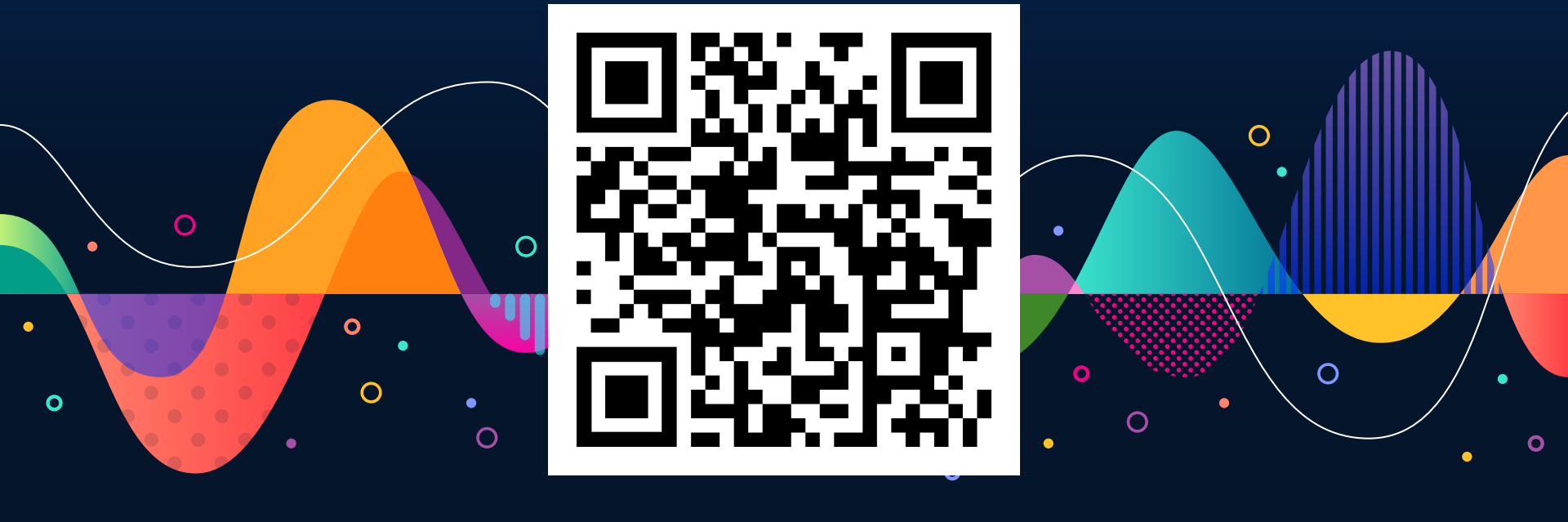
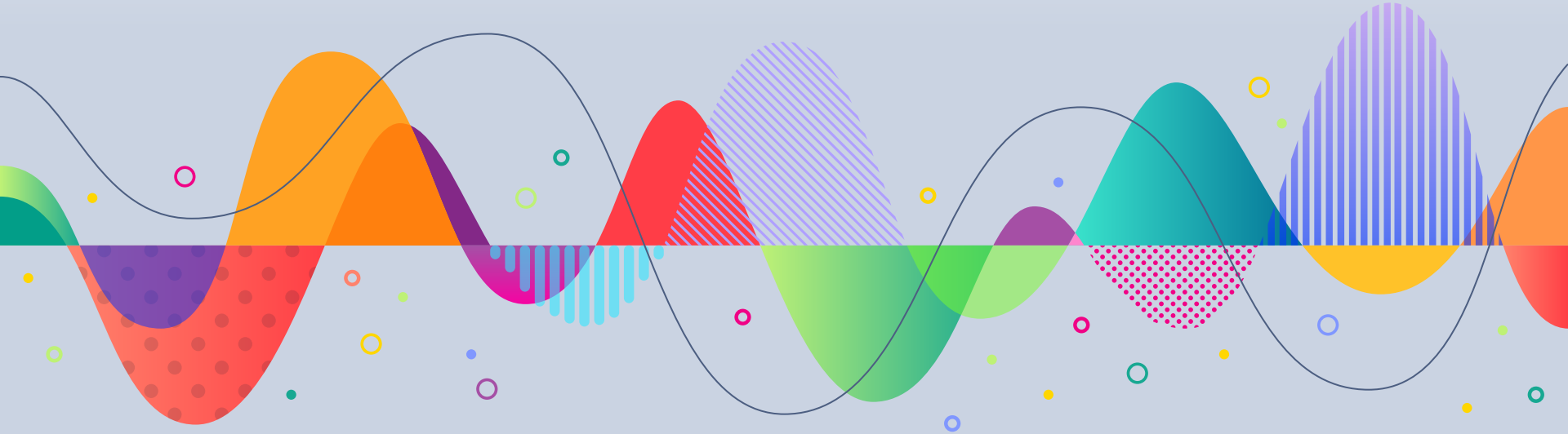
PHYS211
Exam 3 Prep



