^{2\pi} \frac{k\lambda h R d\theta}{(h^2+R^2)^{3/2}} \approx \int_0^{2\pi} \frac{k\lambda h R d\theta}{(h^2)^{3/2}} = \int_0^{2\pi} \frac{k\lambda h R d\theta}{h^3} = \int_0^{2\pi} \frac{k\lambda R d\theta}{h^2}$$

Completing the integral and the electric field becomes

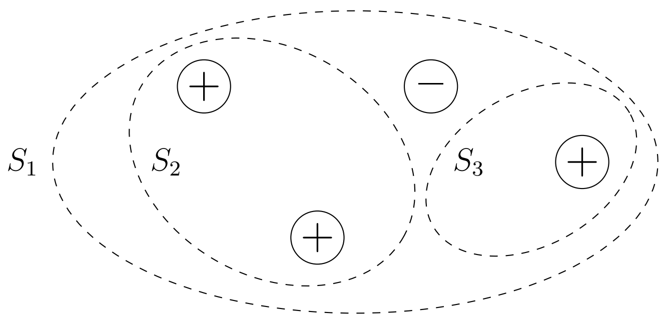
$$E = \int_0^{2\pi} \frac{k\lambda R d\theta}{h^2} = \frac{2\pi R k\lambda}{h^2}$$

By definition, $Q = 2\pi R\lambda$, so our final answer becomes:

$$E = \frac{2\pi Rk\lambda}{h^2} = \boxed{\frac{kQ}{h^2}}$$

This is exactly the electric field by a point charge at the origin carrying a charge Q.

4. All four charges below have the same magnitude Q. The only difference is the sign for each charge. Given the charges and the Gaussian surfaces below, compare the electric flux of the three Gaussian surfaces.



(i) Electric flux is defined to be

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

By Gauss's Law, we know that the electric flux is only proportional to the charge enclosed. Therefore, we can compute the electric flux of the three Gaussian surfaces to be

$$\Phi_{S1} = \frac{Q + Q + Q - Q}{\epsilon_0} = \frac{2Q}{\epsilon_0}$$

$$\Phi_{S2} = \frac{Q + Q}{\epsilon_0} = \frac{2Q}{\epsilon_0}$$

$$\Phi_{S3} = \frac{Q}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

$$\boxed{\Phi_{S1} = \Phi_{S2} > \Phi_{S3}}$$