Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

MATH 257 – Linear Algebra with Computational Applications

Midterm 2 Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: October 16th, 6:30-8PM Alice, Carlos, JD

Session 2: October 17th, 7-8:30PM Danielle, Rishi

Can't make it to a session? Here's our schedule by course:

`https://care.grainger.illinois.edu/tutoring/schedule-by-subject`

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/955
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. Given

$$A = \begin{bmatrix} 4 & -3 & 1 \\ -8 & 14 & -1 \\ 4 & 13 & 4 \end{bmatrix} \qquad \boldsymbol{b} = \begin{bmatrix} 3 \\ 22 \\ 63 \end{bmatrix}$$

a) Compute REF(A) without swapping any rows. Write down all row operations used in the process

b) Express REF(A) as the product of elementary matrices and A (Hint: Use your process from a)

c) Express A as the product of a lower triangular matrix and an upper triangular matrix A = LU

d) Use the LU decomposition of A to solve the linear system $A\boldsymbol{x} = \boldsymbol{b}$

2. Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be defined:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 4 & -1 & 2 & 4 \\ 5 & 4 & 0 & 5 & 5 \\ 8 & 4 & 2 & 6 & 2 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

For the following statements, determine whether they are true or false. If they are false, explain why.

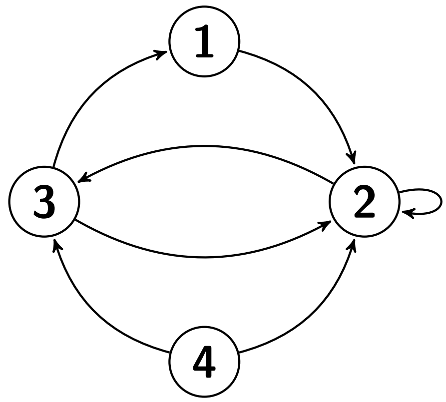
a) Null($\boldsymbol{A}$) is a subspace of $\mathbb{R}^5$

b) $\dim \text{Col}(\boldsymbol{B}) = \dim \text{Null}(\boldsymbol{B})$

c) The vector $\begin{bmatrix} -1 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix}$ spans the nullspace of $\boldsymbol{A}$, and is a solution to the vector equation $\text{A}\boldsymbol{x} = \mathbf{0}$

d) If the row operation $R_4 \to R_4 + R_1$ is performed on matrix $\boldsymbol{B}$, the column space of the resulting matrix is the same as $\text{Col}(\boldsymbol{B})$

3. What is the adjacency matrix for the following graph?



4. For the set S $= \{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\}$, state what is necessary for it to be a basis of $\mathbb{R}^3$

a) For the same set S, state what is necessary to make it a basis of $\mathbb{R}^2$

5. The vectors

$$\boldsymbol{u_1} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \boldsymbol{u_2} = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}, \boldsymbol{u_3} = \begin{bmatrix} -8 \\ 12 \\ -4 \end{bmatrix}, \boldsymbol{u_4} = \begin{bmatrix} 1 \\ 37 \\ -17 \end{bmatrix}, \boldsymbol{u_5} = \begin{bmatrix} -3 \\ -5 \\ 8 \end{bmatrix}$$

span $\mathbb{R}^3$. Find the minimally spanning subset of set A $= \{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}, \boldsymbol{u_4}, \boldsymbol{u_5}\}$.
Hint: the minimally spanning subset of $\mathbb{R}^3$ should have three vectors.

6. Let $S \subseteq M_{3\times 3}(\mathbb{R})$ be all 3×3 symmetric matrices of real entries. Find its dimension.

a) Find a basis of A. (Hint: Any matrix $A \in S$ is symmetric)

7. For the following statements, state whether they are true or false and why.

a) True/False : A vector space cannot have more than one basis

b) True/False : If a vector space has a finite basis, then the number of vectors in each minimally spanning basis is the same.

c) True/False : If S generates V (span(S) = V), then every vector in V can be written as a linear combination of vectors in S in only one way.

8. Let A be defined as $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$. Compute A^{-1} and A^T. What type of matrix is A?

9. Consider the following basis: $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ -3 \\ -3 \end{bmatrix}. \begin{bmatrix} -4 \\ 2 \\ 4 \end{bmatrix} \right\}$ and standard basis $\mathcal{E}$

and $\quad \boldsymbol{V_{\mathcal{E}}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \boldsymbol{W_{\mathcal{E}}} = \begin{bmatrix} -48 \\ 26 \\ 48 \end{bmatrix} \quad \boldsymbol{U_{\mathcal{E}}} = \begin{bmatrix} -23 \\ 16 \\ 26 \end{bmatrix} \quad \boldsymbol{X_{\mathcal{B}}} = \begin{bmatrix} 0 \\ \frac{1}{9} \\ 0 \end{bmatrix} \quad \boldsymbol{Y_{\mathcal{B}}} = \begin{bmatrix} -3 \\ -2 \\ \frac{1}{2} \end{bmatrix}$

Compute $\boldsymbol{V_{\mathcal{B}}},\ \boldsymbol{W_{\mathcal{B}}},\ \boldsymbol{U_{\mathcal{B}}},\ \boldsymbol{X_{\mathcal{E}}},\ \boldsymbol{Y_{\mathcal{E}}}$