



## Center for Academic Resources in Engineering (CARE) Peer Exam Review Session

Math 285 – Intro Differential Equations

### Midterm 1 Worksheet

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*The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.*

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Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: September 23rd from 6:00-8:00pm in 4035 CIF Hayden and Regina

Session 2: September 24th from 4:30-6:30pm in 1035 CIF Charlie and Ben

Can't make it to a session? Here's our schedule by course:

<https://care.grainger.illinois.edu/tutoring/schedule-by-subject>

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: <https://queue.illinois.edu/q/queue/846>
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

**Please be sure to follow the above steps to add yourself to the Queue.**

Good luck with your exam!

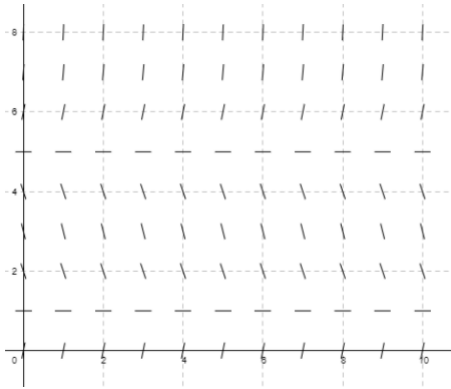
1. Consider the following differential equation and initial condition:

$$(16 - t^2)y' + t^3y = \cos\left(\frac{t}{2}\right) \quad y(1) = -5$$

On what interval is the unique solution certain to exist?

- A)  $(0, 2\pi)$
- B)  $(-2\pi, 0)$
- C)  $(-4, 0)$
- D)  $(-4, 4)$
- E)  $(-\infty, -4)$

2. Which equation produces the direction field below?



A)  $\frac{dy}{dx} = (x - 1)(x - 5)$

B)  $\frac{dy}{dx} = xy$

C)  $\frac{dy}{dx} = (y - 1)(y - 5)$

D)  $\frac{dy}{dx} = (y + 1)(y + 5)$

E)  $\frac{dy}{dx} = y^2$

3. Which of the following equations are linear? Note:  $u_*$  denotes a first order partial derivative and  $u_{**}$  is a second order partial derivative.

$$(I) \frac{d^2y}{dt^2} + e^y = 6t + 5$$

$$(II) (2t^3 + 6)\frac{d^5y}{dt^5} - \frac{d^3y}{dt^3} + 4y = t \cos(t - 1)$$

$$(III) u_y = uu_{xx} - u_{xy}$$

$$(IV) u_{xx} + xu_{xt} + t^2u_{tt} = \sin(x + 2t)$$

- A) (I), (II), (III)  
B) (II) and (IV)  
C) (I) and (IV)  
D) (II)  
E) (IV)

4. What is the order of the following differential equation?

$$\cot(y)y''' + (t^2 + t + 9)y' - \ln(xy^2)y + 6y^9 = \sin(3t^5 + 1)$$

- A) 9  
B) 1  
C) 3  
D) 5  
E) None

5. Consider the following autonomous equation:  $y' = (y^2 - 9)(5 - y)$

List the equilibrium solutions and classify them as stable, semi-stable, or unstable.

6. Prove that the following ODE is exact and find the general solution using the exact equations method with  $\psi$ :  $(3x^2 - 3y^2)\frac{dy}{dx} + (3x^2 + 6xy) = 0$

7. Solve  $x\left(\frac{dy}{dx}\right) = 2y + x$

8. Find the solution to the following initial value problem

$$(x^4 + 1)\left(\frac{dy}{dx}\right) = 2x^3y^2 \quad y(0) = \left(\frac{3}{2}\right)$$