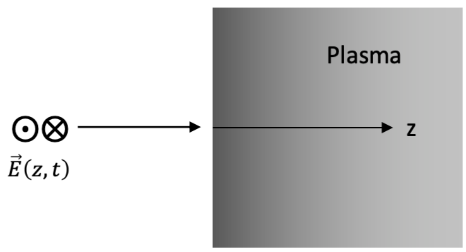
Q2 Consider a model of the ionosphere as an electrically neutral ideal gas of ions and electrons with electron number density n per cubic meter. A plane electromagnetic wave of angular frequency ω and polarized along the x-direction is traveling along the z-direction. It is incident on the plasma which fills the half-space $z \geq 0$.



Assume that the ions remain stationary and treat the electrons as a gas of mobile, non-interacting, classical particles with mass m and charge $-e$. Take the physical fields to be the real parts of complex quantities $\mathbf{E}$, $\mathbf{D}$, $\mathbf{B}$, $\mathbf{H}$ and $\mathbf{J}$ that are $\propto \exp\{i(kz - \omega t)\}$.

a) Find the frequency-dependent complex conductivity $\sigma(\omega)$ defined by $\mathbf{J} = \sigma(\omega)\mathbf{E}$.

b) Use your result from part (a) to derive a wave equation for $E_x(z,t)$. From it show that the dispersion equation for transverse waves in the plasma is $\omega(k) = \sqrt{a^2k^2 + \omega_p^2}$. You should find expressions for both a and the *plasma frequency* ω_p^2 in terms of n, e, m and other physical constants.

c) What are the boundary conditions on $\mathbf{E}$, $\mathbf{D}$, $\mathbf{B}$,and $\mathbf{H}$ at $z = 0$?

d) Use your expression for $\omega(k)$ and your boundary conditions from part (c) to compute the magnitude of the reflection coefficient $R = |E_{\text{reflected}}/E_{\text{incident}}|$ when $\omega \leq \omega_p$ and $\omega > \omega_p$.

e) Now consider oscillations of the plasma with the E field polarized *parallel* to the direction of the wave. Show that for such waves $\omega = \omega_p$ for all k. (You may use your previous result for the conductivity.)