1.

Overview

Quick bits of info to know



Rotational Kinematics

- Same concept as linear kinematics

Translational	Rotational
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$\Delta x = v_0 t + \frac{1}{2} a t^2$	$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a\Delta x$	$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

Variables:
ω = angular velocity
α = angular acceleration
θ = angular displacement

Converting between the two:

v=ω*r

a=α*r

x= θ*r

Rotational Dynamics

- Many equivalencies

TABLE 1 Comparison of Dynamics Equations for Linear and Rotational Motion

	Linear Motion	*Rotational Motion*
Newton's second law	$F = ma$	$t = I\alpha$
Momentum	$p = mv$	$L = I\omega$
Work	$W = F\Delta x$ or $W = \int F \cdot dx$	$W = t\Delta\theta$ or $W = \int t \cdot d\theta$
Kinetic energy	$K \cdot E \cdot = \frac{1}{2} mv^2$	$K \cdot E \cdot = \frac{1}{2} I\omega^2$
Power	$P = Fv$	$P = t\omega$

- Moment of Inertia
  - The farther the distribution of mass from the Axis of Rotation, greater the Moment of Inertia
- Parallel Axis Theorem
  - Moment of Inertia about an axis other than Center of Mass

$$I_{parallel} = I_{CM} + MD^2$$

Torque

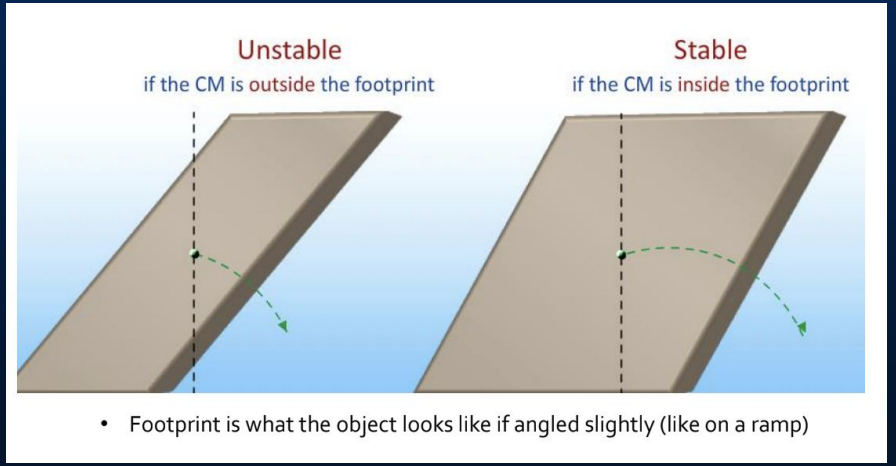
- Use right hand rule
  - Conventionally, thumb towards you is +, towards screen is -
- Visualize how force turns object
  - Clockwise (-)
  - Counterclockwise (+)

$\tau = I\alpha$ (rotation about a fixed axis)

$\tau = \mathbf{r} \times \mathbf{F}$, $|\tau| = rF\sin\phi$

Statics

- During static equilibrium,
  - $\Sigma F_x = 0$
  - $\Sigma F_y = 0$
  - $\Sigma \tau = 0$
- Tipping
  - If Center of Mass is outside vertical footprint of object-> Will tip
  - If Center of Mass is inside vertical footprint of object -> Will slide

