Center for Academic Resources in Engineering (CARE)
Mid-Semester Review Session

Phys211 – University Physics: Mechanics

Midterm 3 Worksheet Solutions

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Sun. 11/16, 7:00-8:50pm Lucy, Teddy, Diego

Session 2: Mon. 11/17, 4:00-5:50pm Alex, Lucas, Andy

Can't make it to a session? Here's our schedule by course:

`https://care.grainger.illinois.edu/tutoring/schedule-by-subject`

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

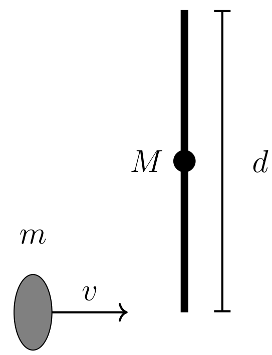
1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/847
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. A wad of gum having mass $m = 0.2$ kg is thrown with speed v = 8 m/s at a perpendicular bar with length d = 1.4 m and mass M. The bar is initially at rest on a table but can rotate freely about a pivot at its center. The gum sticks to the end of the bar and the angular speed of the bar just after the collision is measured to be ω = 3 rad/s.

   Assume that the wad of gum is a point particle and assume that the pivot is frictionless. (You do not have to worry about gravity in this problem)



(a) What is the magnitude of the angular momentum of the gum with respect to the pivot before it collides with the bar?

(b) What is the angular momentum of the gum with respect to the pivot after it collides with the bar?

(c) What is the mass of the bar?

(a)

$$\vec{L} = \vec{r} \times \vec{p}$$

The gum's momentum and the rod are perpendicular so the cross product becomes multiplication

$$L = rmv$$

$$r = \frac{d}{2} = 0.7 \text{ m}, m = 0.2 \text{ kg}, v = 8 \text{ m/s}$$

$$\boxed{L_i = 1.1 \text{ kgm}^2\text{s}^{-1}}$$

(b) Since we are looking just at the gum we can ignore the contributions due to the rod

$$L_{gum} = mr^2\omega$$

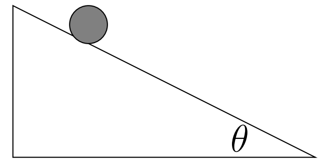
$$\boxed{L_{gum} = .294 \text{ kgm}^2\text{s}^{-1}}$$

(c) By conservation of Angular Momentum we know $L_i = L_f$ and since there are no external torques in this problem we can use this relation. Here L_i is the value we calculated in part (a) and L_{gum} is the value we calculated in part (b)

$$L_i = L_{gum} + L_{rod}$$
$$L_i = L_{gum} + I_{rod}\omega$$
$$L_i = L_{gum} + \frac{1}{12}ML^2\omega$$
$$1.1 = .294 + .49M$$

$$\boxed{M = 1.685 \text{ kg}}$$

2. A solid cylinder ($I = \frac{1}{2}MR^2$) and a solid sphere ($I = \frac{2}{5}MR^2$), of the same radius and mass, roll down an incline of angle $\theta = 30°$. They both start from rest at a distance $D = 2$ meters up the incline, as shown in the diagram, and roll without slipping to the bottom of the incline.



(i) **True or False**
The initial gravitational potential energy of the two objects is partially lost in work done overcoming frictional forces as the objects roll down the incline.

(ii) The ratio of change in translational kinetic energy ($\frac{1}{2}MV^2$) between the top and the bottom of the incline compared to the change in rotational kinetic energy is:

A) Larger for the solid sphere than it is for the solid cylinder
B) The same for both objects
C) Smaller for the solid sphere than it is for the solid cylinder

(iii) What is the value of the ratio of the velocities of the sphere and the cylinder $v_{sphere}/v_{cylinder}$ at the bottom of the incline?

