Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

Math 231 – Calculus II

Midterm 1 Worksheet Solutions

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: September 22nd, 3:00-5:00pm Maya and Sofi

Can't make it to a session? Here's our schedule by course:

`https://care.grainger.illinois.edu/tutoring/schedule-by-subject`

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/844
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. Evaluate the following Integral:

$$\int \sin(3x)e^x \, dx$$

To do this integral we will need to use integration by parts twice. We start with:

$u = \sin(3x)$, $dv = e^x$ and $du = 3\cos(3x)$, $v = e^x$.

Using the integration by parts formula, $\int u \, dv = uv - \int v \, du$, we now write:

$\int \sin(3x)e^x \, dx = e^x \sin(3x) - \int 3e^x \cos(3x) \, dx$

Now we will set up integration by parts for the new integral, $\int 3e^x \cos(3x) \, dx$,

$u = 3\cos(3x)$, $dv = e^x$ and $du = -9\sin(3x)$, $v = e^x$.

Now we can write: $\int \sin(3x)e^x \, dx = e^x \sin(3x) - \Big(3e^x \cos(3x) - \int -9\sin(3x)e^x \, dx\Big)$

If we bring out the factor of 9 (the negatives will cancel) we see that we have our original integral on the left hand side. We can add it to the left hand side.

$10 \int \sin(3x)e^x \, dx = e^x \sin(3x) - 3e^x \cos(3x)$

Now we simply divide both sides by 10 and the integral is solved:

$\boxed{\int \sin(3x)e^x \, dx = \frac{1}{10}\Big(e^x \sin(3x) - 3e^x \cos(3x)\Big) + C}$

2. Evaluate the following Integral:

$$\int \frac{4}{x^2 + 5x - 14} dx$$

To do this integral, we will need to use partial fractions, but first, we will need to factor the denominator and split it into 2 separate fractions.

$$\frac{4}{x^2 + 5x - 14} = \frac{4}{(x+7)(x-2)} = \frac{A}{(x+7)} + \frac{B}{(x-2)}$$

Multiply by the common denominator and solve for A and B.

$A(x-2)+B(x+7)=4$

$Ax+Bx=0; -2A+7B=4$

$A=-4/9; B=4/9$

We can now plug it back into the original integral and solve.

$$\int \frac{4}{x^2+5x-14}dx = \int \frac{\frac{-4}{9}}{x+7} + \frac{\frac{4}{9}}{x-2}dx$$

$\boxed{\frac{4}{9}\ln|x-2| - \frac{4}{9}\ln|x+7| + C}$

3. Evaluate the following Integral:

$$\int t^7 \sin\left(2t^4\right)\, dt$$

The selection of u and dv for this problem may be misleading at first! In order to integrate the $\sin(2t^4)$ term we look for a t^3 to do a u-sub. We can create this term by selecting u and dv carefully:

$u=t^4$, $dv=t^3\sin(2t^4)$, which leads to $du=4t^3$ and $v=-\frac{1}{8}\cos(2t^4)$

Using the integration by parts formula we can now write:

$\int t^7\sin(2t^4)\, dt = -\frac{1}{8}t^4\cos(2t^4) + \frac{1}{2}\int t^3 cos(2t^4)\, dt$

The resulting integral requires only a simple u-sub (using the letter w for clarity):

$w=2t^4$, $dw=8t^3dt$
$\frac{1}{16}\int\cos(w)\, dw = \frac{1}{16}\sin(w)$

Undoing the u-sub and plugging everything back into the original expression, the integral is solved:

$\boxed{\int t^7\sin(2t^4)\, dt = -\frac{1}{8}t^4\cos(2t^4) + \frac{1}{16}\sin(2t^4)}$

4. Evaluate the following Integral:

$$\int \frac{e^{2x}+13x}{e^{2x}+e^x-6}dx$$

For this one you have to manipulate your u-sub in order to do integration by parts.

$u = e^x$, $u^2 = (e^x)^2 = e^{2x}$, $du = e^x dx$

Substitute your values:

$\int \frac{u+13}{u^2+u-6} du$

Factor the denominator and put it into the proper partial fraction decomposition form:

$\int \frac{u+13}{(u+3)(u-2)} du$

$\frac{A}{(u+3)} + \frac{B}{(u-2)}$

Find your A and B values:

$A = -2$, $B = 3$

$\int \frac{-2}{u+3} + \frac{3}{u-2} du$

Now integrate! Don't forget to put your answer back in terms of x

$\int \frac{-2}{u+3} du + \int \frac{3}{u-2} du$

$= -2 \ln |u+3| + 3 \ln |u-2|$

$= -2 \ln |e^x + 3| + 3 \ln |e^x - 2| + c$

5. Evaluate the following Integral:

$$\int_0^{\pi/4} \tan^5(x) \sec^4(x) dx$$

$u = \tan(x)$, $du = \sec^2(x) dx$.