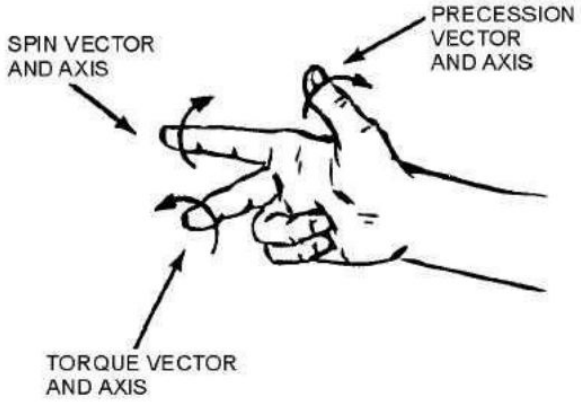


Angular Momentum

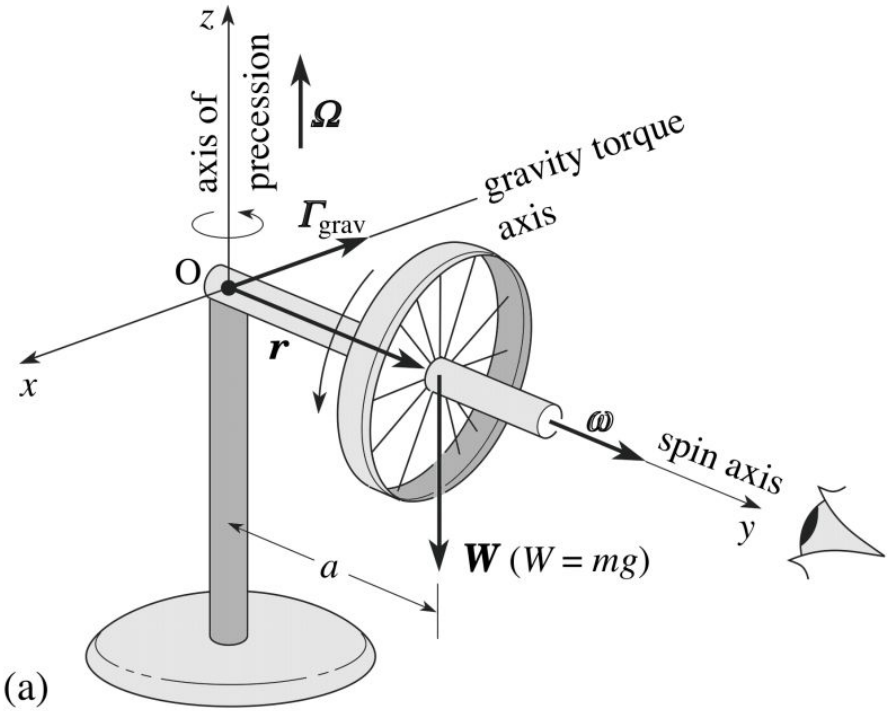
- Right hand rule to find Direction
  - Curl fingers in direction of rotation
- Conserved when net torque = 0.
- $\tau = dL/dt$
- From linear motion
  - $L=mvR \rightarrow R$ = dist from axis of rotation
- Precession = Torque/L
  - Period = 2 pi / precession

