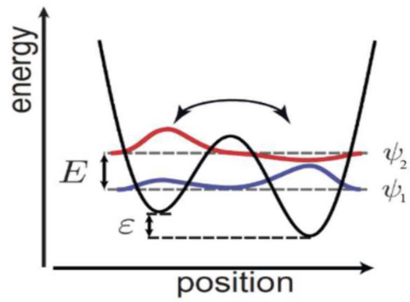
Q3



Consider a particle in an asymmetric double-well potential with Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{T},$$

where $\hat{T}$ describes the tunneling between the two wells. In the absence of tunneling (*i.e.* when $\hat{T} = 0$) the two lowest energy states are $|\psi_L\rangle$ and $|\psi_R\rangle$ with

$$\hat{H}_0|\psi_L\rangle = E_L|\psi_L\rangle, \quad \hat{H}_0|\psi_R\rangle = E_R|\psi_R\rangle,$$

with $E_L - E_R = \varepsilon$. The effects of the higher energy states in the wells can be neglected. When tunneling is present

$$\hat{T}|\psi_L\rangle = \Delta|\psi_R\rangle, \quad \hat{T}|\psi_R\rangle = \Delta|\psi_L\rangle,$$

where Δ is a real, positive number.

a) Find the eigenstates $|\psi_1\rangle$ and $|\psi_2\rangle$ of the system as a linear combination of $|\psi_L\rangle$ and $|\psi_R\rangle$. Also find the associated energy eigenvalues and hence the new energy-level splitting $E = E_2 - E_1$.

For the remaining parts of the problem assume that $E_L = E_R = E_0$ so $\varepsilon = 0$. To save writing, you may also use units in which $\hbar = 1$.

b) At $t = 0$ the particle is in state $|\psi\rangle = |\psi_L\rangle$. Find the probability P_R for the particle to be found in state $|\psi_R\rangle$ at a later time t. Define the probability current to be $J = dP_R/dt$. Find J at time $t = 0$.

c) Now assume that at time t the particle is in the state

$$|\psi\rangle = \alpha|\psi_L\rangle + \beta e^{i\varphi}|\psi_R\rangle, \quad |\alpha|^2 + |\beta|^2 = 1.$$

Use the Schrödinger evolution equation to again compute $J = dP_R/dt$.

d) If at some moment α and β are real numbers, show that at that moment we have $J = J_0 \sin\varphi$ where J_0 is a real number that you should find in terms of E_0, Δ, α and β.