Midterm 3
Welcome to Math 231 Exam Review!

CARE

QR Code to Queue and Worksheet:

Sequences

- A list of numbers in a certain order
  - $a_1, a_2, a_3, a_4, a_5, \ldots, a_n, \ldots$
- Taking the Limit of Sequences:
  - Limit as n approaches infinity
    - If it exists, the sequence is convergent
    - DNE, sequence is divergent
    - If the limit is infinity, a_n diverges to infinity

Ex: $1/5$, $-2/25$, $3/125$, $-4/625$, … , $\mathbf{(-1)^{(n-1)} n/5^n}$

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Series

- Similar to sequences, but now add the terms together!
  - $a_1 + a_2 + a_3 + a_4 + a_5 + \ldots + a_n + \ldots$
- Convergent if $a_1 + a_2 + \ldots + a_n + \ldots = s$
  - Divergent if the sum diverges
- Geometric Series:

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}; |r| < 1$$

What about $|r| >= 1$? It diverges!

- Test for Divergence
  - If the limit of an does not exists or does not equal 0
    - The series is divergent

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Integral Test

- Suppose $a_n = f(n)$

  If converges $\int_1^\infty f(x)dx$ then $\sum_{n=1}^\infty a_n$ converges

  If diverges $\int_1^\infty f(x)dx$ then $\sum_{n=1}^\infty a_n$ diverges

- P-Test

When does this series converge?

$\sum_{n=1}^\infty \frac{1}{n^p}$

When p > 1!

Comparison Test

- If given two series and know the convergence or divergence of one:

If $\sum_{n=1}^{\infty} b_n$ is convergent and $b_n \geq a_n$ then $\sum_{n=1}^{\infty} a_n$ converges

If $\sum_{n=1}^{\infty} b_n$ is divergent and $b_n \leq a_n$ then $\sum_{n=1}^{\infty} a_n$ diverges

- Limit Comparison Test:

If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ and c is finite and > 0:

Either both series converge or both series diverge

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Alternating Series

- The series alternates between positive and negative!
- Convergence:
  - Terms are decreasing, $b_n >= b_{n+1}$
  - $\lim_{n \to \infty} b_n = 0$
- Absolute and Conditional Convergence
  - Absolute Convergence: The absolute value of a series is convergent
  - Conditional Convergence: The absolute value of a series is convergent, but the original series converges
  - If a series is absolutely convergent, then it is convergent

Ex: $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ Conditional or Absolute convergence?

Conditionally Convergent!

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Ratio Test

- Take the limit of absolute value of the ratio of the n^{th} term and the $n^{th}+1$ term

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = L$$

- If L < 1: Absolutely Convergent
- If L > 1 or L = ∞: Divergent
- If L = 1: Inconclusive

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Root Test

- Take the limit of n^{th} root of the absolute value of the n^{th} term

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$$

- If L < 1: Absolutely Convergent
- If L > 1 or L = ∞: Divergent
- If L = 1: Inconclusive

Putting It All Together

A general order for how you might want to go by solving problems:

1. Test for Divergence
2. p-Series Test
3. Geometric Series Test
4. Comparison Test
5. Alternating Series Test
6. Ratio Test
7. Root Test
8. Integral Test