$$L = I\omega = r \times p$$

Direction of Precession



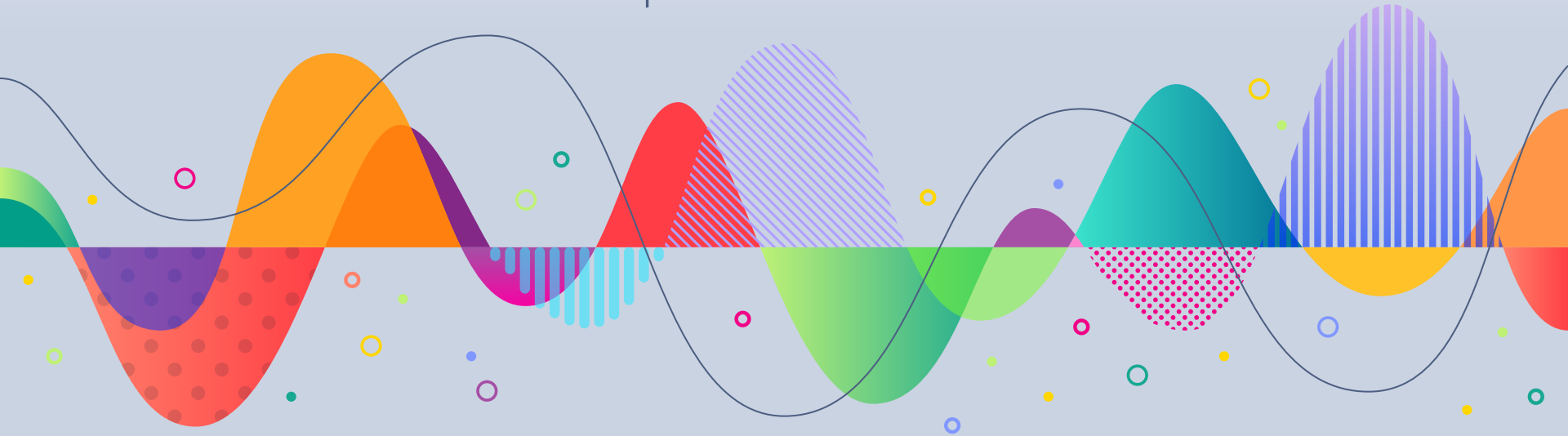
**Or curl fingers from spin vector (ω) to torque vector



2.

Problem Solving

Some Steps to Follow If You're Lost



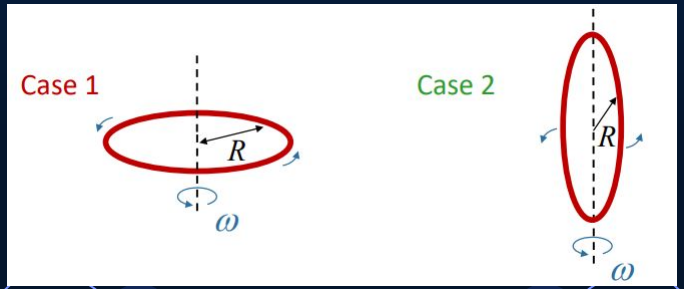
Rotational Kinematics

How to Identify:

- Has rotational variables, notably time
- Likely never first approach

Rotational Kinematics Concept Question

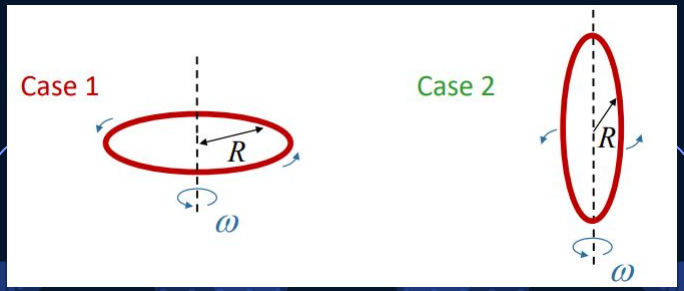
Where does the spinning hoop have the largest kinetic energy?



Rotational Kinematics Concept Question

Where does the spinning hoop have the largest kinetic energy?

Case 1. There is a larger contribution R further away from the center of mass



Rotational Dynamics

How to Identify:

- Find moment of inertia about...
- An object rotating around an axis

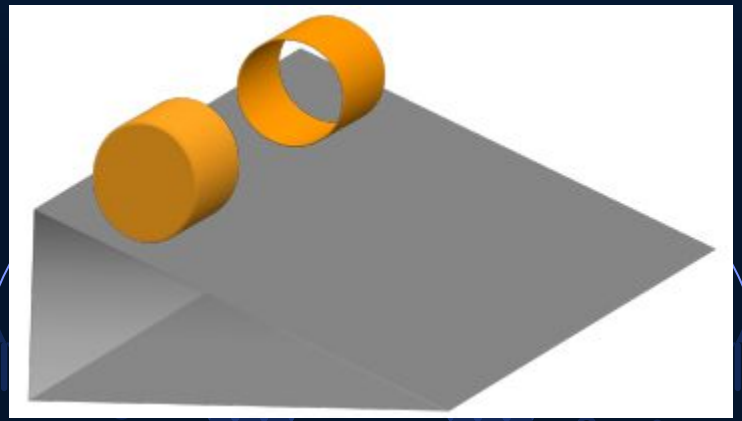
Rotational Dynamics

- Identify what kind of shape the object is and what axis it's rotating around
- Match with a Moment of Inertia equation
- If axis is not center of mass, use parallel axis theorem

Rotational Dynamics Concept Question

A cylinder and a hoop are rolling down a ramp with the same mass and radius. Which reaches the bottom first?