(i) The initial gravitational potential energy is converted into translational and rotational kinetic energy; because they are rolling without slipping, there is no energy lost due to static friction.

$$\boxed{\text{False}}$$

(ii) Let η be the coefficient before a shape's moment of inertia

$$\text{Ratio} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}I\omega^2}$$

Use kinematic relation

$$\omega = \frac{v}{r}$$

$$\frac{\frac{1}{2}mv^2}{\frac{1}{2}I(\frac{v}{r})^2} = \frac{mr^2}{I} = \frac{mr^2}{\eta mr^2} = \frac{1}{\eta}$$

For Sphere: $\eta = \frac{2}{5}$
For Cylinder: $\eta = \frac{1}{2}$

$$\boxed{\text{Ratio}_{sphere} = \frac{5}{2} \text{ and Ratio}_{cylinder} = 2}$$

Interpreting this, these ratios indicate that for every joule of rotational energy, there are $\frac{1}{\eta}$ joules of translational energy given to the object.

(iii) For the Cylinder:

For the Sphere:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\Big(\frac{1}{2}mR^2\Big)\omega^2$$
$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$
$$v^2 = \frac{4}{3}gh$$
$$v = \sqrt{\frac{4}{3}gh}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\Big(\frac{2}{5}mR^2\Big)\omega^2$$
$$mgh = \frac{1}{2}mv^2 + \frac{1}{5}mv^2$$
$$v^2 = \frac{10}{7}gh$$
$$v = \sqrt{\frac{10}{7}gh}$$

$$\boxed{v_{sphere}/v_{cylinder} = \sqrt{\frac{10}{7}gh}\Big/\sqrt{\frac{4}{3}gh} = \sqrt{\frac{15}{14}}}$$

3. A yo-yo begins falling under the influence of gravity at $t = 0$, while the free end of the string is being held steady by a Physics 211 student. The string is wound on the spool of the yo-yo with a constant radius r. The mass of the yo-yo is m and its moment of inertia is I.

(i) What is the magnitude of the downward acceleration a of the yo-yo?

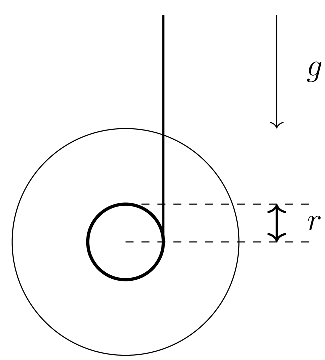
A) $a = g\frac{I}{mr}$

B) $a = g\frac{mr^2}{I}$

C) $a = g(\frac{I}{mr^2})$ - 1

D) $a = \frac{g}{(1-\frac{I}{mr^2})}$

E) $a = g + I - mr^2$



(ii) At some later time, t, while it is still traveling downwards, what is the angular velocity?

i) Let T be the tension in the string, and let downwards be the negative direction.

Torque Equation:

$$\tau = I\alpha = r \times F$$
$$I\alpha = Tr$$
$$\alpha = \frac{a}{r}$$
$$T = I\frac{a}{r^2}$$

Force Equation:

$$\Sigma F = ma$$
$$T - mg = ma$$
$$mg = T - ma$$

$$mg = I\frac{a}{r^2} - ma$$
$$mg = a(\frac{I}{r^2} - m)$$
$$a = \frac{mg}{(\frac{I}{r^2} - m)}$$

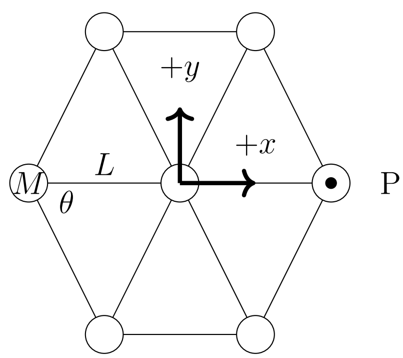
$$\boxed{a = \frac{g}{(\frac{I}{mr^2} - 1)}}$$

ii)

$$v_f = v_i + at$$
$$v_f = \frac{g}{(1 - \frac{I}{mr^2})}t$$
$$v = \omega r$$

$$\boxed{\omega = \frac{g}{(\frac{I}{mr^2} - 1)}\frac{t}{r}}$$

4. Seven identical point particles of mass M are arranged in the x-y plane, with one at the origin and the other six equally spaced into a hexagon as shown. The particles are connected into a rigid assembly by twelve identical massless rods of length L. Would the moment of inertia for rotations through point P be larger, smaller, or the same compared to a moment of inertia for rotations passing through the middle particle?



