Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

PHYS 212 – University Physics: Electricity and Magnetism

Mid-semester Review Worksheet Solutions

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1:

Can't make it to a session? Here's our schedule by course:

`https://care.grainger.illinois.edu/tutoring/schedule-by-subject`

Solutions will be available on our website after the last review session that we host.

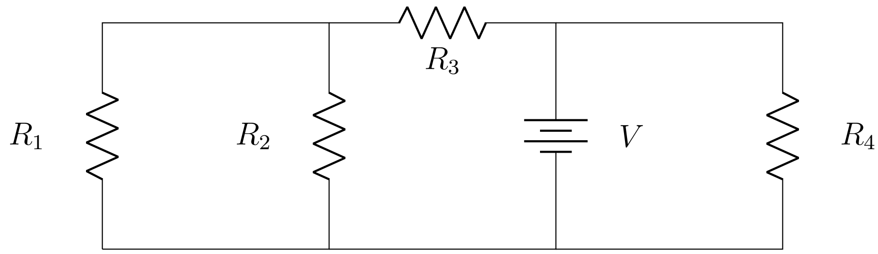
Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/848
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

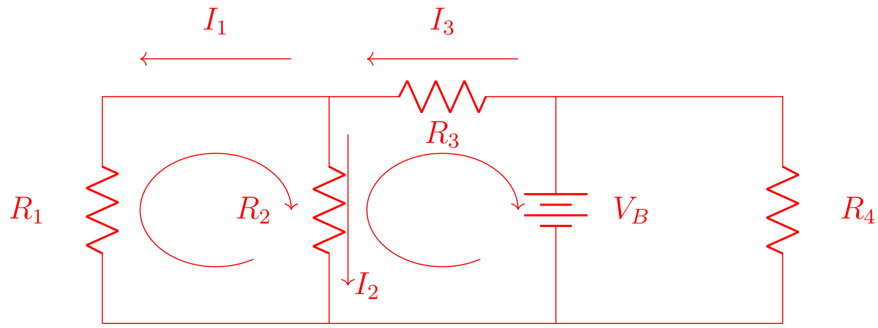
1. A circuit is constructed with four resistors and a battery as shown below. The battery voltage is 18V. $R_1 = 3\Omega$, $R_2 = 6\Omega$, $R_3 = 12\Omega$, $R_4 = 5\Omega$.



(i) What is the voltage across R_2?

(ii) What is the current through R_4?

(i) Using the two loops shown below, we get two KVL equations:



$$V_B - I_2R_2 - I_3R_3 = 0$$
$$-I_1R_1 + I_2R_2 = 0$$

From the node law, we know that

$$I_1 + I_2 = I_3$$

Solving this system of equations, we find that $I_2 = 0.4286A$. Plugging into Ohm's Law

$$V = IR_2 = 0.4286 \times 6 = \boxed{2.571\ V}$$

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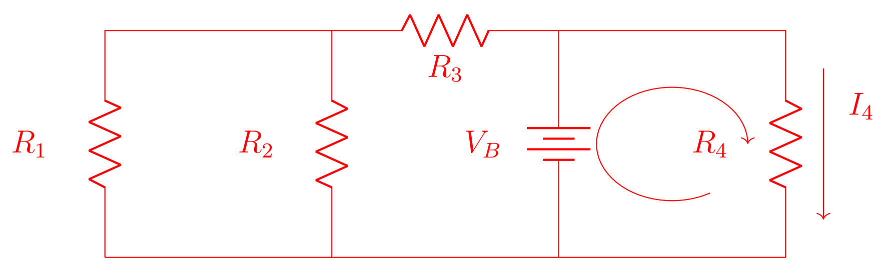
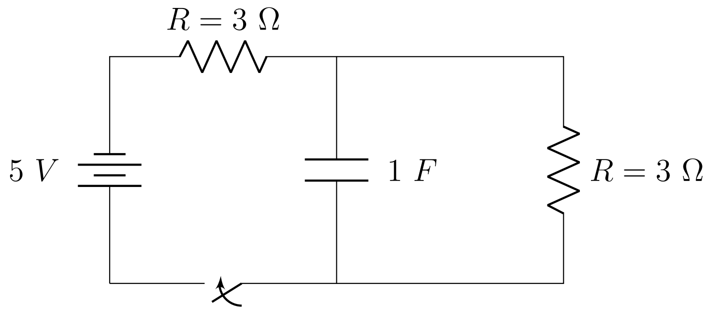
(ii) To solve for the current through R_4, we do KVL using the circuit loop below:



From this KVL, $V_B = I_4R_4$.

$$I_4 = \frac{V_B}{R_4} = \frac{18}{5} = \boxed{3.6\ A}$$

2. A circuit with a 1F capacitor, two 3Ω resistors, and a 5V battery is shown below. The capacitor is initially uncharged and the switch is open.



