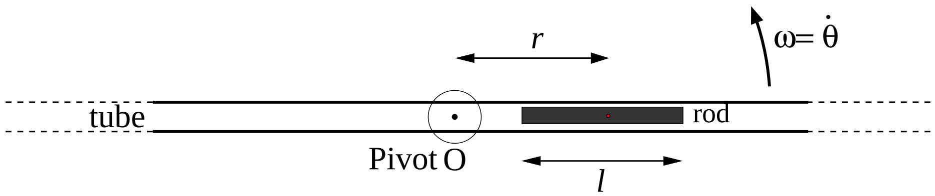
Q1 A long, thin tube of negligible mass is pivoted about its central point O so that it may rotate without friction in a horizontal plane. A thin rod of mass M and length l slides without friction in the tube.



a) Start by calculating the moment of inertia I of the thin rod about its center of mass.

b) Use your result from part (a) to find the Lagrangian for this system. Use coordinates in which r is the distance of the center of mass of the rod from the pivot point and θ is the angle through which the tube (and hence the rod) has rotated.

c) Find the two Euler-Lagrange equations that determine $r(t)$ and $\theta(t)$ for this system. Hint: One of the equations can be written as a conservation law $d(f(r,\dot{\theta}))/dt = 0$ for some quantity $f(r,\dot{\theta})$.

The rod is initially centered over the pivot (*ie.* $r = 0$) and the tube is rotating with angular velocity ω_0. We wish to show that this situation is unstable to small perturbations.

d) Show that for very small r the equation of motion is approximately $\ddot{r} = \Omega^2 r$, where Ω is real. Determine Ω and deduce that any initially small displacement from $r = 0$ will grow rapidly.

e) What are the radial and angular velocities after a *long* time? Assume that the tube is long enough so that the rod remains inside.