Which way does friction point? Is it static or dynamic?



Rotational Dynamics Concept Question

A cylinder and a hoop are rolling down a ramp. Which reaches the bottom first? The cylinder. The hoop has a larger moment of inertia. If they both start with the same amount of potential energy, the hoop would have more rotational kinetic energy than the cylinder which means it is travelling slower.

Friction is static and points up the ramp to produce a speeding up torque, yet oppose the translational motion.

Torque

How to Identify:

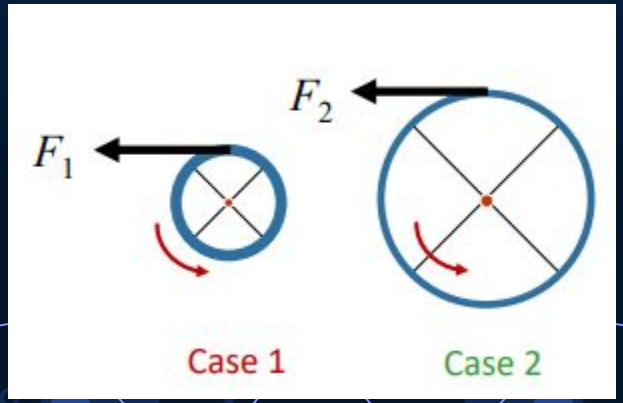
- “Find sum of torques”
- Static problems

Torque

- Find sum of torques in
  - X direction
  - Y direction
- Determine if object is moving ($\Sigma\tau = I\alpha$) or is still ($\Sigma\tau = 0$)
- Use equation to find missing variable or quantity

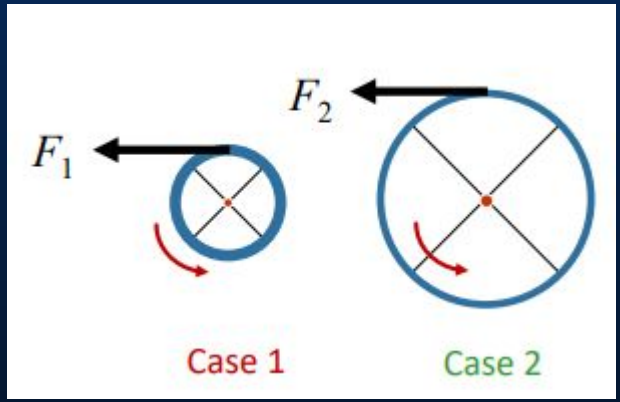
Torque Concept Question

If the hoops below have the same mass and the same force, case 1 has radius R and case 2 has radius 2R, how does the angular acceleration relate? What is the direction of torque?



Torque Concept Question

If the hoops below have the same mass and the same force, case 1 has radius R and case 2 has radius 2R, how does the angular acceleration relate? What is the direction of torque? Torque is out of the page and positive



Formulas:

