

Unified Constitutive Model for Engineering- Pavement Materials and Computer Applications

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Participation in Pavements.

- Material Modeling and Computer Methods for Rail Road Track Structures, DOT Office of University Research
- Consultant : Review of Superpave I (SHRP).
- Consultant Airfield Track Structures, Cambridge , Mass.
- Participant in Superpave II.
- Participant in AASHTO Design Guide project.

NEED FOR UNIFIED MODEL

- Material under mechanical and environmental loading can experience various responses simultaneously

Elastic, plastic, creep deformations,

- Microcracking leading to softening, fracture and failure,
- And Healing or strengthening

Important for Pavement Materials

Hence,

- “Unified” model that account for various responses is desirable and needed.

Research in US

Significant effort and expenditure have been spent with the objective to develop a Unified model for pavement materials in various projects, e.g. Superpave in USA

However, it appears such a model has been NOT available, yet.

Resilient Modulus Model

- Used Widely
- However, it is essentially nonlinear (piecewise) **elastic** model
- Could provide vertical Displacement; i.e. its “universal” version may be valid for this quantity
- But does not provide two- and three-dimensional stress,
Stress and displacements,
- Hence, it can not account for other important factors such as cracking, fracture, softening, i.e
- **Rutting or permanent deformation,**
- **Microcracking and Fracture**
- **Thermal Cracking**
- **Reflection Cracking**

DISTURBED STATE CONCEPT (DSC)

DSC is a unified and hierarchical model that can account for various behavioral aspects, *simultaneously*.

Its basic mathematical framework allows modeling for both SOLIDS, and INTERFACES and JOINTS

The hierarchical property allows user to adopt a version(s) of the DSC for specific material need of the user.

Approaches for Pavement Analysis, Design and Maintenance

- Empirical (E)
- Mechanistic Empirical (M-E)
- Full Mechanistic (M)
- DSC is capable of both M-E and M

Bound (Concrete and Asphalt Concrete) and Unbound Materials

Are very often “Discontinuous”

- Initially and/or During deformation
- Therefore, conventional models based on continuum theories such as elasticity and plasticity may not be valid.
- Hence, a Unified Model should account for discontinuities such as due to microcracking or microslips and relative motions, leading to fracture and failure. In fact, such a model need to account for both continuity and discontinuity, at the same time.

CONTINUUM – ASSUMPTION

“ALL PERVASIVE” DISCONTINUITY

- Initial
- Induced

Continuity and discontinuity, order and disorder, positive and negative exist simultaneously; they are not separate, and they are contained and culminate in each other. They produce the holistic material world. The material world, matter, is a projection or manifestation of the complex and mysterious universe, which we have to deal with and comprehend. “Engineering material” is but a subset of the material world and carries with it the complexity and consciousness of the whole. The metaphysical and physical comprehension of matter entails interconnected phenomena at the macroscopic, microscopic, atomic, and subatomic levels, and beyond.

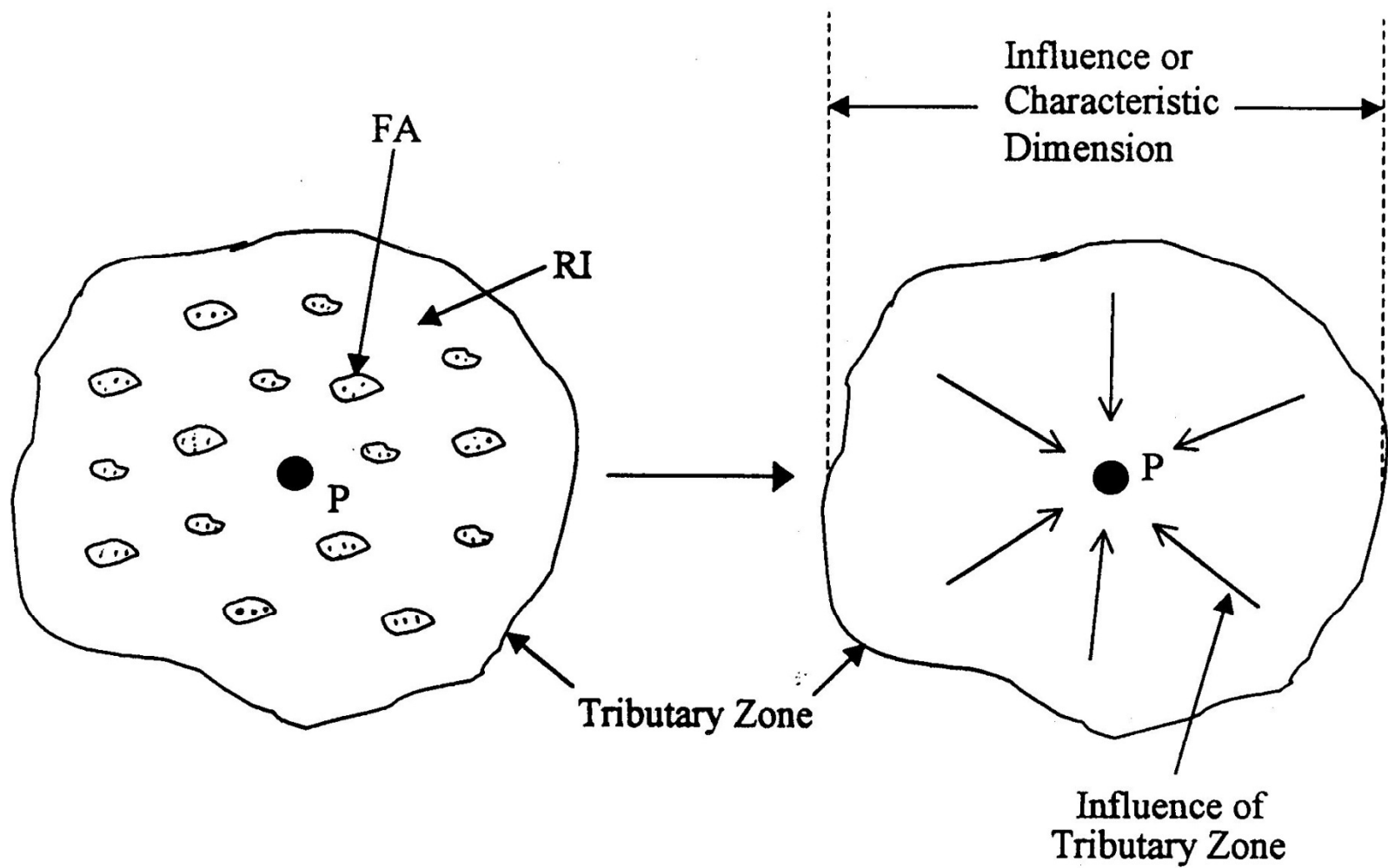
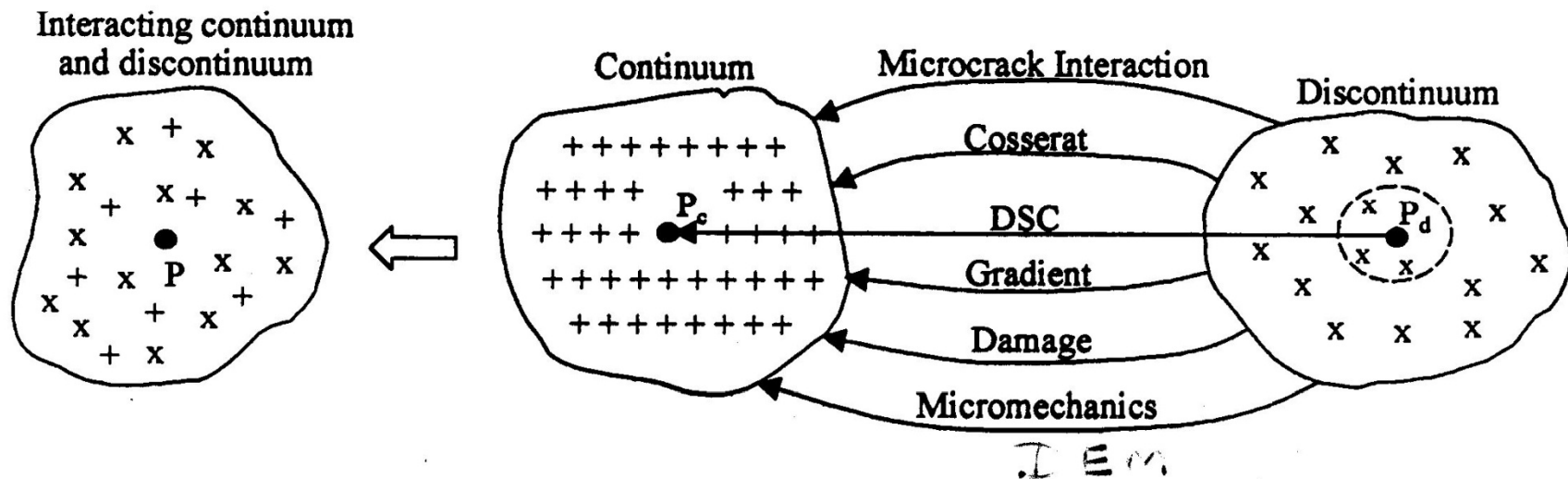


Figure 12-6. Influence of Tributary Zone Around a Point



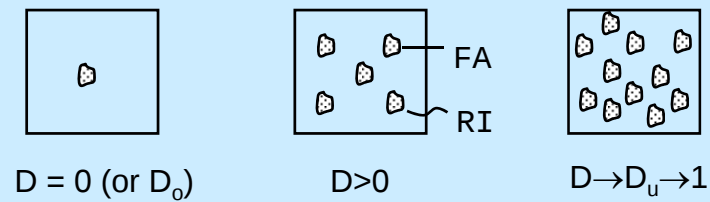
$$B(P) \Leftarrow B(P_c) \Xi B(P_d)$$

Coupling

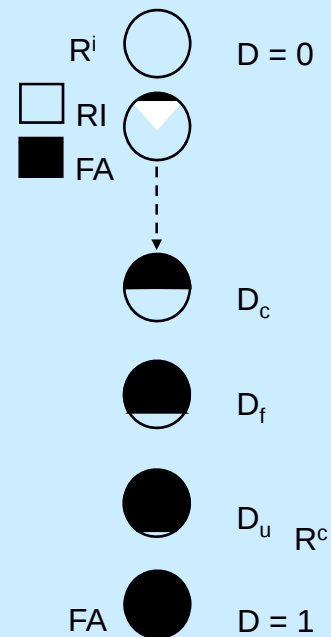
Figure 12-8. Symbolic Representation of Nonlocal Models

DISTURBED STATE CONCEPT

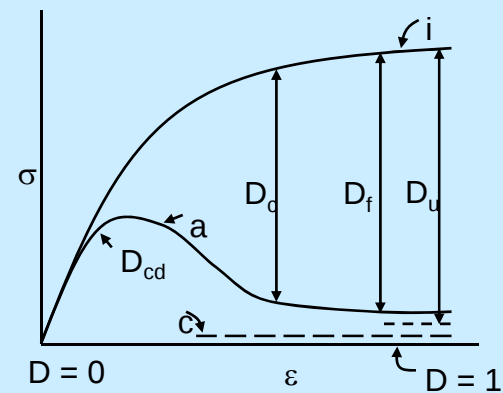
- BASIC IDEA
 - MIXTURE
 - Relative Intact (RI) - Continuum
 - MANIFEST
 - Fully Adjusted (FA):
 - UNMANIFEST
 - Interaction: RI and FA
- ⇒ DISTURBANCE (D)
- For Interacting or Coupled Mechanisms



(a) Clusters of RI and FA parts



(b) Symbolic Representation of DSC



$i \Rightarrow$ relative intact
 $a \Rightarrow$ observed
 $c \Rightarrow$ fully adjusted

(c) Schematic of Stress-strain Response

Schematic Representation of the Disturbed State Concept

- Unified and Hierarchical: Solids and Interfaces
- Allows for Elastic, Plastic and Creep Behavior, Microcracking, Fracture and Degradation, Healing and Stiffening.

DSC Equations

$$d \underset{\sim}{\sigma}^a = (1 - D) \underset{\sim}{C}^i d \underset{\sim}{\varepsilon}^i + D \underset{\sim}{C}^c d \underset{\sim}{\varepsilon}^c + dD (\underset{\sim}{\sigma}^c - \underset{\sim}{\sigma}^i)$$

where

$d \underset{\sim}{\sigma}^a$ = Observed Incremental Stress

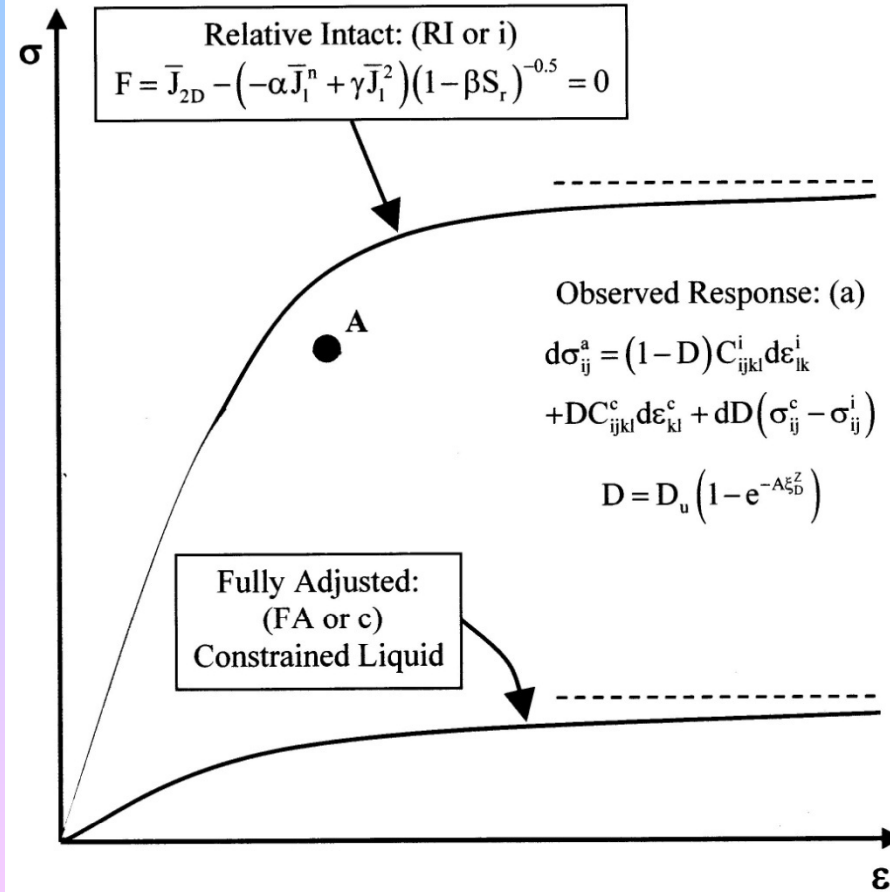
D = Disturbance

$\underset{\sim}{C}^i$ and $\underset{\sim}{C}^c$ = RI and FA Constitutive Matrices

$d \underset{\sim}{\varepsilon}^i$ and $d \underset{\sim}{\varepsilon}^c$ = RI and FA Incremental Strains

dD = Increment or Rate of Disturbance

The DSC/HISS Constitutive Model



Relative Intact (RI) Response

HISS as RI response

$$F = \bar{J}_{2D} - (-\alpha \bar{J}_1^n + \gamma \bar{J}_1^2)(1 - \beta S_r)^m = 0$$