(i) Immediately after the switch is closed, what is the current across the capacitor?

(ii) What is the charge on the capacitor 2s after the switch is closed?

(i) Immediately after the switch is closed, the capacitor acts as a wire, so no current goes through R_2. Doing KVL using the left loop then gives

$$5\ V = 3\ \Omega \times I \Rightarrow \boxed{I = 1.67\ A}$$

(ii) Since the capacitor is charging, we use the equation

$$Q(t) = Q(\infty)(1 - e^{-\frac{t}{\tau}})$$

From the point of view of the capacitor, the two resistors are in parallel during the charging process, so

$$R_{eq} = (\frac{1}{3} + \frac{1}{3})^{-1} = 1.5\ \Omega$$
$$\tau = R_{eq} \times C = 1.5\ s$$

At time $t = \infty$, the capacitor must have no current going through it and acts as an open. Using a KVL on the out most loop, we then get

$$5V = 3I + 3I \Rightarrow I = 0.833\ A$$

Since the capacitor is in parallel with R_2, we know that the voltage across the capacitor must be the same as the voltage across R_2 at all times. Using this information, we can calculate Q_∞:

$$V_C = IR_2 = 0.833(3) = 2.5\ V$$

$$Q_\infty = C \times V_C = 2.5\ C$$
$$Q(t) = 2.5(1 - e^{-\frac{t}{1.5}})$$

Plugging $t = 2$ into our equation,

$$Q(2) = 2.5(1 - e^{-\frac{2}{1.5}}) = \boxed{1.84\ C}$$

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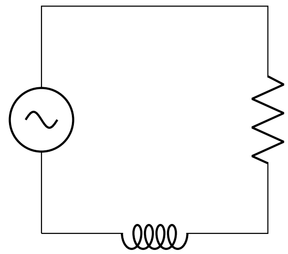
3. An alternating current circuit is constructed with an inductor, a resistor and a generator, but no capacitor.



(i) If we increase the frequency of the generator, what happens to the magnitude of the peak voltage across the resistor?

(ii) What happens to the energy stored in the inductor?

(i) There is no capacitor in this circuit, so $X_C = 0$. Therefore, the total impedance of the circuit is

$$Z = \sqrt{R^2 + X_L^2}$$

Since $X_L = \omega L$, increasing ω will increase X_L, which will then increases Z. Additionally, the relationship between V_{max} and I_{max} is:

$$V_{max} = I_{max} Z \Rightarrow I_{max} = \frac{V_{max}}{Z}$$

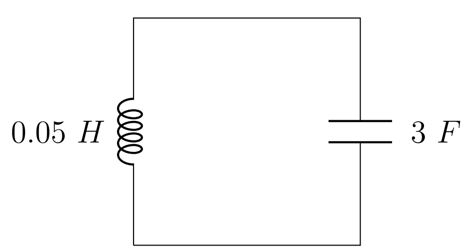
Therefore, increasing Z decreases I_{max}.
Finally, $V_{R,max} = I_{max} R$, so increasing ω decreases the peak voltage across the resistor.

(ii) The energy stored in the inductor is

$$U_L = \frac{1}{2} L I^2$$

Since increasing ω decreases I_{max}, the energy stored in the inductor would also decrease by increasing ω.

4. A circuit with a 0.05H inductor and a 3F capacitor is shown below. At t = 0, the capacitor is fully charged to 4 C and there are no currents.



(i) Calculate the resonant frequency of this circuit.

(ii) What is the energy stored in the inductor at t = 2 s?

(i) The resonance frequency of an LC circuit is $\omega_0 = \frac{1}{\sqrt{LC}}$. Using this formula, we are able to get

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.05 \times 3}} = \boxed{2.582\ s^{-1}}$$

(ii) We know that the current in the LC circuit behaves like a sine wave or a cosine wave. Since there is no current going through the inductor at time $t = 0$, the current must behave like a sine wave. Therefore,

$$I(t) = I_{max} \sin(\omega t)$$

To calculate I_{max}, we use energy conservation inside the LC circuit:

$$\frac{1}{2} L I_{max}^2 = \frac{1}{2} \frac{Q_{max}^2}{C}$$

$$I_{max}^2 = \frac{Q_{max}^2}{LC} \Rightarrow I_{max} = \frac{Q_{max}}{\sqrt{LC}} = \omega_0 Q_{max} = 10.328\ A$$

Therefore, the current function is:

$$I(t) = 10.328 \sin(\omega_0 t)$$

$$I(2) = 10.328 \sin(2.582 \times 2) = -9.29\ A$$

Finally, plugging this current into the equation for the energy stored in the inductor

$$U_L = \frac{1}{2} L I^2 = \boxed{2.16\ J}$$