$\tau = I\alpha$, $\tau = R \times F$, $I = MR^2$

$\rightarrow R \times F = MR^2\alpha$, $R \times F$ at $90°$

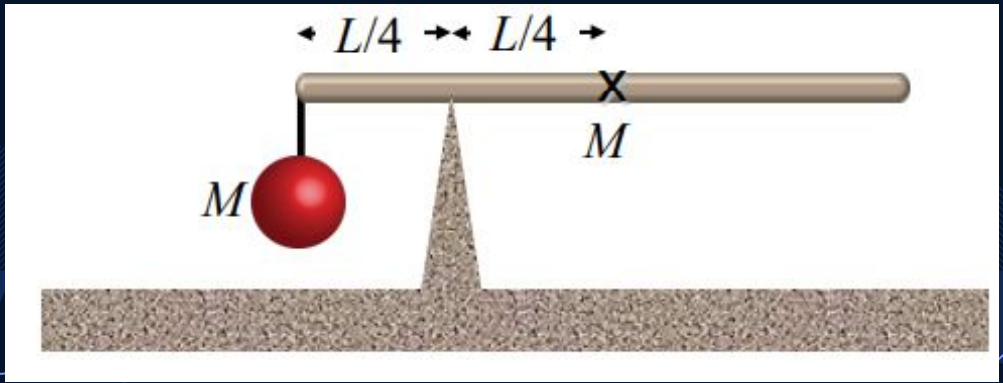
$\alpha = \frac{F}{MR}$

case 1	case 2
$\alpha_1 = \frac{F}{MR}$	$\alpha_2 = \frac{F}{M \cdot 2R}$

$\alpha_1 = 2\alpha_2$

Static Equilibrium Concept Question

An object is made by hanging a ball of mass M from one end of a plank and having the same mass and length L. The object is then pivoted at a point a distance L/4 from the end of the plank supporting the ball. Is the system balanced?



Static Equilibrium Concept Question

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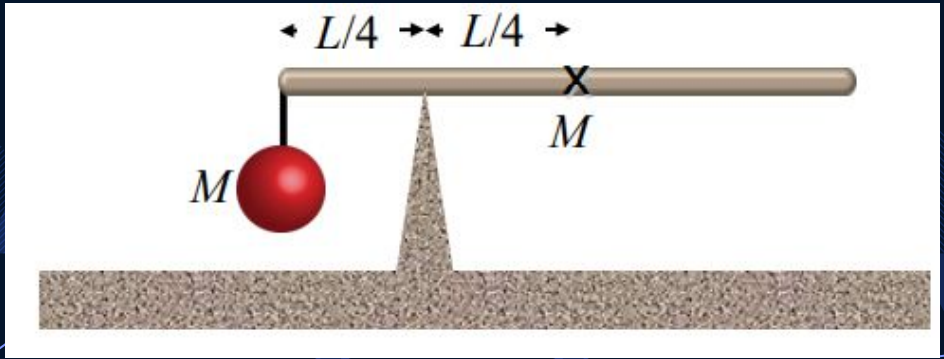


Angular Momentum

How to Identify:

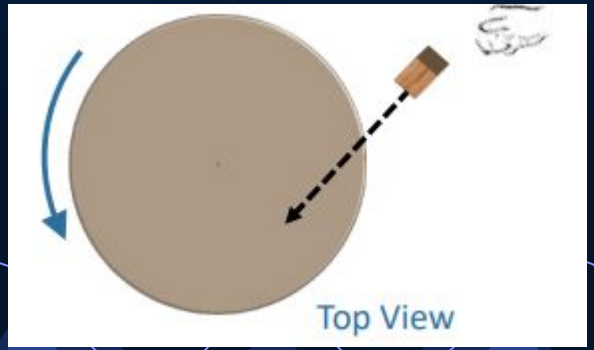
- Gyroscopes
- Precession
- Rotating object

Angular Momentum

- Identify what kind of motion, moment of inertia, angular velocity, etc
  - If linear motion, use L =mvR
  - Use correct moment of inertia
- Use right hand rules accordingly

Angular Momentum Concept Question

The initial magnitude of the angular momentum of a freely rotating disk is L. You toss a heavy block onto the disk along the direction shown. Friction acts between the disk and the block so that eventually the block is at rest on the disk and rotates with it. Is the total angular momentum of the disk-block system conserved during this event (after the block has left your hand)?



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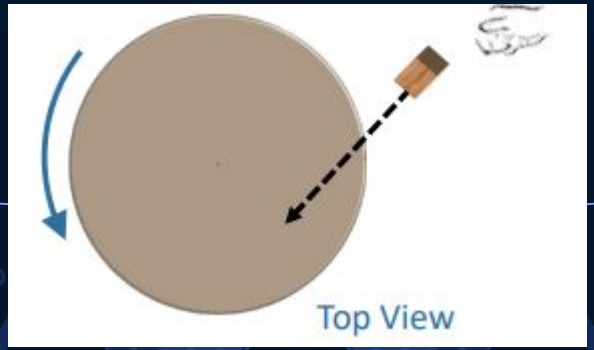
Angular momentum is conserved as there are no external torques.



Please sign in

Another Precession Problem