Where,

$$= \bar{J}_{2D} - F_b F_s = 0$$

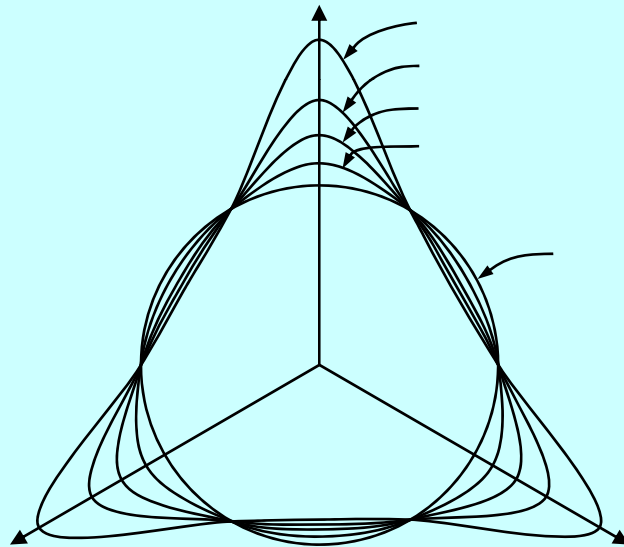
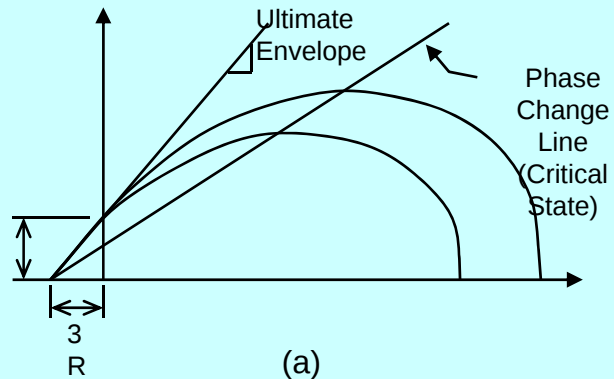
$$\bar{J}_{2D} = \frac{J_{2D}}{p_a^2} \quad \bar{J}_1 = \frac{(J_1 + 3R)}{p_a^2} \quad S_r = \left(\frac{\sqrt{27}}{2} \right) \left(\frac{J_{3D}}{J_{2D}^{3/2}} \right)$$

$$\alpha = \frac{a_1}{\xi^{n_1}} \quad \xi = \int (d\epsilon_{ij}^p d\epsilon_{ij}^p)^{1/2}$$

Used also by
Bonaquist and Witzak;
and Scarpas

$$\xi = \xi_D + \xi_v = \int (dE_{ij}^p \cdot dE_{ij}^p)^{1/2} + \frac{1}{\sqrt{3}} |\epsilon_{ii}^p|$$

E_{ij}^p is the deviatoric plastic strains tensor = $\epsilon_{ij}^p - (1/3)\epsilon_{ii}\delta_{ij}$



(b) Octahedral plane; (for convexity)

Figure 8. Plots of Yield Surface, F , in Stress Spaces

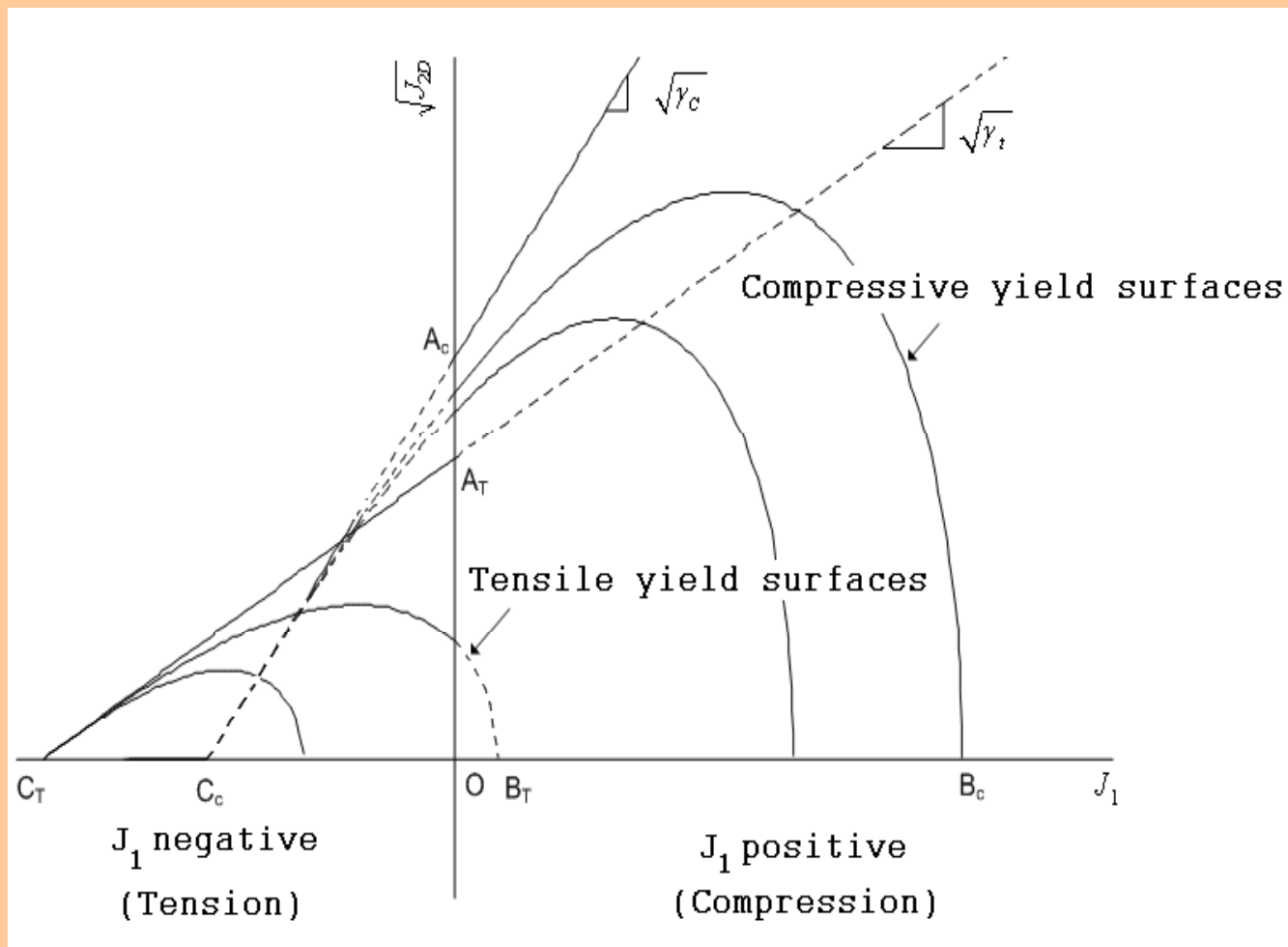
Conventional Plasticity Models,
e.g. Von Mises, Mohr Coulomb
and Drucker Prager: and
Continuous hardening Models,
e.g. Critical state and Cap

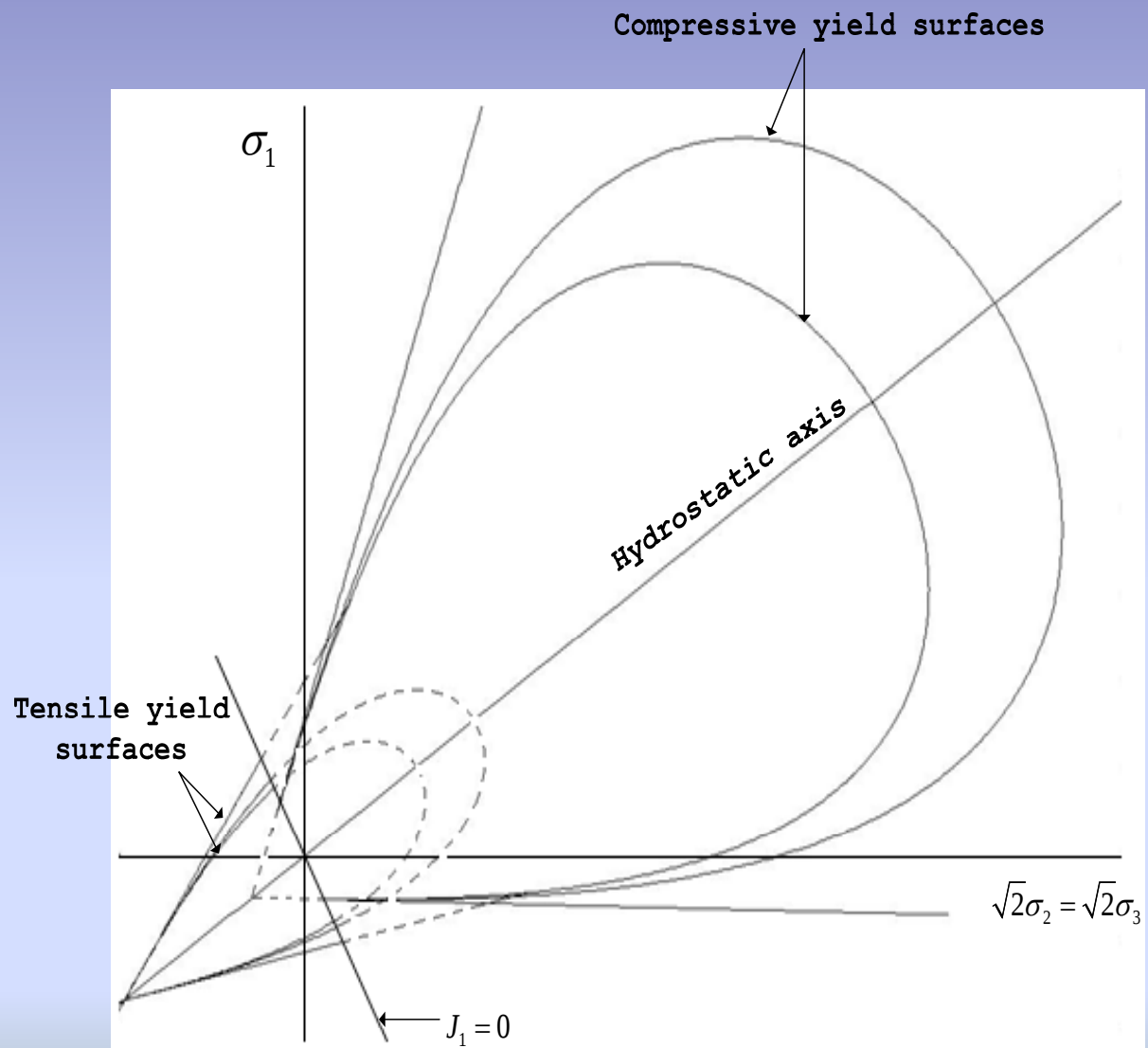
SPECIAL CASES of HISS

Used by
Scarpas;
Bonaquist ,
and others

Compressive and Tensile Yields

- Usually models for pavement (also geologic) materials are defined from compression tests. Hence, the yield behavior in plasticity (HISS) model is valid for compressive yield. Often, when tensile (extension) stress develops, different, *ad hoc*, models are used, such as Stress transfer and Hoffman (Scarpas)
- It is usually difficult to use a model for compressive yield to define tensile yield, because the behavior and parameters in compression and tension are different.





It is possible to develop the HISS model for both compressive and tensile yield.

- **Determine yield parameters both from compression and tension tests**
- **Input them in computer code**
- **Use them depending on the state (compressive or tensile) of stress induced.**

FULLY ADJUSTED (FA) STATE

Zero Strength :

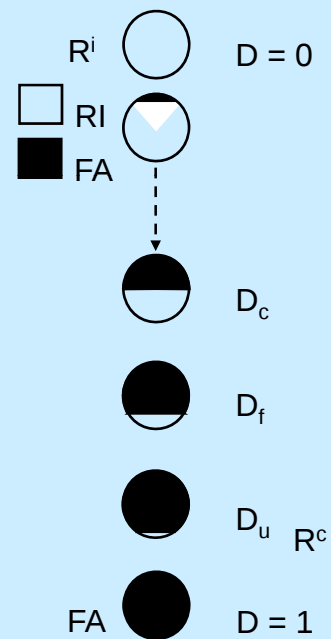
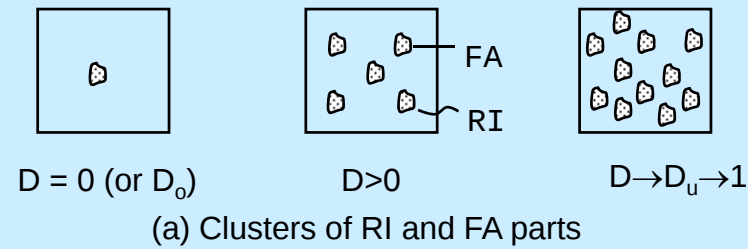
$$\sigma_{\sim} = 0$$

Critical State : $J_{2D}^c = \overline{m}.\overline{J}_1, e^c = e_0^c - \lambda \ln\left(\frac{J_1^c}{3p_a}\right)$

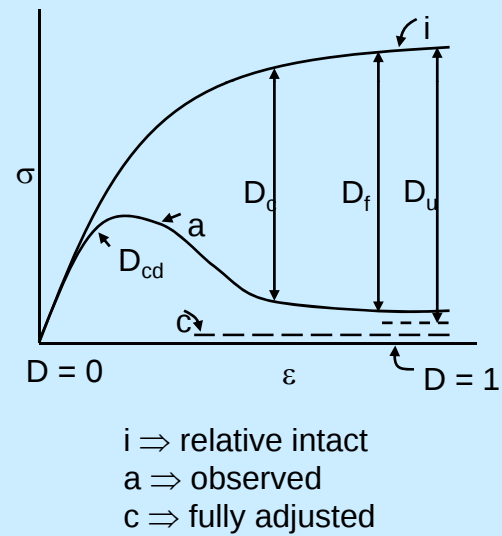
Partially Saturated Soil: Saturated State

$$\sigma_{\sim} = C_{\sim} \varepsilon_{\sim}$$

DISTURBANCE



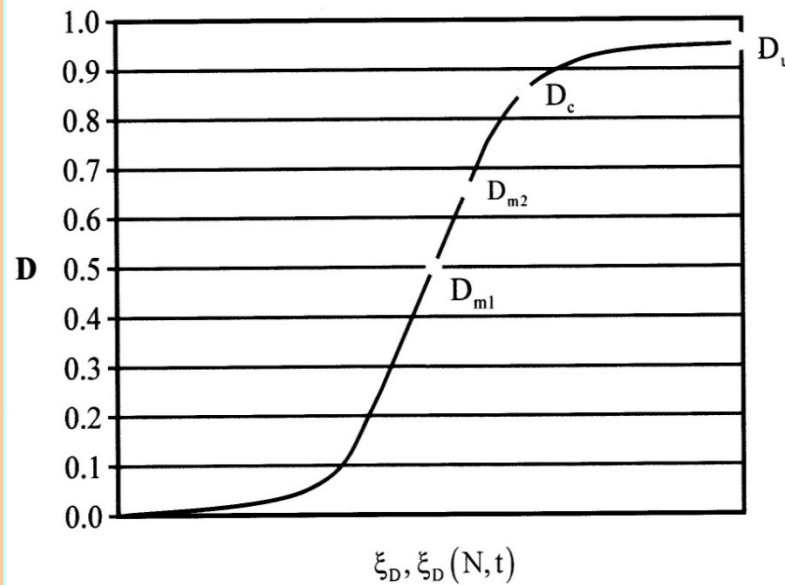
(b) Symbolic Representation of DSC



(c) Schematic of Stress-strain Response

Figure 5. Schematic Representation of the Disturbed State Concept

Variation of D with ξ_D or $\xi_D(N, t)$



Approximate Definitions:

- $D_{m1} \equiv$ Initiation of Microcracking
- $D_{m2} \equiv$ Intermediate Microcracking
- $D_c \equiv$ Initiation of Fracture/Failure
- $D_u \equiv$ Ultimate Disturbance

Can be defined from Stress-Strain, Volumetric, Pore Water Pressure, Nondestructive (P-, S. Wave etc) Measurements

IMPORTANT:

*****Critical Disturbances, e.g. microcracking starts D_{m1} , and Fracture initiates, D_c .**

DISTURBANCE FUNCTION

$$D = D_u \left(1 - \bar{\theta}^{\lambda_1} \xi_D^{12} \right)$$

$$D_\sigma = \frac{\bar{\sigma}^I - \bar{\sigma}^a}{\bar{\sigma}^I - \bar{\sigma}^c} \quad \text{stress - strain}$$

$$D_V = \frac{\bar{V}^I - \bar{V}^a}{\bar{V}^I - \bar{V}^c} \quad \text{velocity (ultrasonic)}$$

Strains

Pore Water pressure

Leads to

$$d\sigma_{ij}^a = \bar{C}_{ijkl}^{p\theta v}(D) \cdot d\epsilon_{kl}^I$$

Function of D

DISTURBANCE (Chapter 3)

- can be defined on the basis of:
 - Stress-strain Response
 - Volumetric Response
 - Pore Water Pressure/Effective Stress Response
 - Nondestructive Response

Weibull type

Expression for D

$$D = D_u \left(1 - e^{-A \xi_D^2} \right)$$

ξ_D = plastic (deviatoric) strain trajectory

$$D = D_u \left(1 - e^{-A (w^p)^2} \right)$$

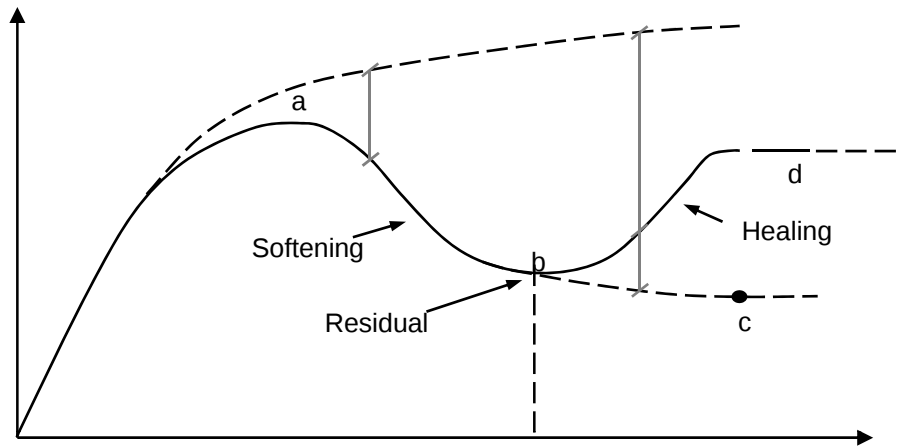
w^p = plastic work on dissipated energy

$$D = D(\phi)$$

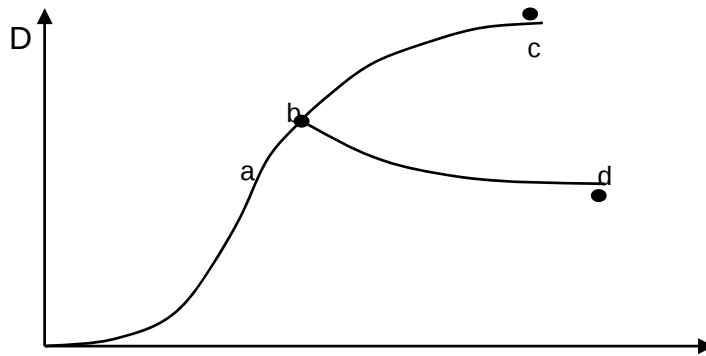
ϕ = free energy, Entropy

HEALING OR STIFFENING

- **Chemical**
- **Temperature**
- **Loading**



(a) Stress-strain Response



(b) Disturbance

Figure 7. Representation of Softening and Healing (Stiffening) Response in DSC

DSC used for silicon with impurities for stiffening behavior, J. of Applied Physics.

THERMAL EFFECTS (Page 44)

– As “Initial strains”

$$\begin{aligned} d\tilde{\sigma} &= \tilde{C}^e d\tilde{\varepsilon}^e \\ &= \tilde{C}^e [d\tilde{\varepsilon} - d\tilde{\varepsilon}(T)] \end{aligned}$$

$$\tilde{\varepsilon}(T) = \alpha_T dT$$

– Parameters

$$p = p_r \left(\frac{T}{T_r} \right)^\lambda$$

p = any parameters: Elastic, plastic, creep, disturbance

r denotes reference temperature

λ = parameter

– Thermoplastic Behavior (Page 46)

– Thermoviscoplastic Behavior (Page 47)

CREEP BEHAVIOR

- Multicomponent DSC (MDSC) Models for Creep

- Elastic (e)
- Visco-elastic (ve)
- Visco-elastic-plastic (evp- Perzyna)
- Visco-elastic-visco-plastic (vevp)

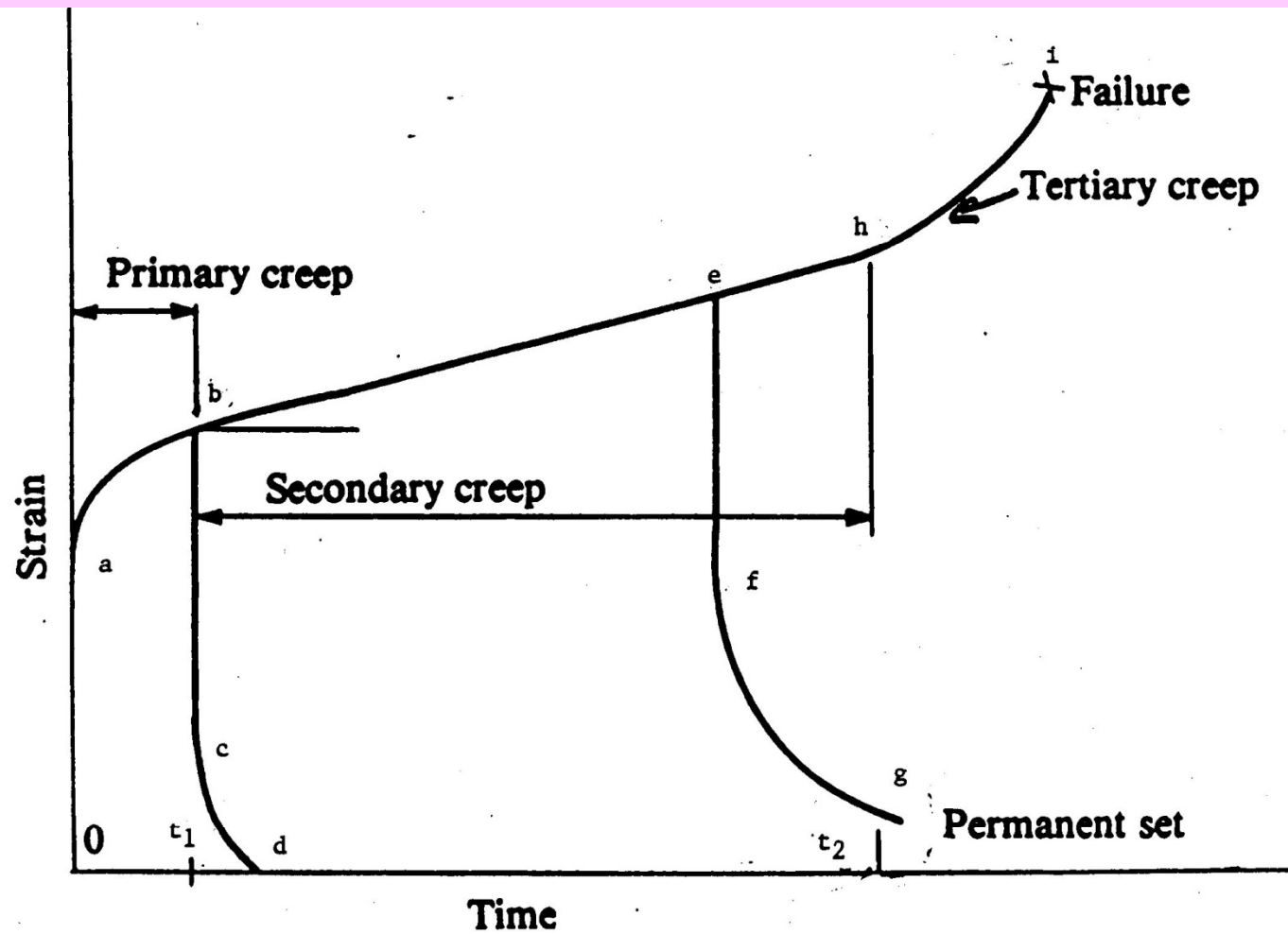
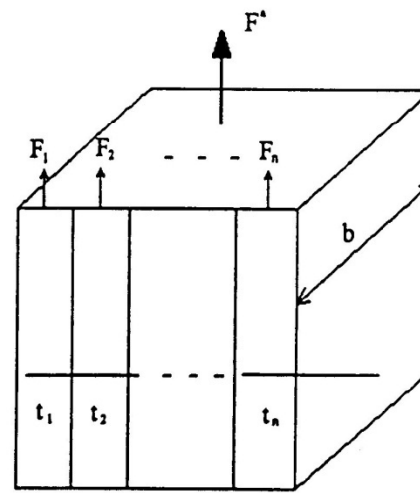
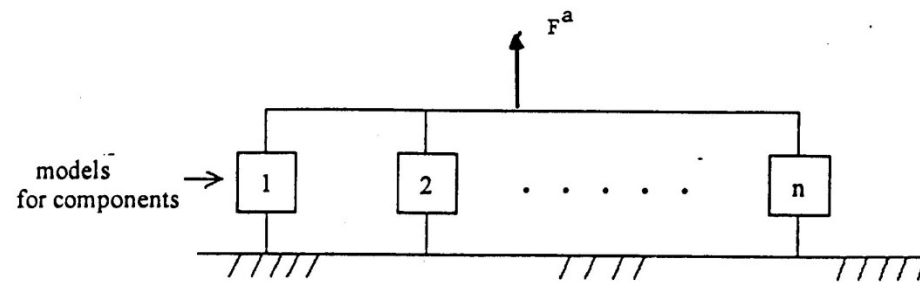


Figure I.5 Schematic of Strain-Time Response Under Constant Stress.



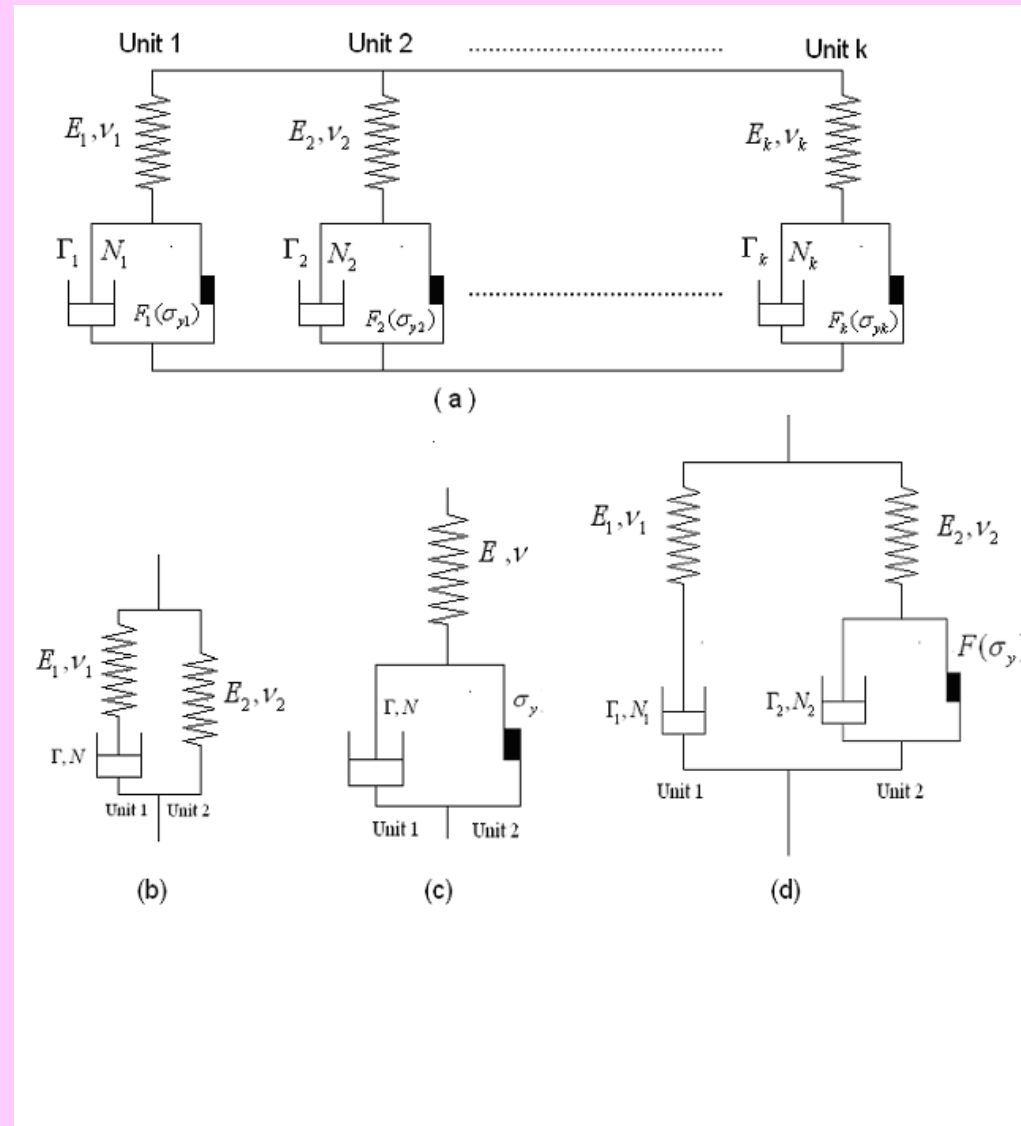
A

(a)



(b)

