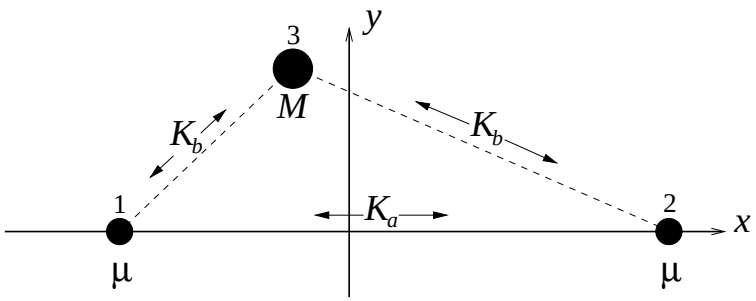
Q1



Consider a system of three particles moving in the x-y plane. Particles 1 and 2 have mass μ and are constrained to slide frictionlessly along the x-axis. Particle 3 has mass M and can move anywhere in the plane. Each particle is attracted to the others by forces directed along the line joining them, and with a magnitude that is proportional to their separation. The force between the two particles of mass μ has constant of proportionality K_a, while the force between either of the two particles of mass μ and the particle of mass M has constant of proportionality K_b. For example, with $\mathbf{r}_1 = (x_1, y_1)$, *etc.* the force on particle 2 due to particles 1 and 3 is

$$\mathbf{F}_{21} + \mathbf{F}_{23} = K_a(\mathbf{r}_1 - \mathbf{r}_2) + K_b(\mathbf{r}_3 - \mathbf{r}_2).$$

Assume also that the particles are able to pass by each other without colliding.

a) Write down a Lagrangian that encodes the motion of this system and write the corresponding equations of motion for the three particles.

b) Show that the evolution of the y-component of the position of the third particle is decoupled from the rest of the motion.

c) There are three remaining degree of freedom for the system and therefore three normal modes. Two of them should be easy to find. Describe these two normal modes and find their frequencies in terms of μ, M, K_a and K_b.

d) Obtain the remaining normal mode and find its frequency.