A) Larger

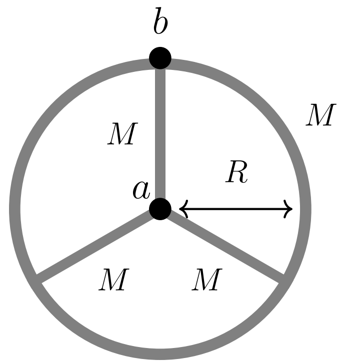
B) Smaller

C) The Same

The answer is **(A)**. The moment of inertia for rotation through the center of mass will always be the smallest possible moment of inertia. The center of mass in this case is the middle particle, so a rotation around the z-axis through one of the outer points will always be larger. This can be seen using the parallel axis theorem

$$I_{parallel} = I_{CM} + ML^2$$

5. A wheel is made by combining a hoop of radius R and mass M with three spokes, each a thin rod of length R and mass M



What is the moment of inertia of the wheel for rotations through the point labeled b in the diagram?

First we need to find the total moment of intertia of the whole system:

$$I_{total} = \Sigma I_{objects}$$

First we need to find the moment about the center of mass. The center of mass for the hoop is about point a, and this is at the ends of the 3 rods

$$\begin{aligned} I_{hoop} &= MR^2 \\ I_{rod,end} &= \frac{1}{3}MR^2 \\ I_{CM} &= I_{hoop} + 3I_{rod} \\ I_{CM} &= MR^2 + 3\left(\frac{1}{3}\right)MR^2 \\ I_{CM} &= 2MR^2 \end{aligned}$$

Now we can use the parallel axis theorem to find I_b

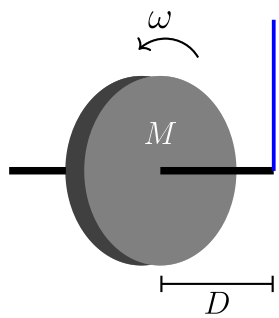
$$I_b = I_{CM} + MR^2$$

When we use the parallel axis theorem, the MR^2 term assumes we're using the *total* mass of the whole system, which in this case is $4M$

$$I_b = 2MR^2 + 4MR^2$$

$$\boxed{I_b = 6MR^2}$$

6. A gyroscope made from a solid disk of mass $M = 6$ kg and radius R hangs from a vertical rope attached to the ceiling. The disk spins with angular velocity $\omega = 21$ rad/s around a horizontal axle through its center in the direction shown by the arrow, and the rope is attached to one end of this axle at a distance D from the center of mass of the disk.



The angular momentum of the spinning disk is $L = 124$ kgm^2/s. The time it takes the gyroscope to make one complete revolution in the horizontal plane (its precession period) is 22.2 seconds.

i) What is the moment of inertia of the spinning disk?

ii) What is the distance D between the gyroscope and the rope?

iii) Suppose the same gyroscope is moved to the surface of a new planet where the acceleration of gravity on the surface is smaller than it is on the Earth. How does the precession period change?

   A) It increases
   B) It decreases
   C) It stays the same

iv) **True or False**, the Angular momentum vector 'follows' the torque vector as the gyroscope precesses?

v) Which way does the disk precess?