MULTICOMPONENT DSC: OVERLAY MODEL

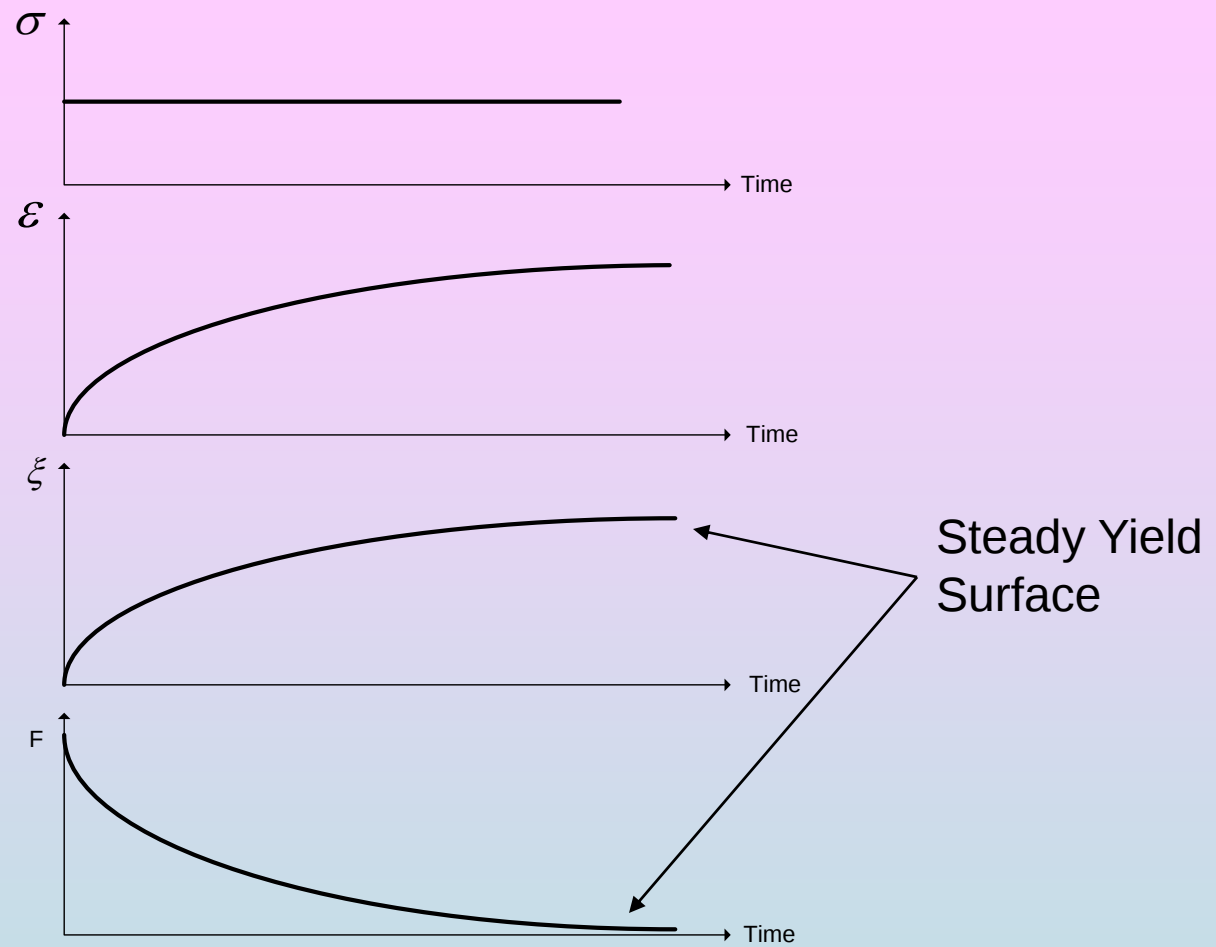


(a) MDSC Overlay model specializations: (b) viscoelastic (ve) (c) elastoviscoplastic (evp) or Perzyna (d) viscoelastic viscoplastic (vevp) models

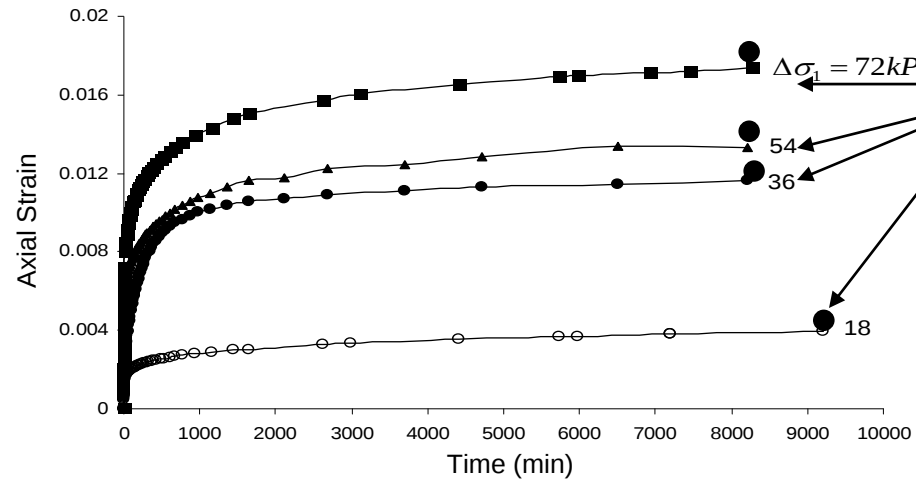
Rate Effect

“Static” Yield Surface in evp (Perzyna) Model

- In the evp model the yield surface refers to the static surface, i.e. related to the test data at the lowest (possible) strain rate
- Usually test data are available at strain rate higher than the static rate.
- A new method is proposed to determine the stress-strain behavior at static state based on available creep test data.

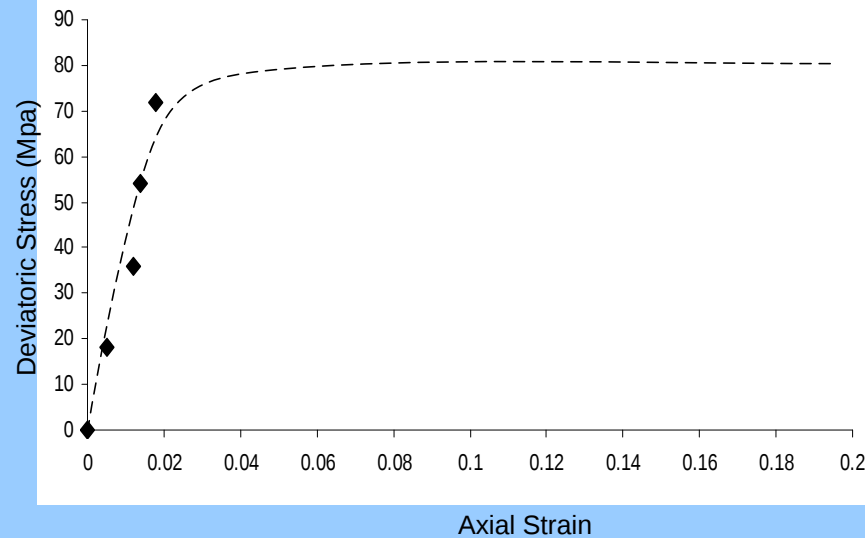


Mechanics of viscoplastic solution



Steady States

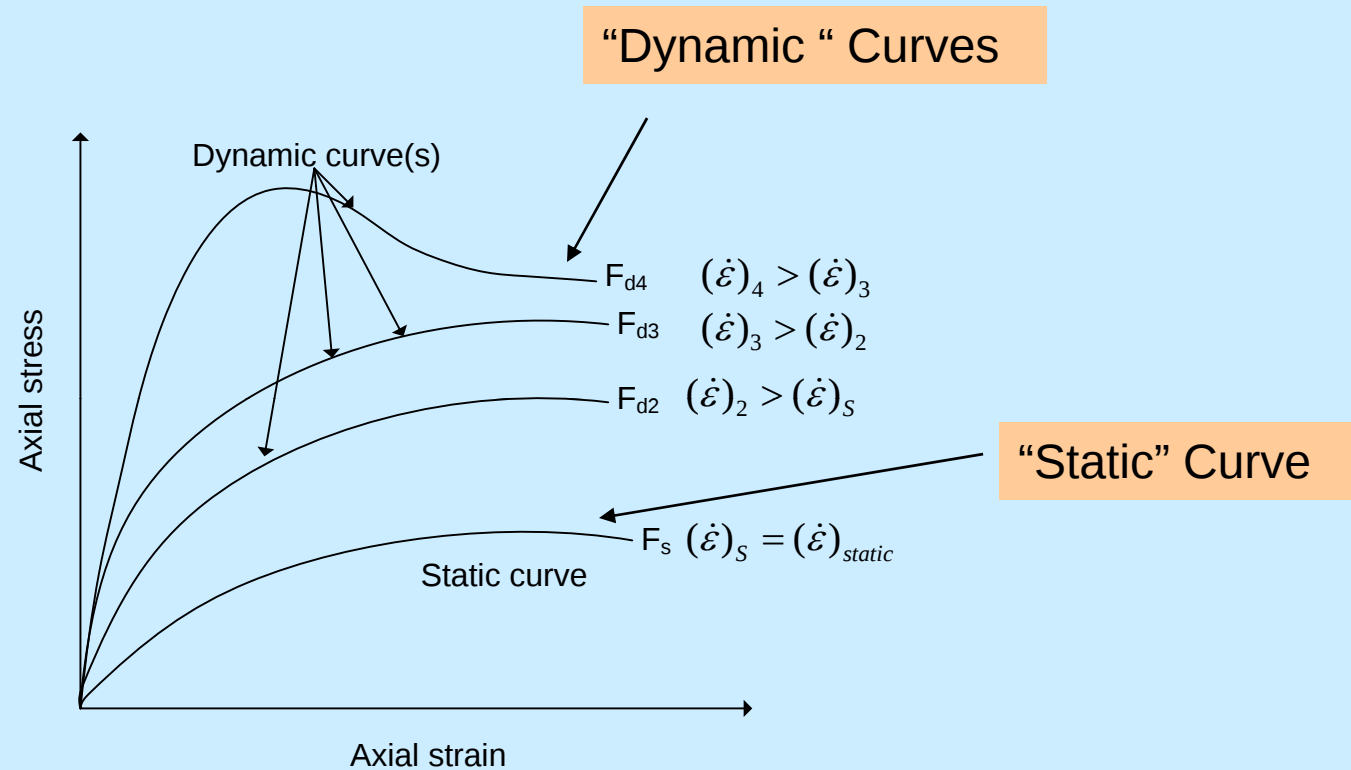
(c) Creep test data at various axial loads for Sky Pilot till at 100 kPa confining pressure



(d) Constructed Static curve for Sky Pilot till at 100 kPa confining pressure

Construction of static curve for Sky Pilot till at 100 kPa.

Rate Dependent Behavior



Schematic of rate dependent material behavior

$$F_d(\sigma, \alpha, \dot{\epsilon}) = \frac{F_s(\sigma, \alpha) - F_R(\dot{\epsilon})}{F_0} = 0$$

**F_d = plastic yield surface for “dynamic “
rate**

F_o = non-dimesionalizing factor

F_s = static yield surface

Derivation of $\mathbf{F}_d$

$$\Gamma\left(\frac{F_s}{F_0}\right)^N = \sqrt{\frac{\left(\frac{d\varepsilon_{ij}^{vp}}{dt}\right)^T \cdot \left(\frac{d\varepsilon_{ij}^{vp}}{dt}\right)}{\left(\frac{\partial F_s}{\partial \sigma_{ij}}\right)^T \cdot \left(\frac{\partial F_s}{\partial \sigma_{ij}}\right)}}$$

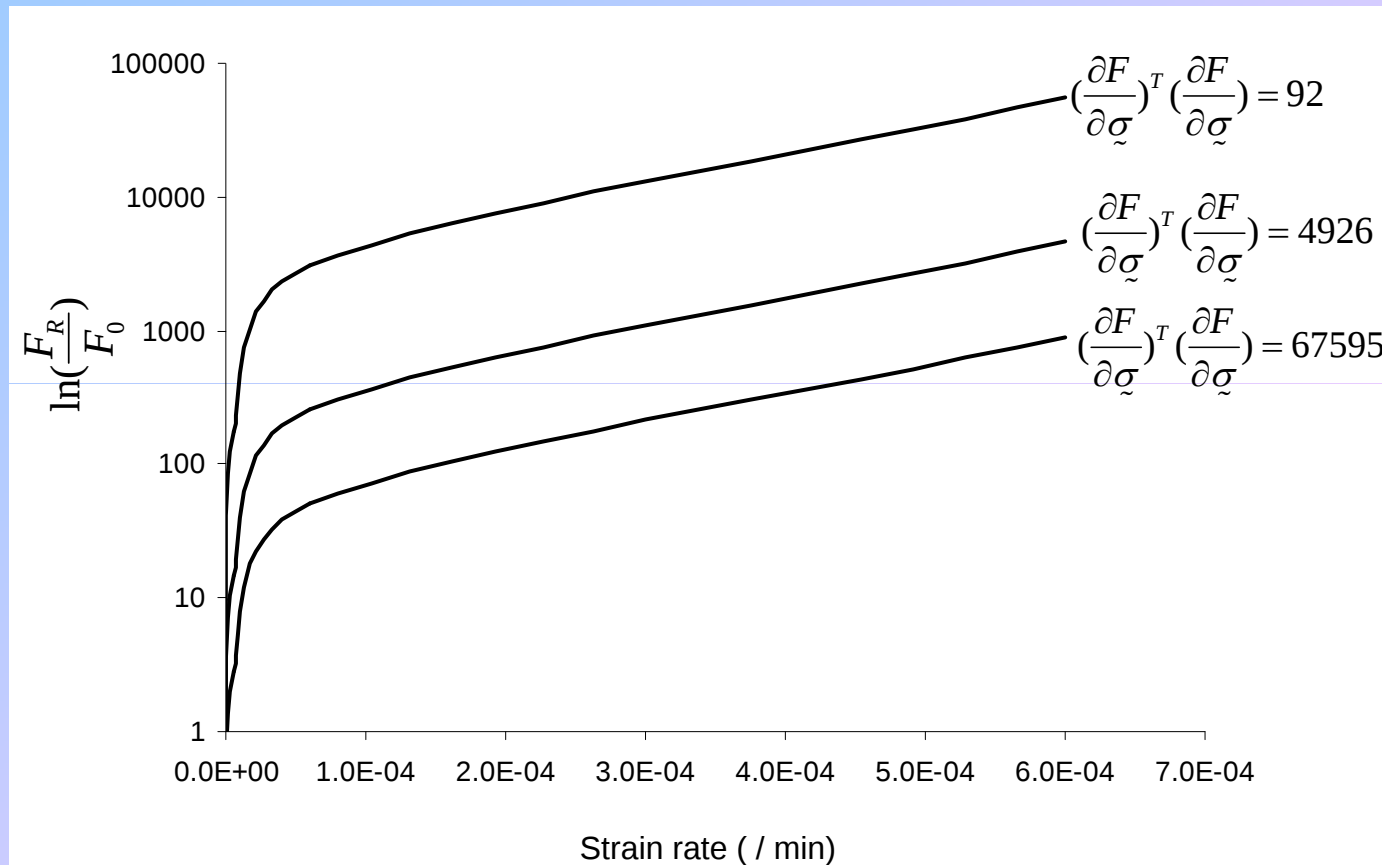
$$\frac{F_s}{F_0} = \left\{ \left(\frac{1}{\Gamma} \right) \cdot \sqrt{\frac{\left(\frac{d\varepsilon_{ij}^{vp}}{dt}\right)^T \cdot \left(\frac{d\varepsilon_{ij}^{vp}}{dt}\right)}{\left(\frac{\partial F_s}{\partial \sigma_{ij}}\right)^T \cdot \left(\frac{\partial F_s}{\partial \sigma_{ij}}\right)}} \right\}^{\frac{1}{N}}$$

Then

$$\frac{F_s}{F_0} - \left\{ \left(\frac{1}{\Gamma} \right) \cdot \sqrt{\frac{\left(\frac{d\varepsilon_{ij}^{vp}}{dt} \right)^T \cdot \left(\frac{d\varepsilon_{ij}^{vp}}{dt} \right)}{\left(\frac{\partial F_s}{\partial \sigma_{ij}} \right)^T \cdot \left(\frac{\partial F_s}{\partial \sigma_{ij}} \right)}} \right\}^{\frac{1}{N}} = 0$$

Comparing above equations:

$$\frac{F_R}{F_0} = \left\{ \left(\frac{1}{\Gamma} \right) \cdot \sqrt{\frac{\left(\frac{d\varepsilon_{ij}^{vp}}{dt} \right)^T \cdot \left(\frac{d\varepsilon_{ij}^{vp}}{dt} \right)}{\left(\frac{\partial F_s}{\partial \sigma_{ij}} \right)^T \cdot \left(\frac{\partial F_s}{\partial \sigma_{ij}} \right)}} \right\}^{\frac{1}{N}}$$



Variation of Rate dependent function FR with strain rate for SAC alloy.

