Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

Math 285 – Intro Differential Equations

Midterm 3 Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Wed., Apr. 24th 6:00-7:30pm 2039 CIF Tutors: Hayden and Suleymaan

Session 2: Thurs., Apr. 25th 5:00-6:30pm 1035 CIF Tutors: Charlie and Eric

Can't make it to a session? Here's our schedule by course:

`https://care.grainger.illinois.edu/tutoring/schedule-by-subject`

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/846
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. Find the eigenvalues and corresponding eigenvectors for the following matrix A:

$$\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

2. Find the matrix exponential using Putzer's Method for the following matrix A:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 2 & 0 & 0 \end{bmatrix}$$

3. Solve the ODE with given boundary value conditions. How many solutions does it have?

$$y'' + y = 0 \qquad y(-\pi) = 0, \qquad y(\pi) = 2$$

4. Find the matrix exponential using diagonalization for the following matrix A:

$$\begin{bmatrix} 5 & 4 \\ 2 & 3 \end{bmatrix}$$

5. Match all the sets of boundary conditions with the solution type they produce when imposed on the following homogeneous ODE:

$$y'' - 2y' = 0$$

A) $y'(0) = 1$ and $y'(1) = 0$	(I) Unique Solution
B) $y'(1) = 2$ and $y''(1) = 4$	(II) Infinitely Many Solutions
C) $y(0) = 5$ and $y'(0) = 2$	(III) Trivial Solution
D) $y(0) = 0$ and $y'(0) = 0$	(IV) No Solution

6. Compute all the eigenvalues and corresponding eigenfunctions for the boundary value problem

$$y'' - \lambda y = 0 \qquad y'(-2) = 0, y(0) = 0$$

If a certain range of the real numbers does not include any eigenvalues, show why there are none in that range

7. Functions f, g, h and k are 6-periodic. Their values on $[-3,3)$ are given below. For which of these functions does the Fourier series converge at $x=0$ to the value 1?

$$f(x)=\begin{cases} 2+x & -3\le x<0 \\ 1 & x=0 \\ -2 & 0<x<3 \end{cases} \qquad g(x)=\begin{cases} 1+x & -3\le x<0 \\ 4 & x=0 \\ 2-x^2 & 0<x<3 \end{cases}$$

$$h(x)=\begin{cases} x^2-1 & -3\le x<0 \\ -1 & x=0 \\ 3 & 0\le x<3 \end{cases} \qquad k(x)=\begin{cases} 3+x & -3\le x<1 \\ 1 & x=1 \\ x-8 & 1<x<3 \end{cases}$$

A) f

B) h

C) None

D) g and k

E) f and h

8. Consider the function $f(x) = 1 - x$ defined on the interval $x \in [-1, 1)$

   (a) Sketch the 2-periodic Classical extension of f(x) on the interval $x \in [-3, 3]$

   (b) Compute the 2-periodic Classical Fourier series representation of f(x)

9. Find the matrix exponential using Putzer's Method for the following matrix A:

$$\begin{bmatrix} 9 & 8 \\ 6 & 7 \end{bmatrix}$$