$= \int_0^1 u^5 \sec^2(x) du$

$= \int_0^1 u^5 (1 + \tan^2(x)) du$

$= \int_0^1 u^5 (1 + u^2) du$

$= \int_0^1 u^5 du + \int_0^1 u^7 du$

$= \int_0^{\frac{\pi}{4}} \tan^5(x)dx + \int_0^{\frac{\pi}{4}} \tan^7(x)dx$

$= \frac{1}{6}\tan^6(x)|_0^{\frac{\pi}{4}} + \frac{1}{8}\tan^8(x)|_0^{\frac{\pi}{4}}$

$= \frac{1}{6} + \frac{1}{8} = \boxed{\frac{7}{24}}$

6. Evaluate the following Integral:

$$\int \frac{1}{\sqrt{25+x^2}}dx$$

$x = 5\tan(\theta)$, $dx = 5\sec^2(\theta)$.

$= \int \frac{5\sec^2(\theta)d\theta}{\sqrt{25+5\tan(\theta)}}$

$= \int \frac{5\sec^2(\theta)d\theta}{\sqrt{25\sec^2(\theta)}}$

$= \int \sec(\theta)d\theta$

$= \ln|\sec(\theta) + \tan(\theta)| + C$

$= \boxed{\ln|\frac{\sqrt{x^2+25)}}{5} + \frac{x}{5}| + C}$

7. Evaluate the following Integral:

$$\int_0^4 \frac{1}{(x^2+16)^{\frac{3}{2}}}dx$$

$x = 4\tan(\theta)$, $dx = 4\sec^2(\theta)$.

$= \int_0^4 \frac{4\sec^2(\theta)d\theta}{4^3\sec^3(\theta)}$

$= \frac{1}{16}\int_0^4 \frac{1}{\sec(\theta)}d\theta$

$= \frac{1}{16}\int_0^4 \cos(\theta)d\theta$

$= \frac{1}{16}\sin\theta|_0^4$

$= \frac{1}{16}\frac{x}{\sqrt{x^2+16}}|_0^4 = \frac{1}{16}(\frac{4}{\sqrt{32}} - 0) = \boxed{\dfrac{1}{16\sqrt{2}}}$

8. Evaluate the following Integral:

$$\int \frac{2x^2 - x + 4}{x^3 + 4x} dx$$

$x^3 + 4x = x(x^2 + 4x)$

$\frac{2x^2-x+4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$

$2x^2 - x + 4 = A(x^2 + 4) + (Bx + C)x$

$2x^2 - x + 4 = (A + B)x^2 + Cx + 4A$

$A + B = 2$, $C = -1$, and $4A = 4$

$A = 1, B = 1, C = -1$

$\int \frac{2x^2-x+4}{x^3+4x} dx = \int \frac{1}{x} + \frac{x-1}{x^2+4} dx$

$\int \frac{1}{x} + \frac{x}{x^2+4} - \frac{1}{x^2+4} dx = \boxed{\ln|x| + \dfrac{1}{2}\ln\big(x^2 + 4\big) - \dfrac{1}{2}\tan^{-1}(\dfrac{x}{2}) + C}$

For the second term, use U-sub where $u = x^2 + 4$. For the third term in the integral, use trig integral for tangent.

9. Evaluate the following Integral:

$$\int_0^{16} \frac{\sqrt{x}}{x+1} dx$$

Let $u = \sqrt{x}$, let $du = \frac{dx}{2\sqrt{x}} \rightarrow 2udu = dx$.

$= \int_0^4 \frac{2u^2du}{u^2+1}$

$= 2\int_0^4 \frac{u^2+1-1}{u^2+1} du$

$= 2\int_0^4 1 - \frac{1}{u^2+1} du$

$= \boxed{2(u - \tan^{-1}(u))|_0^4 = 8 - 2\tan^{-1}(4)}$

10. Evaluate the following Integral:

$$\int \arctan\left(\frac{2}{x}\right)dx$$

Let $u = \tan^{-1}(\frac{2}{x})$**,** $du = \frac{-2}{x^2}(\frac{1}{1+(\frac{2}{x})^2})dx$**,** $dv = dx$**, and** $v = x$**.**

$= \int \tan^{-1}(\frac{2}{x}) = x\tan^{-1}(\frac{2}{x}) + 2\int \frac{x}{x^2+4}dx$

Let $u = x^2 + 4$ **and** $du = 2xdx$**.**

$= \boxed{\int \tan^{-1}(\frac{2}{x})dx = x\tan^{-1}(\frac{2}{x}) + \ln|x^2+4| + C}$