DSC Parameters and Testing

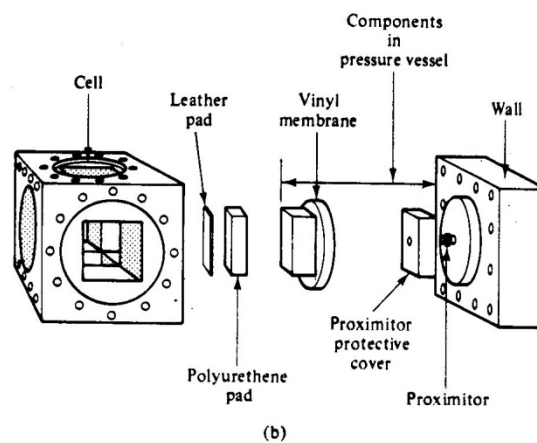
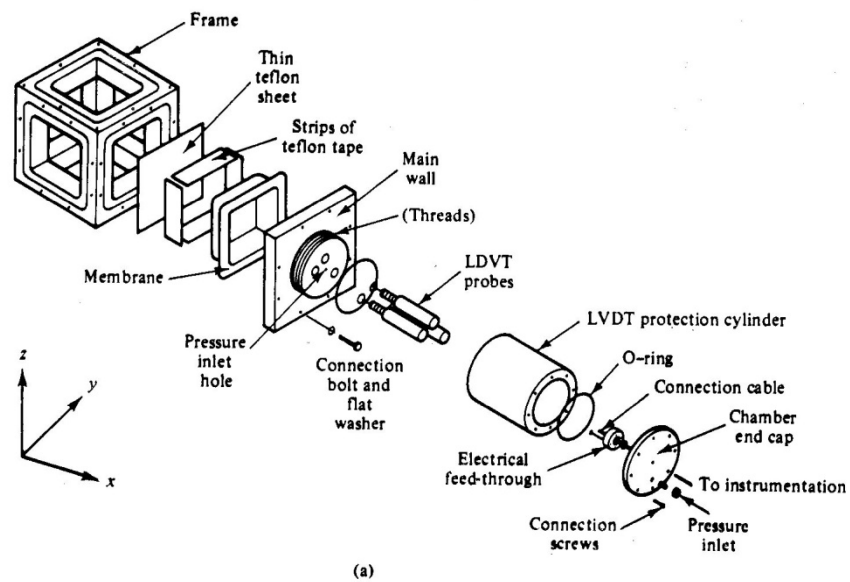
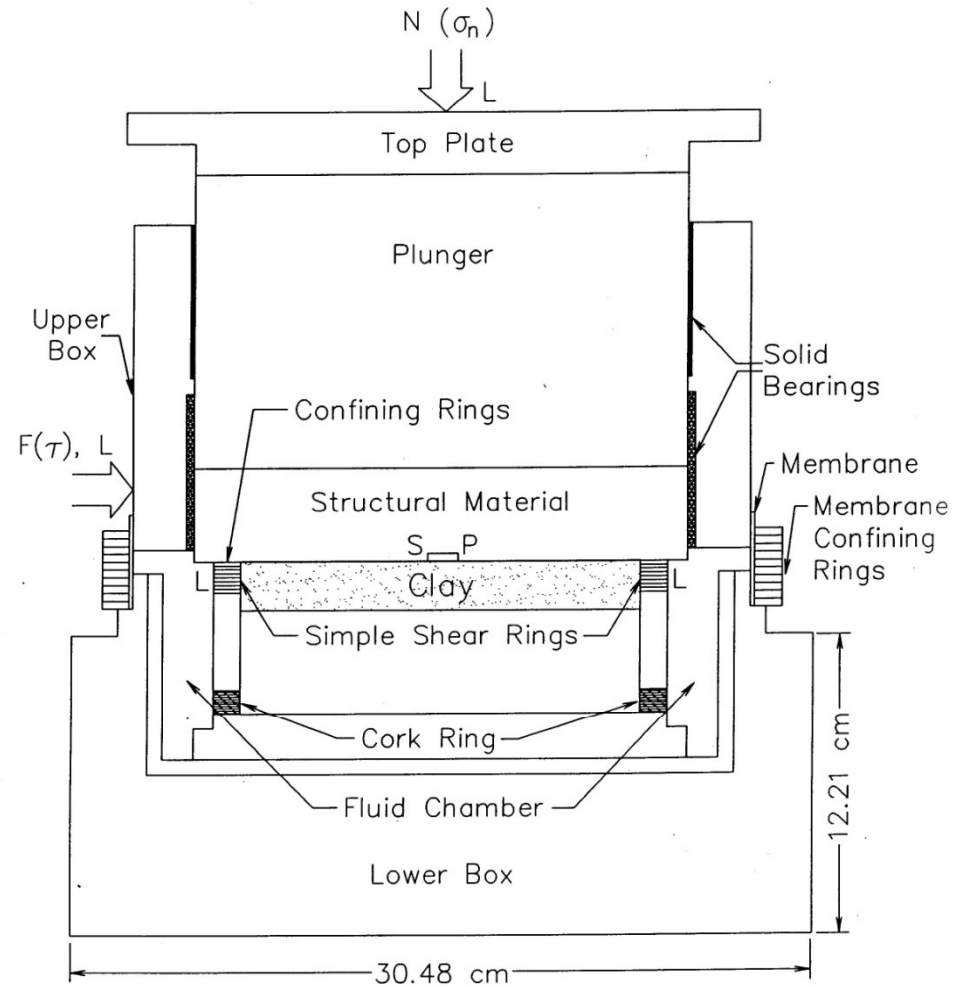


Figure 5-8 Truly triaxial or multiaxial devices: (a) capacity, 200 psi (1.38 MPa); (b) capacity, 20,000 psi (138 MPa) (Ref. 15, 16). (Copyright ASTM, reprinted with permission.)



(a) Sectional Details

Figure 2. Details of CYMDOF test box.

DSC/HISS Parameters for Sky Pilot Till

Category	Symbol	Sky-Pilot Till
Elastic	E	50470
	ν	0.45
Plastic – HISS	γ	0.0092
	β	0.52
	n	6.85
		16.67
Plastic – hardening		1.35e-7
		0.14
FA (critical) state		0.09
	λ	0.16
		0.52
Disturbance	A	5.5
	Z	1.0

Parameters for vevp Model

<i>vevp</i> creep parameters22	Confining Pressure		
	100 kPa	200 kPa	400 kPa
E_1 (kPa)	6666	24244	30000
E_2 (kPa)	42500	83333	63333
	0.45	0.45	0.45
	0.45	0.45	0.45
Γ_1 (kPa⁻¹min⁻¹)	0.001446	0.001841	4.24E-06
Γ_2 (kPa⁻¹min⁻¹)	0.006	0.006	0.006
N_1	2.28	4.00	1.40
N_2	1.63	2.20	2.0

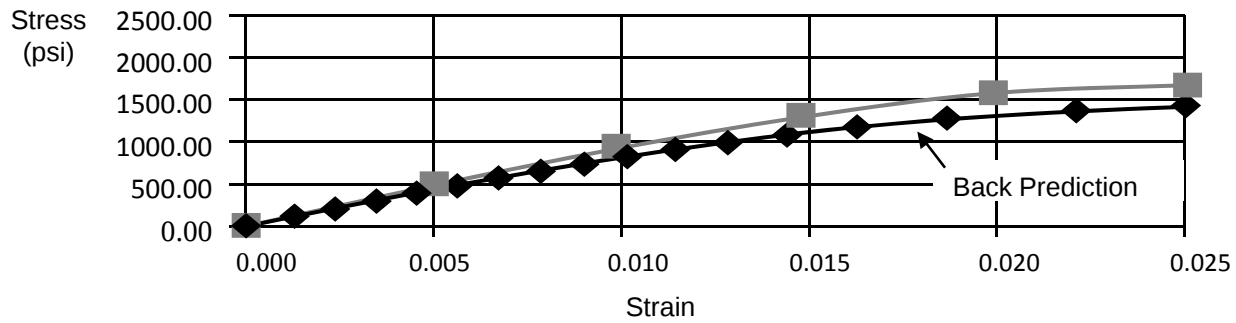
Validations at Specimen Level

Level 1: Predictions of test results used for determination of parameters

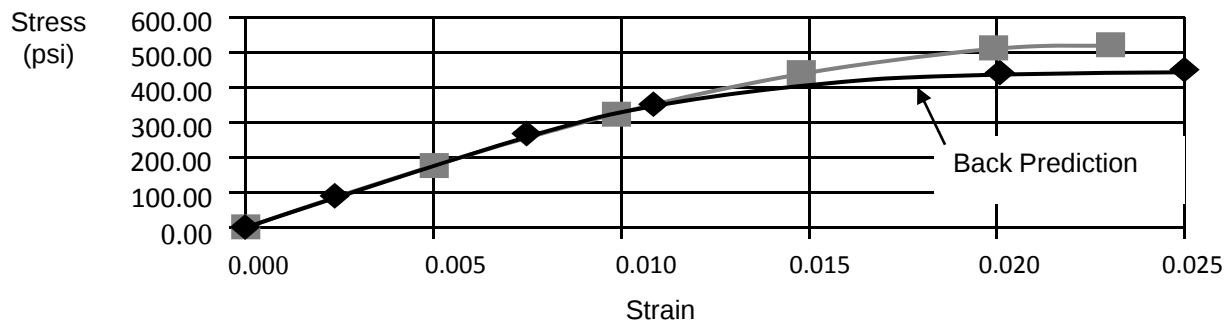
Level 2: Prediction (Independent) of tests NOT used to find parameters

Validation for Stress-Strain Behavior of Asphalt Concrete

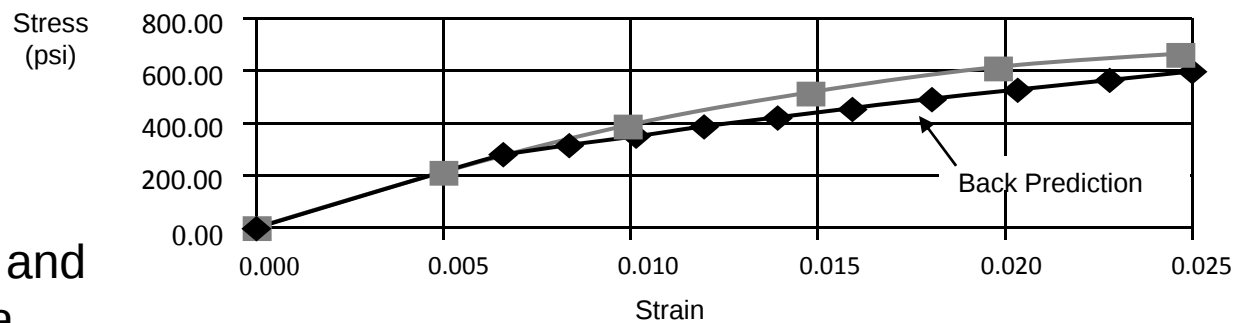
- Test Data from Monismith and Secor (1962) : Viscoelastic Behavior of Asphalt Concrete
- Test Data from Scarpas et al.
- Under Different Strain Rates and Temperatures



(a) $T = 40\text{ }^{\circ}\text{F}$, = 43.8 psi



(b) $T = 77\text{ }^{\circ}\text{F}$, = 0.0 psi



(c) $T = 140\text{ }^{\circ}\text{F}$, = 250 psi

Monismith and
Secor Data

Strain Rate= 1
in/min

Figure 11. Comparison Between DSC Predictions and Test Data for Various Temperatures and Confining Stresses (1 psi = 6.89 kPa)

