Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

Math 221 – Calculus I

Midterm 1 Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Sep. 22nd, 5:00pm - 6:30pm -Zac, Sophia

Session 2: Sep. 23rd, 5:00pm - 6:30pm -Meredith, Erik

Can't make it to a session? Here's our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/1056
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. Let $f(x) = \frac{2}{\sqrt{x-1}}$. Use the definition of a derivative as a limit to prove that $f'(x) = \frac{-1}{(x-1)^{3/2}}$. Show each step in your calculation and be sure to use proper terminology in each step of your proof.

2. Compute the following limit:

$$\lim_{x\to\infty} \frac{91\sqrt[8]{x}+3}{5-7\sqrt[8]{x}}$$

3. Write an equation for each horizontal asymptote on the graph of the following function. Use limits to justify your answer. We will learn L'Hopital's Rule and other shortcuts for obtaining limits later. For now, you are not allowed to use these approaches.

$$\frac{56e^{-5x}-30}{7e^{-5x}+10}$$

4. Compute the following limits:

(a)

$$\lim_{x\to 0} \frac{19x-5sin(x)}{2x}$$

(b)

$$\lim_{x\to 0} \frac{e^{6x}-1}{e^{3x}-1}$$

5. Determine whether the following statements are always true.

(a) A function which is continuous at point (a) must also be differentiable at (a).

(b) The limit $\lim_{x\to a} f(x)$ depends only on the values of the function near a, not $f(a)$ itself.

(c) The function $y = \frac{9x-63}{x^2+6x-91}$ has a vertical asymptote at $x = 7$.

6. A car starts from rest and travels along a straight highway for 30 minutes. At some point during the trip, the car reaches a speed of 60 miles per hour. The car's speed is a continuous function of time. Prove that there is a moment during the trip when the car's speed was exactly 30 miles per hour. (Hint: Use Intermediate Value Theorem)

7. Evaluate the following limits and write your answers in simplified form.

(a)
$$\lim_{x\to\sqrt{2}} \frac{120\arcsin\left(\frac{x}{2}\right)}{x^2+4}$$

(b)
$$\lim_{x\to\infty} \frac{13+5\sin(9e^{3x}+6)}{x^{10}}$$

8. Use implicit differentiation to find $\frac{dy}{dx}$ for the equation $xy + y^4 = x^3y$.

9. Compute the following derivatives:

a.)

$$y = \frac{x^2 + 1}{x - 1}$$

b.)

$$f(x) = sec^{-1}(x^4)$$

c.)

$$f(x) = \frac{3x}{5 - sec(x)}$$

d.)

$$y = \frac{cos(4x) - 1}{e^x x^3}$$