



MATH 285

Midterm 1 Review

CARE

Disclaimer

- These slides were prepared by tutors that have taken Math 285. We believe that the concepts covered in these slides could be covered in your exam.
- HOWEVER, these slides are NOT comprehensive and may not include all topics covered in your exam. These slides should not be the only material you study.
- While the slides cover general steps and procedures for how to solve certain types of problems, there will be exceptions to these steps. Use the steps as a general guide for how to start a problem but they may not work in all cases.



Topics

- I. Classifying Differential Equations
- II. Slope Fields
- III. Existence and Uniqueness
- IV. Autonomous Equations
- V. Solving Methods:
 - I. Separable
 - II. Exact
 - III. Integrating Factor

Differential Equations

- “A differential equation is any relationship between a function (usually denoted $y(t)$) and its derivatives up to some order.”
- **Slope Fields:** Help visually model a differential equation
 - Lines parallel to the derivative at each point
 - Can show overall direction and shape of the solution, as well as equilibrium values

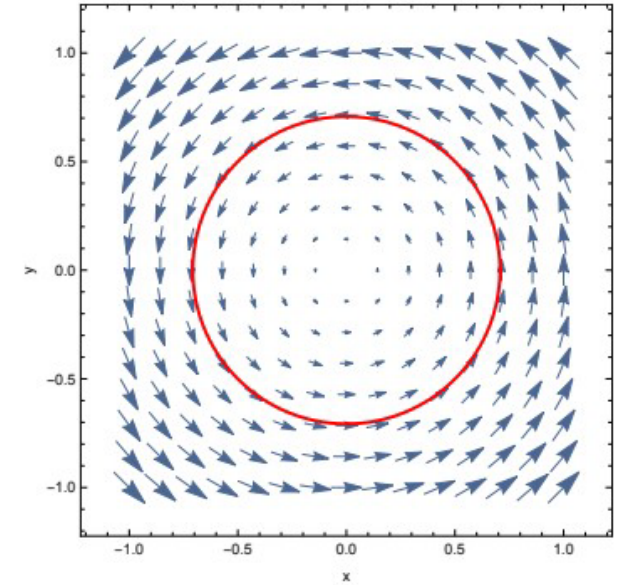



Figure 1.4: A slope field for $\frac{dy}{dt} = -\frac{t}{y}$ (blue) together with a solution curve (red).

Differential Equations. Bronski J., Manfroi A., Figure 1.4

Classifications

2nd order  **Linear** **Ordinary**

$$\frac{d^2 y}{dt^2} + \sin(t) \frac{dy}{dt} + 15y = e^t$$

Ordinary vs Partial

- ODE's involve only standard derivatives
- PDE's involve partial derivatives

Linear vs Nonlinear

- Linear differential equations only have linear terms of the function and its derivatives
- Nonlinear equations are everything else

Order

- The order of a differential equation is the degree of the highest derivative it contains

Existence and Uniqueness Theorem

$$\frac{dy}{dt} = f(y, t) \quad y(t_0) = y_0$$

- A solution to the differential equation is **guaranteed to exist** in the **interval** in which the **first derivative is continuous around the initial value**
- That solution is **guaranteed to be** unique if $\frac{\partial f(y, t)}{\partial y}$ is also **continuous around the initial value**

Autonomous Equations

- Autonomous equation: **does not explicitly involve independent variable**

$$\frac{dy}{dt} = f(y)$$

- Draw a **phase line**, identify points where the **derivative is 0**, and then **identify equilibria**
- Types of equilibria:
 - **Stable**: nearby points converge to the equilibrium
 - **Semi-stable**: points converge from one direction
 - **Unstable**: points diverge away from the equilibrium

Separable Equations

- Separable Equations can be written as:

$$\frac{dy}{dt} = f(y)g(t)$$

- If your equation is separable, it can be solved **directly through integration**:

$$\int \frac{dy}{f(y)} = \int g(t)dt + C$$

Exact Equations

- Exact equations have the form of:

$$N(x, y) \cdot y' + M(x, y) = 0$$

- An equation is exact if the **partial derivatives of the two coefficient terms are equal:**

$$\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$$

Solving Exact Equations

1. **Partially integrate** either N or M :

$$\int N \partial y \quad \text{or} \quad \int M \partial x$$

2. **Set equal to Ψ** + a constant of integration function:

$$\Psi = \int N \partial y + f(x) \quad \text{or} \quad \Psi = \int M \partial x + f(y)$$

3. Take the **derivative** with respect to the **opposite variable**:

$$\frac{\partial \Psi}{\partial x} \quad \text{or} \quad \frac{\partial \Psi}{\partial y}$$

4. **Set equal** to the **other term** you didn't integrate:

$$\frac{\partial \Psi}{\partial x} = M \quad \text{or} \quad \frac{\partial \Psi}{\partial y} = N$$

5. **Integrate** to solve for $f(x)$ or $f(y)$ and plug back into step 2

$$\int f'(x) dx \quad \text{or} \quad \int f'(y) dy$$

Exact Equation Example

Solve the following differential equation:

$$(5x^2y + 2x + 4) \frac{dy}{dt} + (5xy^2 + 2y + 7) = 0$$

Integrating Factor Method

1. Make sure your equation looks like:

$$\frac{dy}{dt} + p(t)y = q(t)$$

2. Calculate the **integrating factor**:

$$\mu(t) = e^{\int p(t)dt}$$

3. **Multiply the entire equation** by the integrating factor:

$$\mu(t) \frac{dy}{dt} + p(t)\mu(t)y = \mu(t)q(t)$$

4. Re-write the left-hand side as the **result of product rule**:

$$\frac{d}{dt}(\mu(t)y) = \mu(t)q(t)$$

5. **Integrate both sides** and rearrange to solve for $y(t)$

$$\mu(t)y = \int \mu(t)q(t)dt$$

Integrating Factor Example

Solve the following differential equation:

$$y' + 3y = 2$$

Thanks for
Coming!