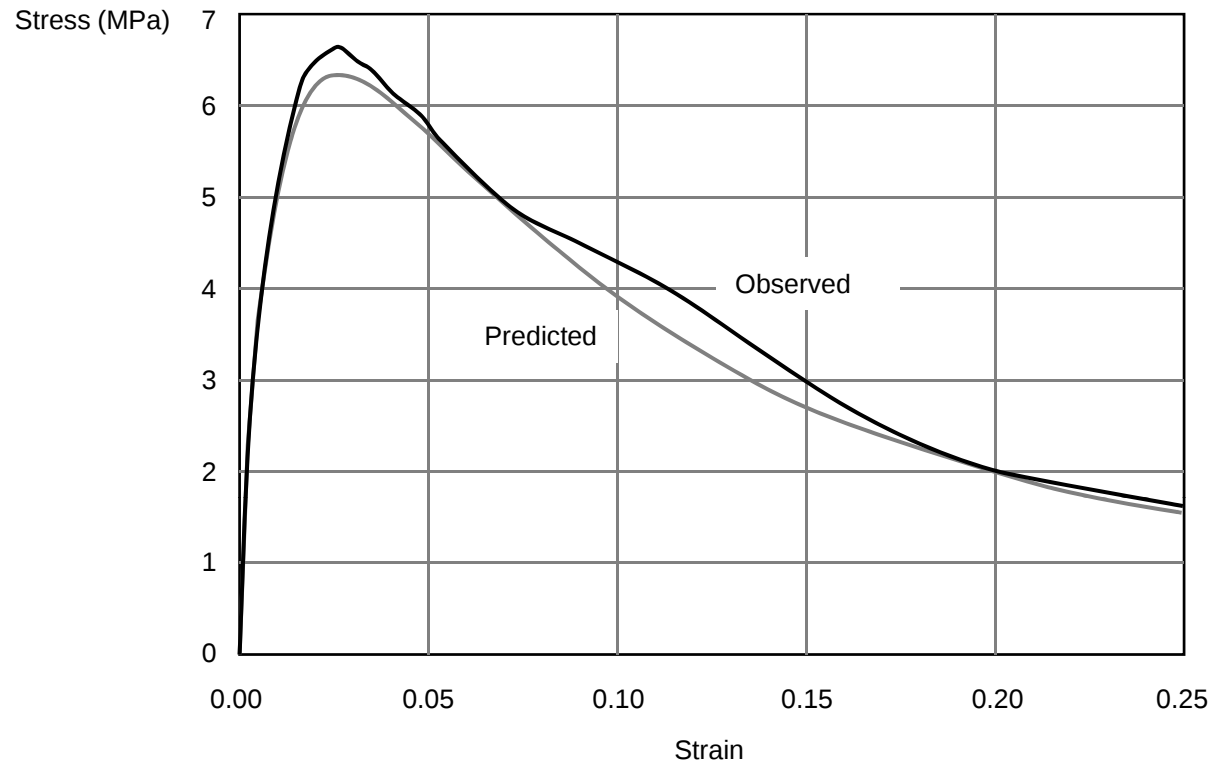
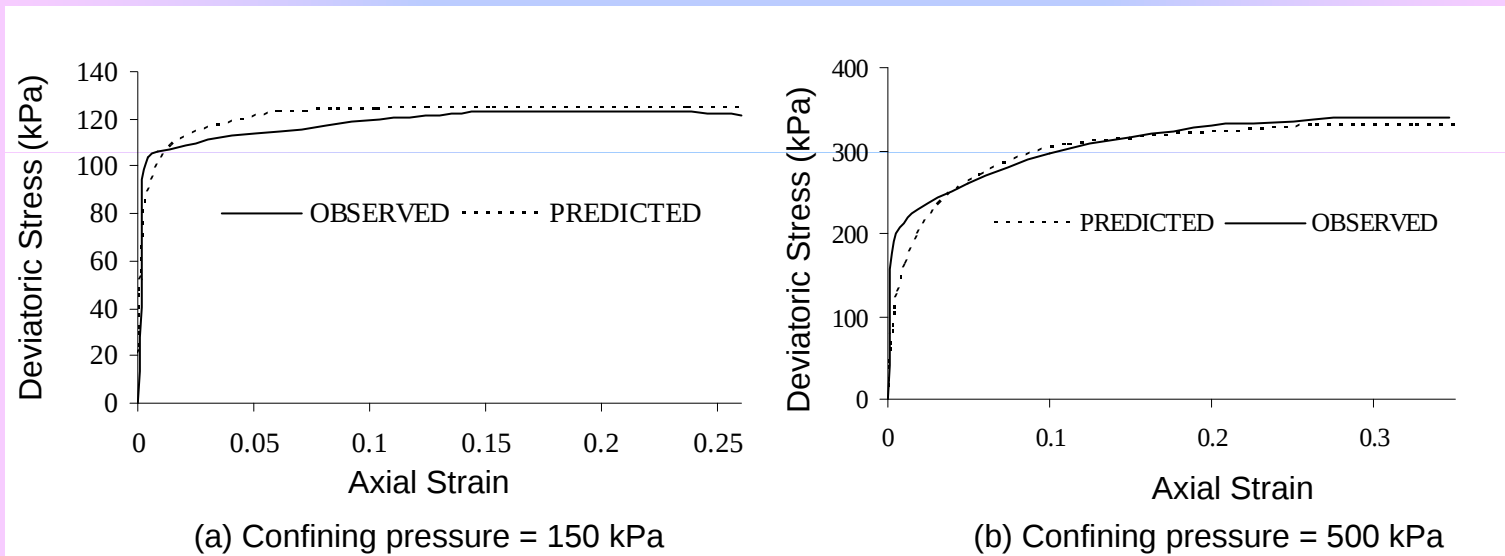


Figure 12. Comparison Between Predicted and Observed Test
Data for $T = 25\text{ }^{\circ}\text{C}$, $= 5\text{ mm / sec}$.

Scarpas ,A., Al-Khoury, R., Van Gurp, C.A., P.M., Erkens, S. M.J.G. (1997) .

Comparisons for Temperature = 25 Deg. C; Strain Rate = 5 mm/s

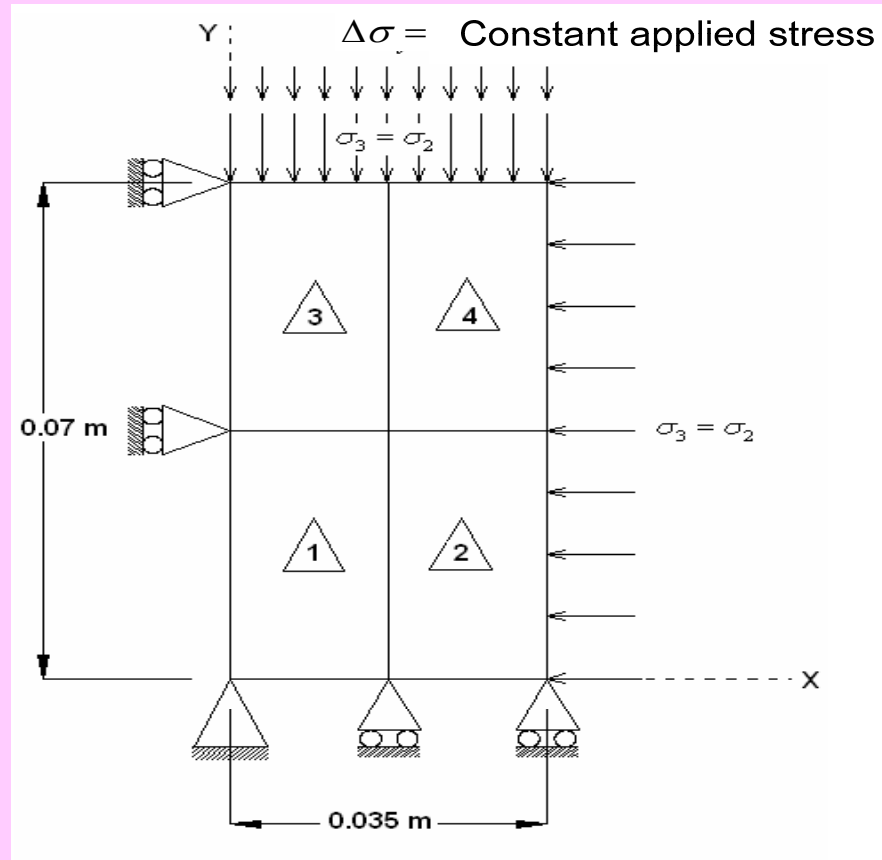
Validations , Specimen Level for Glacial Till

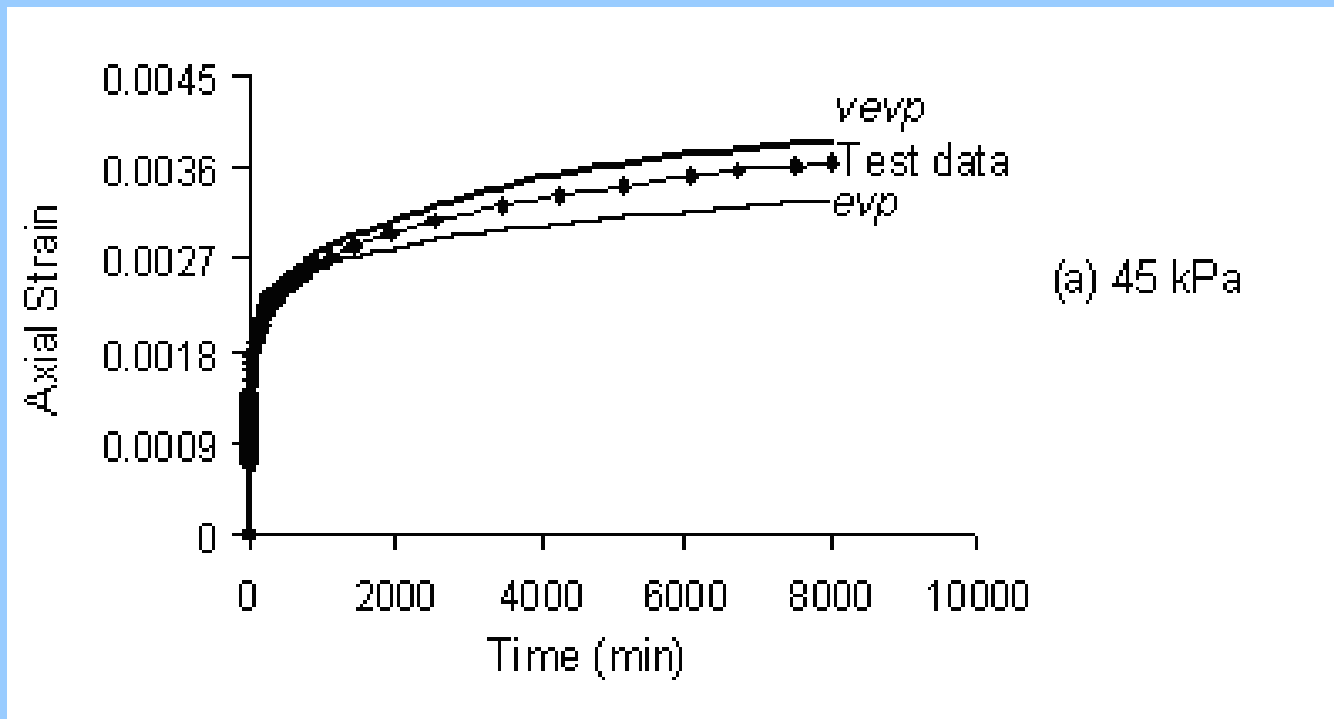


Level 2 (Independent) validation results for Sky Pilot till

Validation of creep (*evp* and *vevp*) model

Using Two-Dimensional Finite Element code for the Test Specimen as
Boundary Value Problem

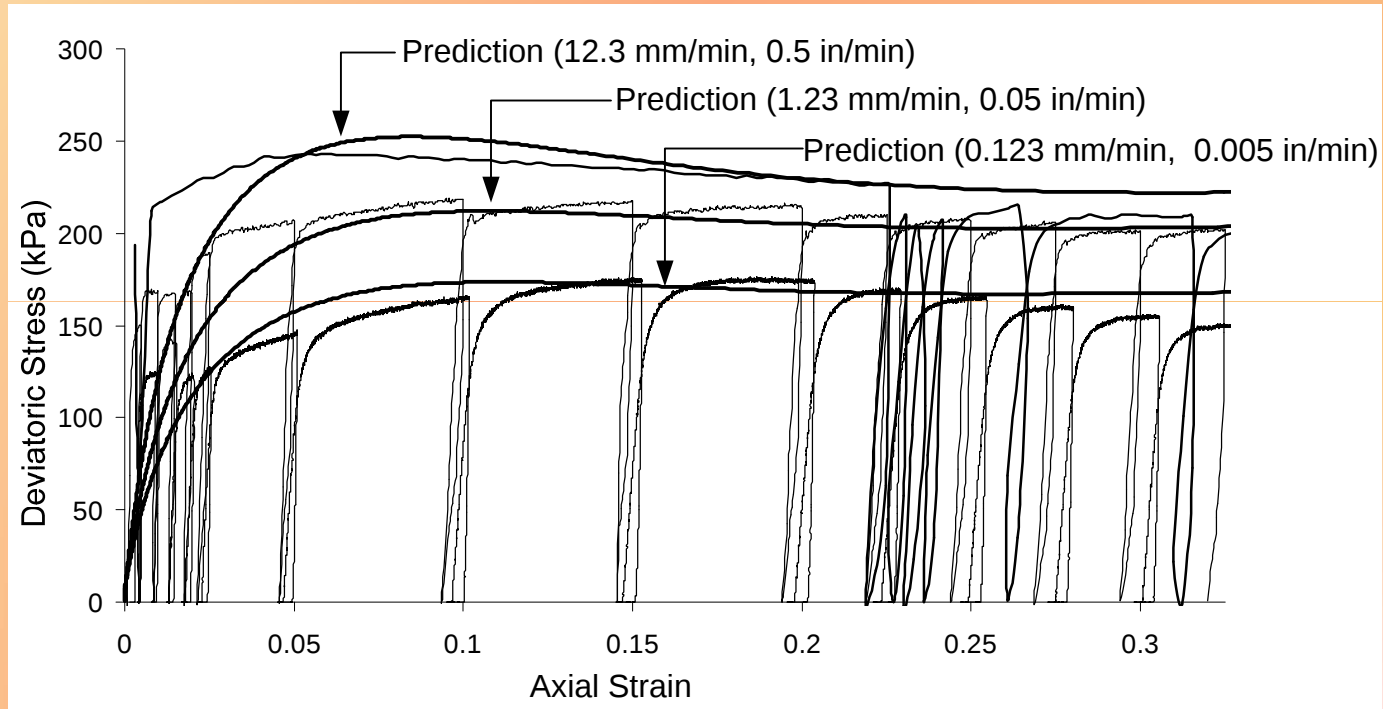




(a) 45 kPa

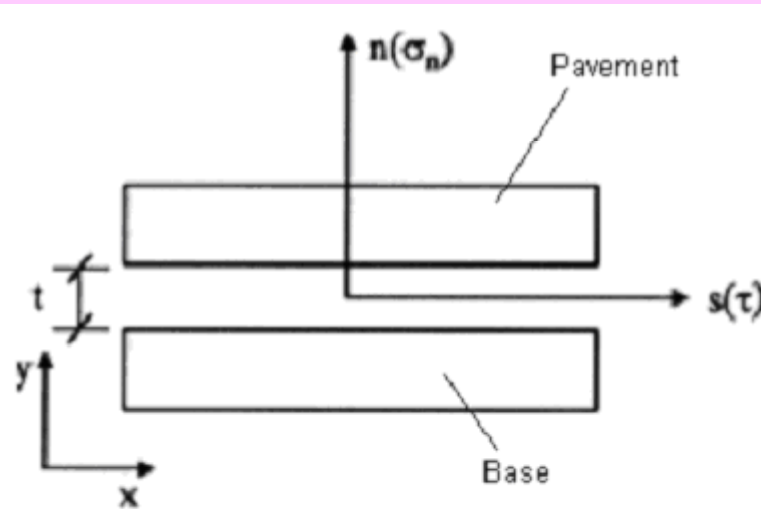
Level 2 FE Predictions for confining pressure = 300 kPa , stress = 45 kPa