   A) It does not precess
   B) Clockwise as seen from above (looking down the rope)
   C) Counterclockwise as seen from above (looking down the rope)

i)

$$L = I\omega$$
$$I = \frac{L}{\omega}$$
$$\boxed{I = 5.905 \text{ kgm}^2\text{rad}^{-1}}$$

ii)

$$\Omega_{precession} = \frac{\tau}{L} = \frac{2\pi}{T}$$
$$\tau = Dmg$$
$$\frac{Dmg}{L} = \frac{2\pi}{T}$$
$$D = \frac{2\pi L}{Tmg}$$
$$\boxed{D = 0.596 \text{ m}}$$

iii)

$$\text{Since D } = \left(\frac{2\pi L}{Tmg}\right) \text{ is constant, T} \propto \frac{1}{g}$$

Therefore, as g decreases, T increases

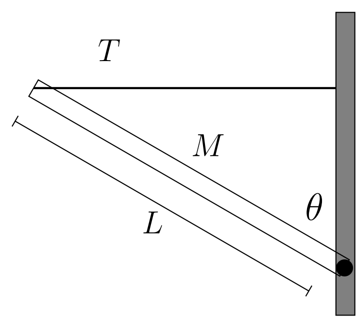
iv) $\boxed{\text{True}}$

This is because of the fact that

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

v) Answer based on the solution to (d) and the diagram on the previous page. The angular momentum vector points rightward while the torque from gravity vector points out of the page. The direction of precession is downwards so the wheel will rotate into the page (B) as this is the direction of the angular momentum taking into consideration the torque.

7. A beam of mass $M = 5$ kg and length $L = 3$ m is attached to a vertical wall by a hinge at its lower end and a horizontal massless wire at its top end, as shown in the diagram. The angle between the wall and the beam is $\theta = 69°$.



(i) What is the tension in the wire? (Assume the axis of rotation is the hinge.)

(ii) If the attachment point of the wire on the wall were moved upward by half a meter, but M, L and θ were the same as in the above question, how would the tension in the wire change? (Note that a longer wire is required to move the attachment point this way)

A) It would decrease

B) It would increase

C) It would stay the same

(iii) Now suppose the wire breaks and the beam starts to rotate around the hinge. What is α_0, the magnitude of the angular acceleration of the beam about the hinge immediately after the wire breaks?

(iv) If the beam were shorter, but M and θ were the same as above, how would the answer to the above question change?

A) The magnitude of α_0 would be bigger

B) The magnitude of α_0 would be smaller

C) The magnitude of α_0 would be the same

(i)

$$\tau_T + \tau_{mg} = 0$$

$$\tau_T = -TL\cos(\theta)$$
$$\tau_{Mg} = \frac{1}{2}LMg\sin(\theta)$$
$$TL\cos(\theta) = \frac{1}{2}LMg\sin(\theta)$$
$$T = \frac{1}{2}Mg\tan(\theta)$$

$$\boxed{T = 63.9\text{ N}}$$

(ii) The answer is **(A)**. Point 1: As the string was attached higher and higher on the wall the component of the torque due to tension is going to have to remain the same in order to balance the beam.

Point 2: Our equation for the torque, $\tau = r \times F$, states that as we make the force "more perpendicular" to our axis we increase the torque.

Here we know that we don't need to increase the torque (by point 1) and in order to keep the torque constant we need to either decrease r or F (by point 2). The problem states that L is the same as the previous problem, thus the force needs to be decreased.

(iii)

$$\tau_{Mg} = \frac{1}{2}LMg\sin(\theta) = I\alpha_0$$

$$I_{rod} = \frac{1}{3}ML^2$$
$$\frac{L}{2}Mg\sin(\theta) = \frac{1}{3}ML^2\alpha_0$$
$$\frac{1}{2}g\sin(\theta) = \frac{1}{3}L\alpha_0$$

$$\boxed{\alpha_0 = \frac{3g\sin(\theta)}{2L} = 4.58\text{ rad}}$$

(iv) The answer is **(A)**.

$$\alpha_0 = \frac{3g\sin(\theta)}{2L}$$
$$\sin(\theta)\text{ is a constant}$$
$$\alpha_0 \propto \frac{1}{L}\text{ so as }L\text{ gets smaller, }\alpha_0\text{ gets bigger}$$