ILLINOIS

Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

MATH 257 – Linear Algebra with Computational Applications

Midterm 3 Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Nov. 12, 7 - 9pm Ryan, Rishi, Danielle

Session 2: Nov. 13, 7 - 9pm Alice, Ryan, JD

Can't make it to a session? Here's our schedule by course:

`https://care.grainger.illinois.edu/tutoring/schedule-by-subject`

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/955
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. Compute the least squares solution of

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \hat{\boldsymbol{x}} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

a)
$$\begin{bmatrix} \frac{4}{3} \\ \frac{2}{3} \\ \frac{1}{2} \end{bmatrix}$$

b)
$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

c)
$$\begin{bmatrix} 1 \\ \frac{1}{3} \end{bmatrix}$$

d)
$$\begin{bmatrix} -\frac{1}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$$

e) None of these

2. Suppose $\boldsymbol{B}$ is a 3×3 matrix and $\det(\boldsymbol{B}) = 5$. Determine the following quantities:

a) $\det(2\boldsymbol{B})$

b) $\det(\boldsymbol{B}^T\boldsymbol{B})$

c) $\det(\boldsymbol{B}^{-1})$

3. Every week at Monster's University the rate of students attending Scaring 101 follows this pattern: 75% of students who attended the 1PM lecture the week before come to class, 15% switch to the 10AM section, and 10% don't attend lecture at all. On the other hand, 20% of the students who went to the 10AM section the week before come back to class, 60% switch to the 1PM lecture, and 20% skip class. Finally, 65% of students who skipped class attend the 1PM lecture, 30% come to the 10AM section, and 5% skip once again.

a) Set up a Markov matrix to represent the attendance of Scaring 101 at Monster's University.

b) If on the first week of the semester, 50% of students attend the 1PM lecture and 50% attend the 10AM lecture, how would you calculate the lecture attendance after 3 weeks?

c) Set up, but do not solve the matrix for finding the steady state probability vector.

4. Given the following matrix:

$$A = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & -3 & 17 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find the characteristic polynomial for the matrix.

a) Find the eigenvalues of A.

b) Find the eigenvector, algebraic multiplicity, and geometric multiplicity for the largest eigenvalue of A.

5. Given the matrix

$$A = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}$$

a) Calculate the diagonalization of A, PDP^{-1}.

b) Calculate e^{At}.

c) Suppose the following is a system of linear ordinary differential equations:

$$\frac{du_1}{dt} = 6u_1 - u_2$$
$$\frac{du_2}{dt} = 2u_1 + 3u_2$$

What is the general solution? (Hint: use your work from the previous part).

d) Find the particular solution for the previous system if the initial condition is $\mathbf{u}(0) = \begin{bmatrix} 13 \\ 21 \end{bmatrix}$.

6. Given an unknown 2x2 matrix A where det(A) = 5, and a 3x3 matrix

$$B = \begin{bmatrix} -2 & 3 & 3 \\ -4 & 0 & -6 \\ 5 & -1 & 8 \end{bmatrix}$$

a) What can you conclude about rank(A) from the information given above?

b) Calculate the determinant of B using cofactor expansion across the second column.

7. For the following statements, determine if they are true or false and why.

a) There exists a square matrix with no eigenvectors.

b) If a matrix A has a eigenvector $\mathbf{v}$, then it has infinitely many eigenvectors.

c) The $\mathbf{0}$ vector is a possible eigenvector.

8. Given a set of 5 data points that fits the function $\boldsymbol{y} = \beta_1 + \beta_2\boldsymbol{x} - \beta_3\boldsymbol{x}^2 + \beta_4 tan(\boldsymbol{x})$, what would the design matrix for a linear regression look like?