Validation for Rate Dependent Tests for Glacial Till

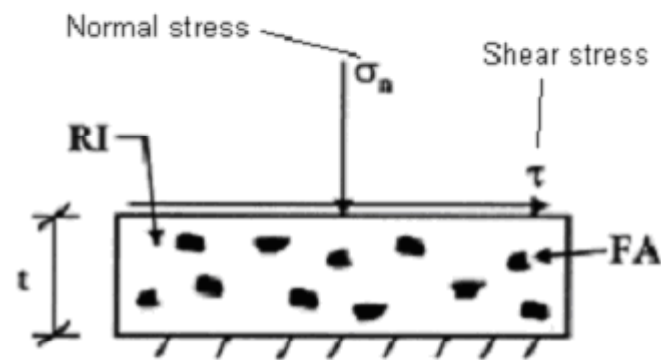


**Comparison of predicted and observed rate dependent curves
for Sky Pilot till**

DSC for Interfaces and Joints



(a) Two-dimensional interface with thickness t



(b) RI and FA states in interface

DSC Incremental Equations for Interfaces and Joints

Two-Dimensional

Observed shear stress

Relative Intact Shear stress

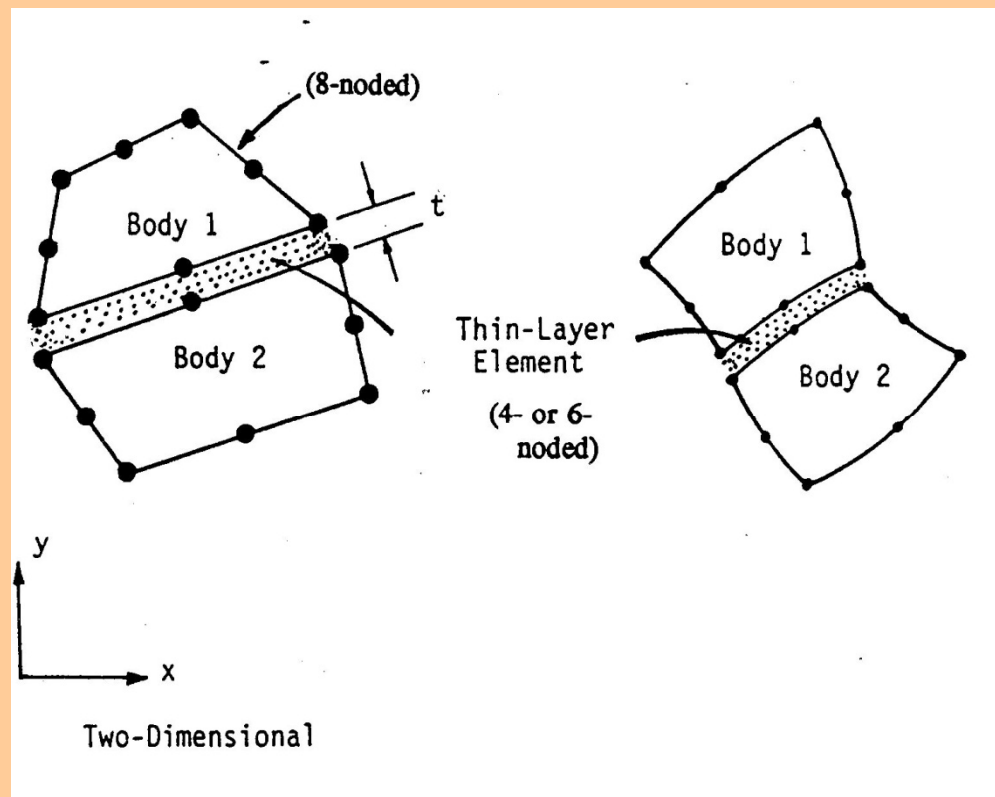
Fully Adjusted Shear stress

$$d\tau^a = (1-D)d\tau^i + D\tau^c + dD(\tau^c - \tau^i)$$

$$d\sigma_n^a = (1-D)d\sigma_n^i + Dd\sigma_n^c + dD(\sigma_n^c - \sigma_n^i)$$

Disturbance

COMPUTER Implementation



BACK PREDICTIONS AND FINITE ELEMENT IMPLEMENTATION

Stress-Strain-Volumetric

$$\underset{\sim}{d}\sigma = \underset{\sim}{C}^{DSC} \underset{\sim}{d}\varepsilon$$

Finite Element

$$\underset{\sim}{k}^{DSC} \underset{\sim}{\Delta}q = \underset{\sim}{\Delta}Q + \underset{\sim}{\Delta}Q_{\underset{\sim}{r}}$$

PREDICTION OF BEHAVIOR OF PACKAGING PROBLEMS

- Other papers
- Book: Mechanics of Materials and Interfaces: The Disturbed State Concept
C. S. Desai, CRC Press, 2001

VALIDATIONS FOR PRACTICAL PROBLEMS

Why Plasticity?

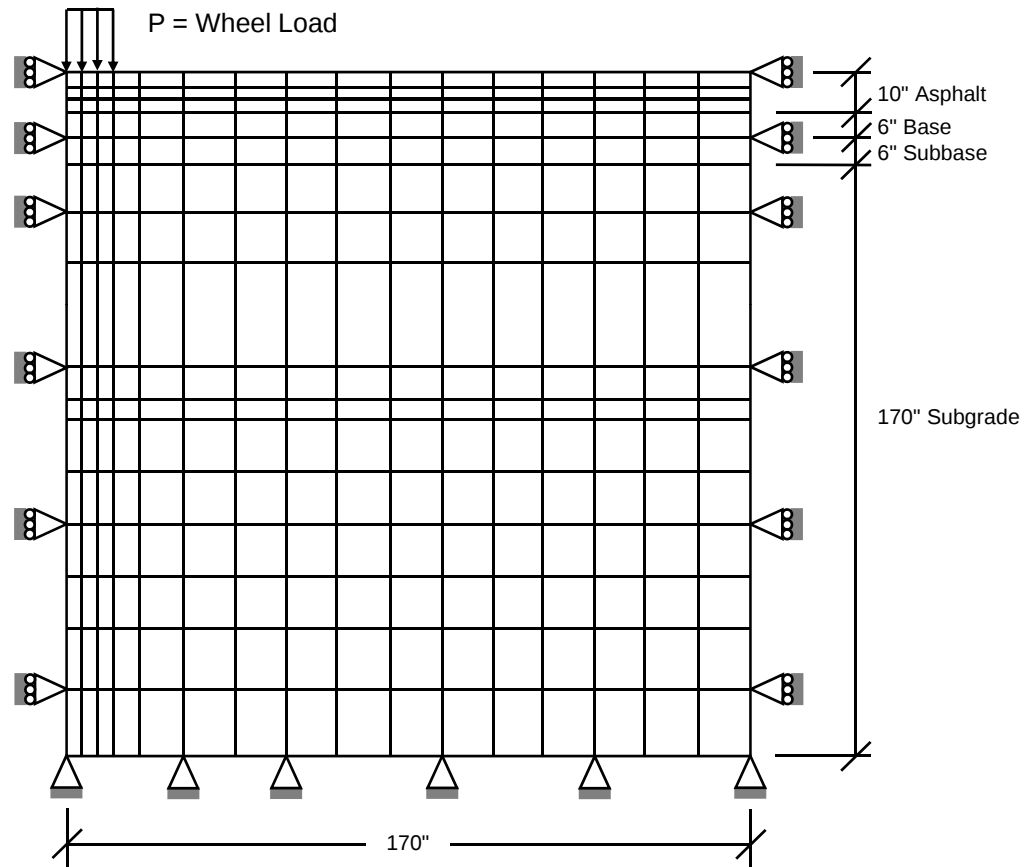
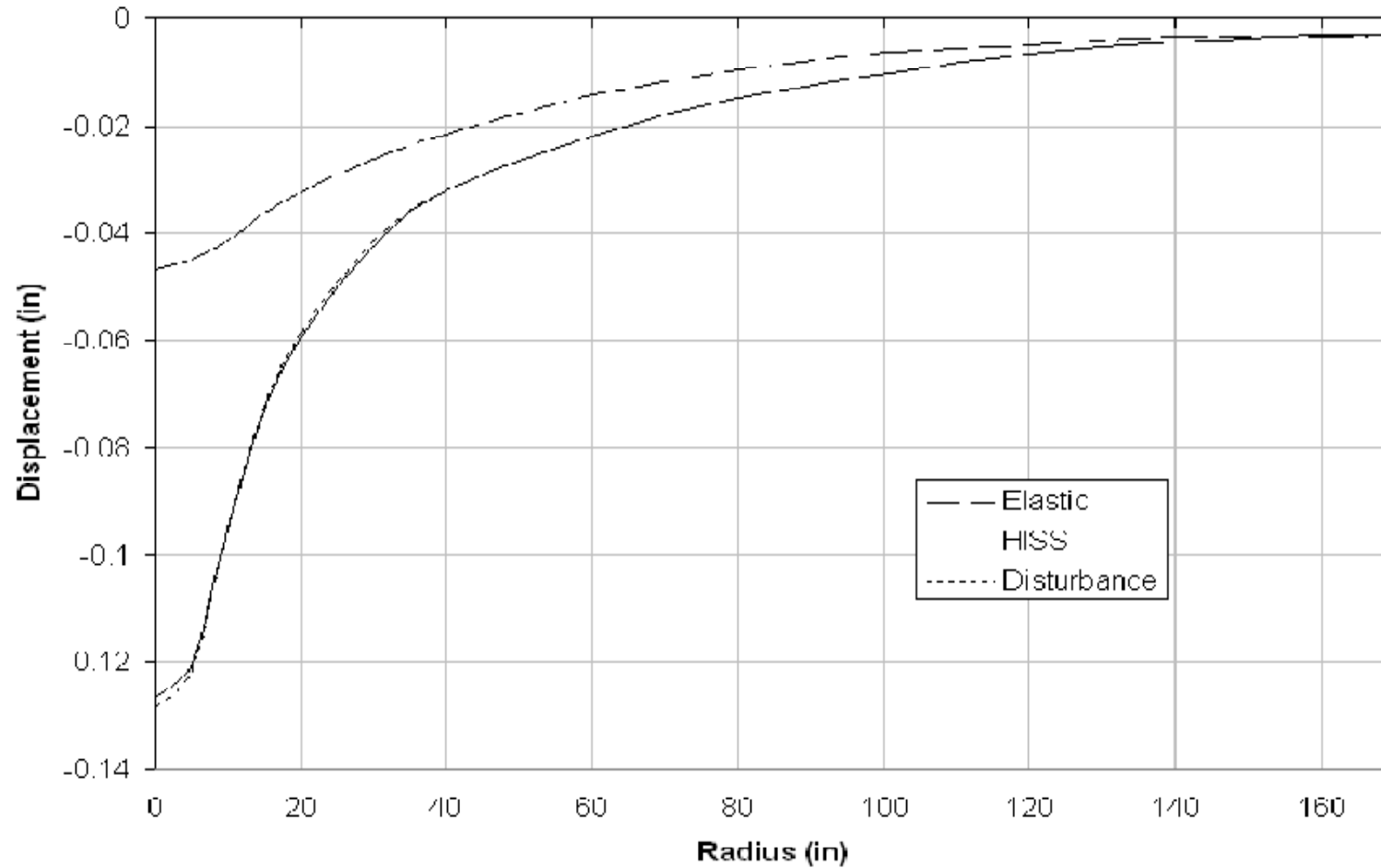


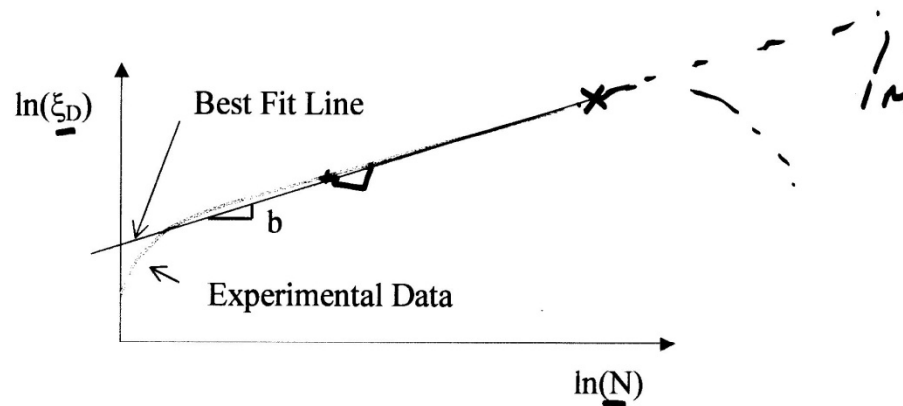
Figure 13. Finite Element Mesh for Plasticity Analysis (1 inch = 2.54 cm)



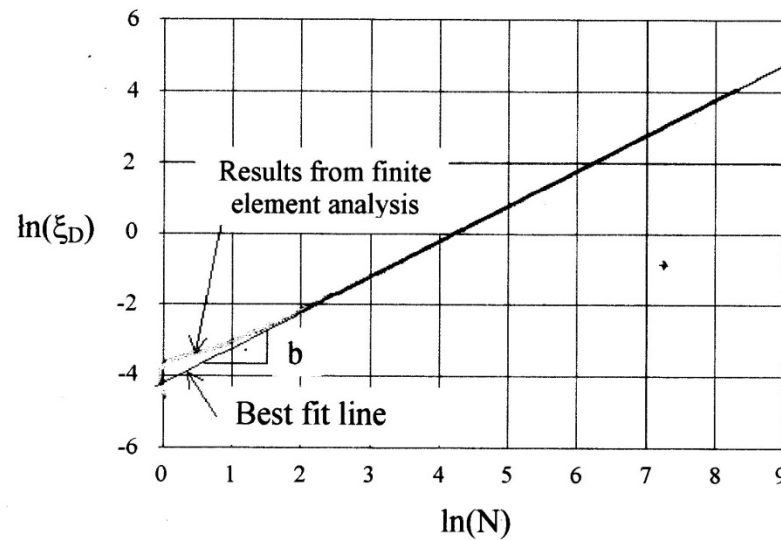
Surface Displacements Using Elasticity and Plasticity (HISS) Models

ACCELERATED APPROACH

Compute response at very large cycles based on a limited (about 10) complete finite element Analyses



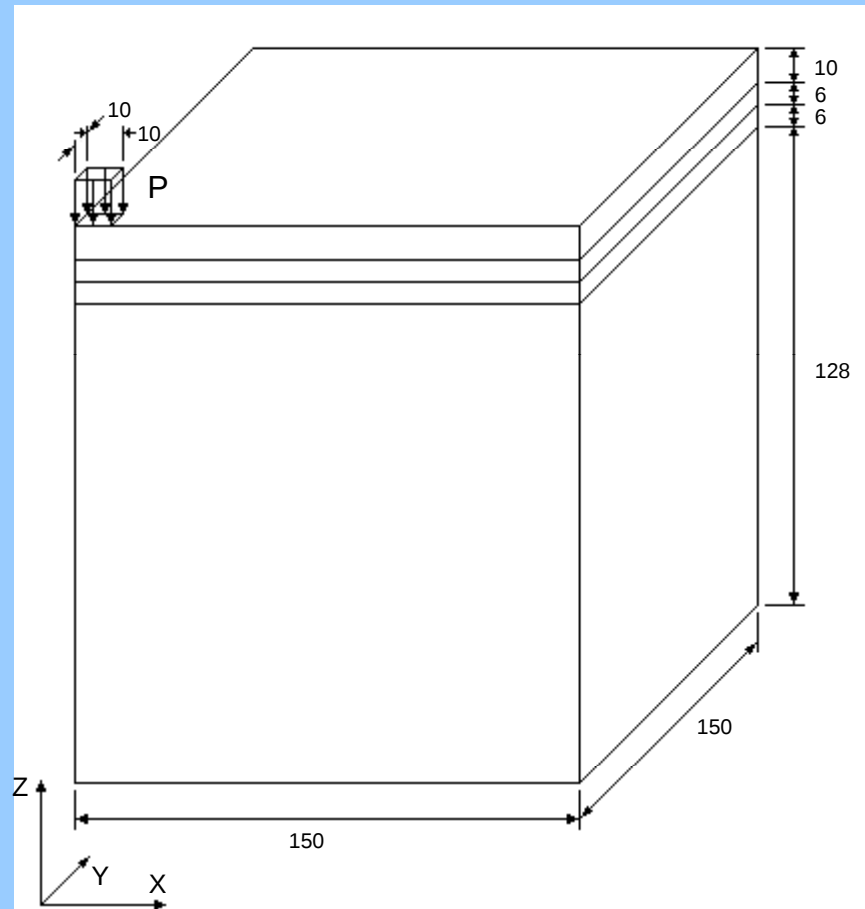
(a) Relation Between Deviatoric Strain Trajectory, ξ_D , and Cycle Number, N .



