Q4 Consider a paramagnetic material consisting of a volume V containing N non-interacting spin-1 particles each with magnetic moment μ. In a magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$ each spin can have energy $\varepsilon = \mu B m$, where $m = -1, 0, +1$.

a) Assuming a temperature T, write down the partition function Z for the system of spins.

b) From the partition function in part (a), compute the average thermal energy U and hence the magnetization M which is defined by $U = -MBV$.

c) Show that at high temperatures your expression for M reduces to

$$M = \alpha/k_\mathrm{B}T + O[T^{-2}],$$

where you should find the constant α. Similarly show that at low temperature

$$M \approx M_0(1 - e^{-\kappa/k_\mathrm{B}T}),$$

where you should find M_0 and κ. Sketch a graph of M versus $\mu B/k_\mathrm{B}T$ and give a physical explanation as to why it has this form.

d) Use your partition function and your expression for U to write down an expression for the entropy S of the system of spins. Show that it depends on B and T only through the ratio $\mu B/k_\mathrm{B}T$.

Now suppose that the spins reside in, and are in thermal equilibrium with, a crystal lattice that is itself in good thermal contact with a heat bath at temperature T_1. The system is cold enough that the phonon contribution to the specific heat of the crystal lattice is negligible. We apply a strong magnetic field B_1 to the spin-lattice system and allow it to come to equilibrium with the heat bath.

e) The spin-lattice system is now isolated from the heat bath. After this thermal isolation the magnetic field is reduced to a much smaller value B_2. What is the final temperature T_2 of the spin-system (and by the assumption of its negligible specific heat, the temperature of the crystal lattice)?