(b) Determination of Parameter b From Finite Element Analysis.

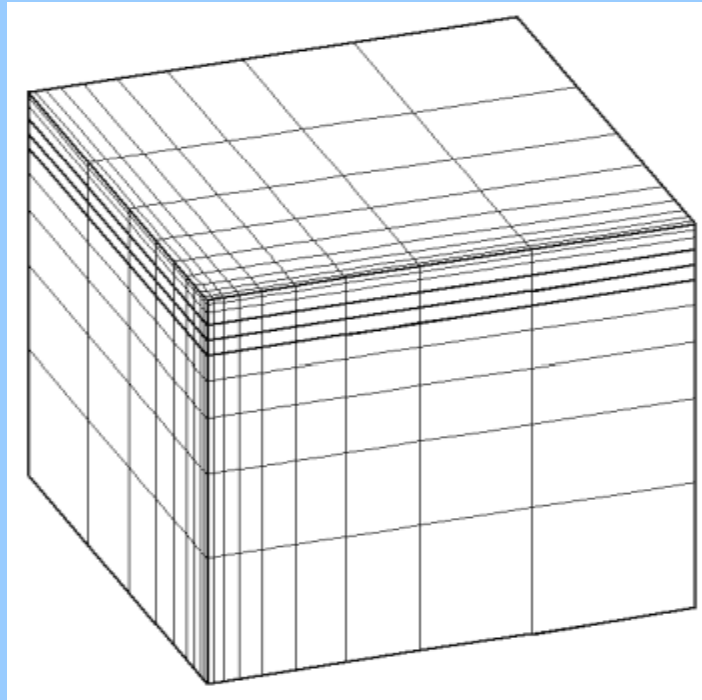
Relation Between ξ_D , N , and Parameter b .

2- and 3- Dimensional Analyses for Pavements

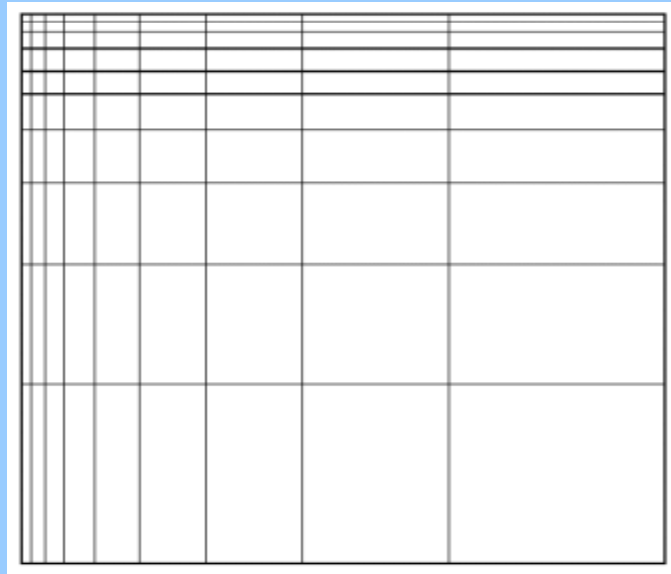


(a) Layered System (dimensions in inches): 1 inch=2.54 cm.

Figure 15. 2- and 3-D Analysis: Four Layered Pavement



(b) 3-D Mesh



(c) 2-D Mesh

Repetitive Cycles

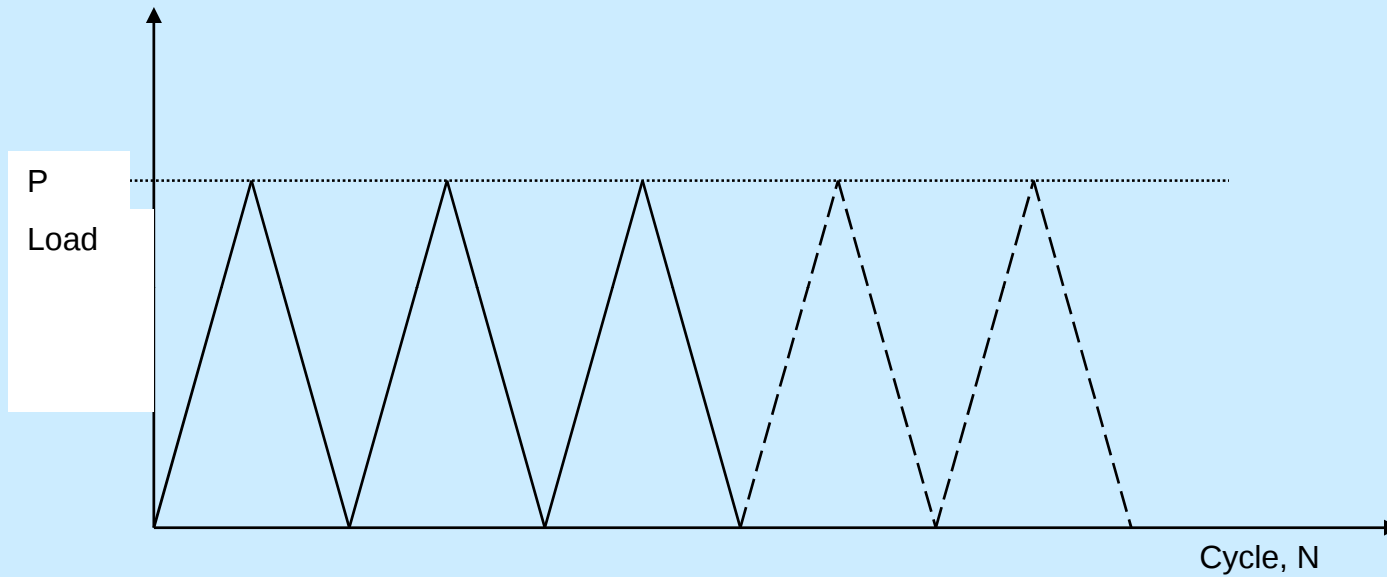
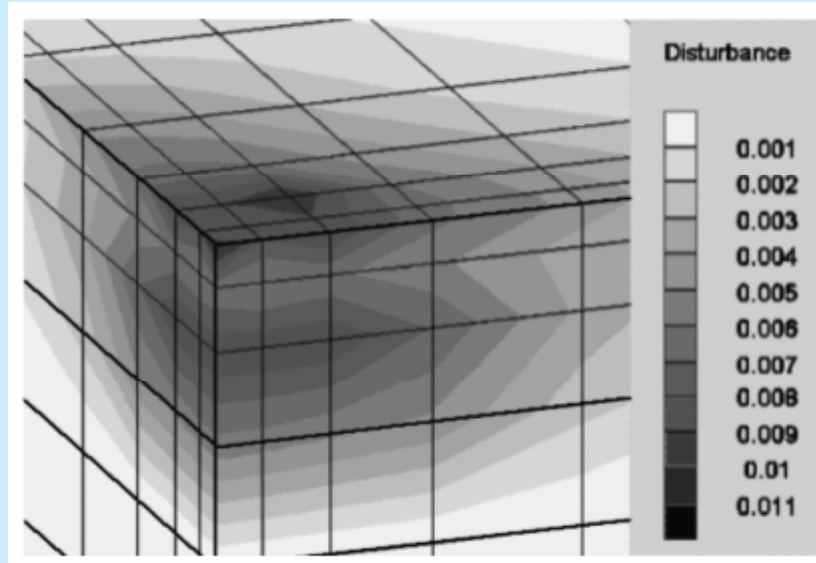
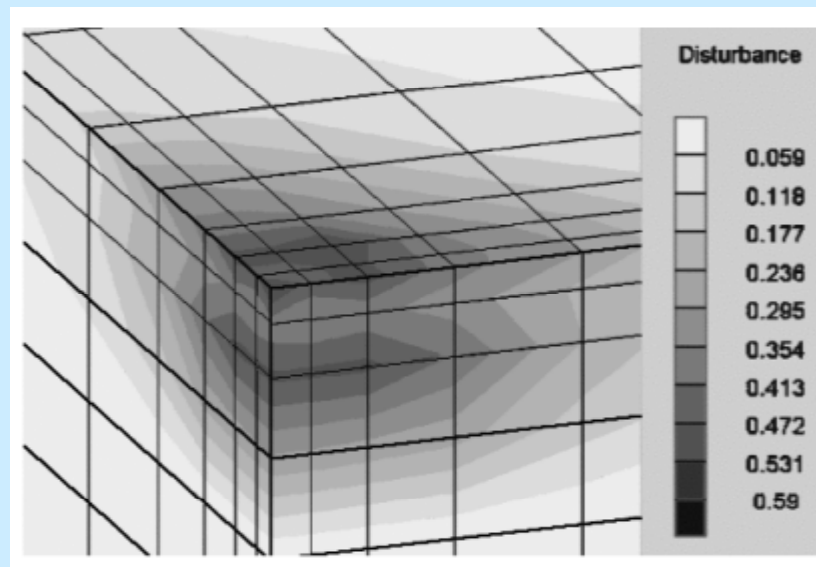


Figure 17. Schematic of Repetitive Loading



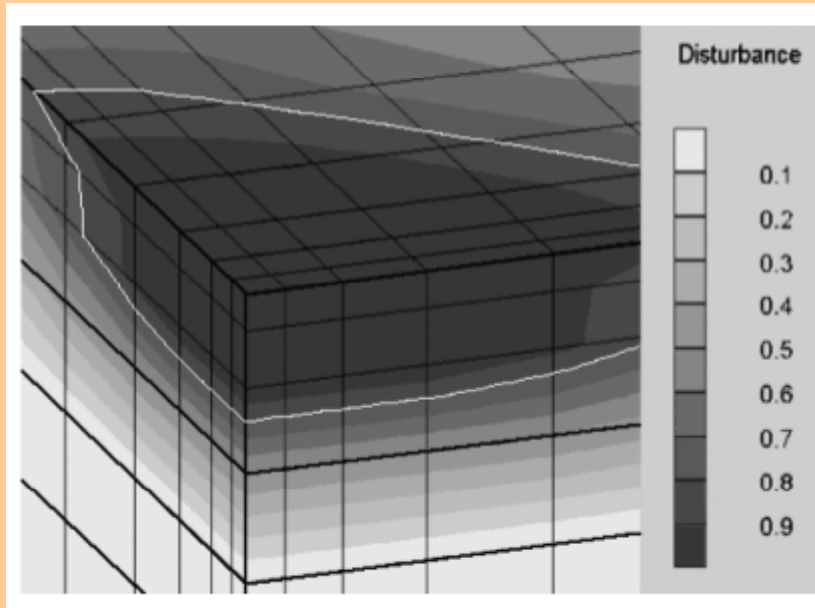
(a) $N = 10$ Cycles



(b) $N = 1000$ Cycles

**Contours of
Disturbance at
different cycles**

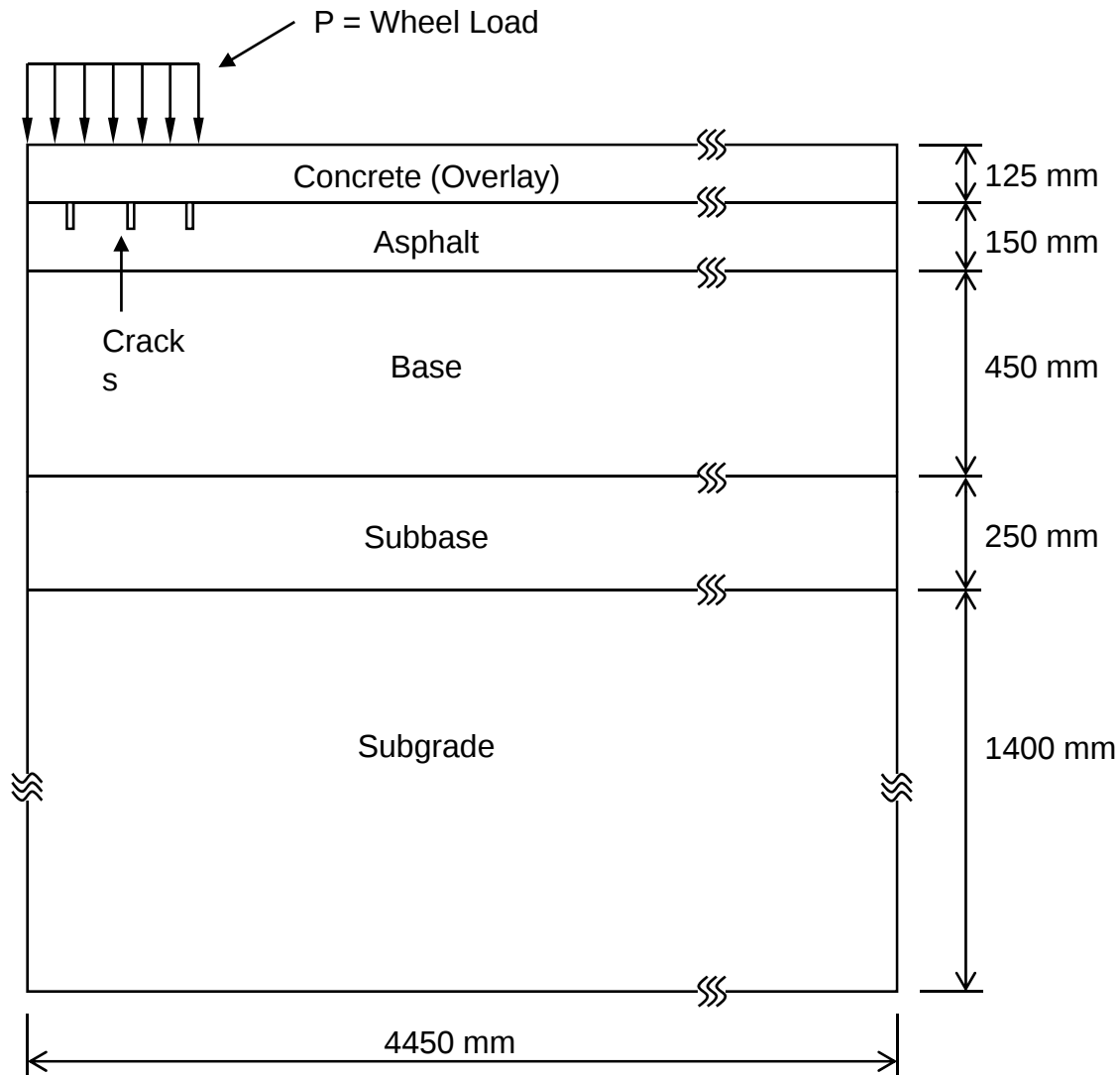
**Microcracking
leading to
fracture is
identified based
on critical
disturbance , D_c**



**N = 20,000
cycles**

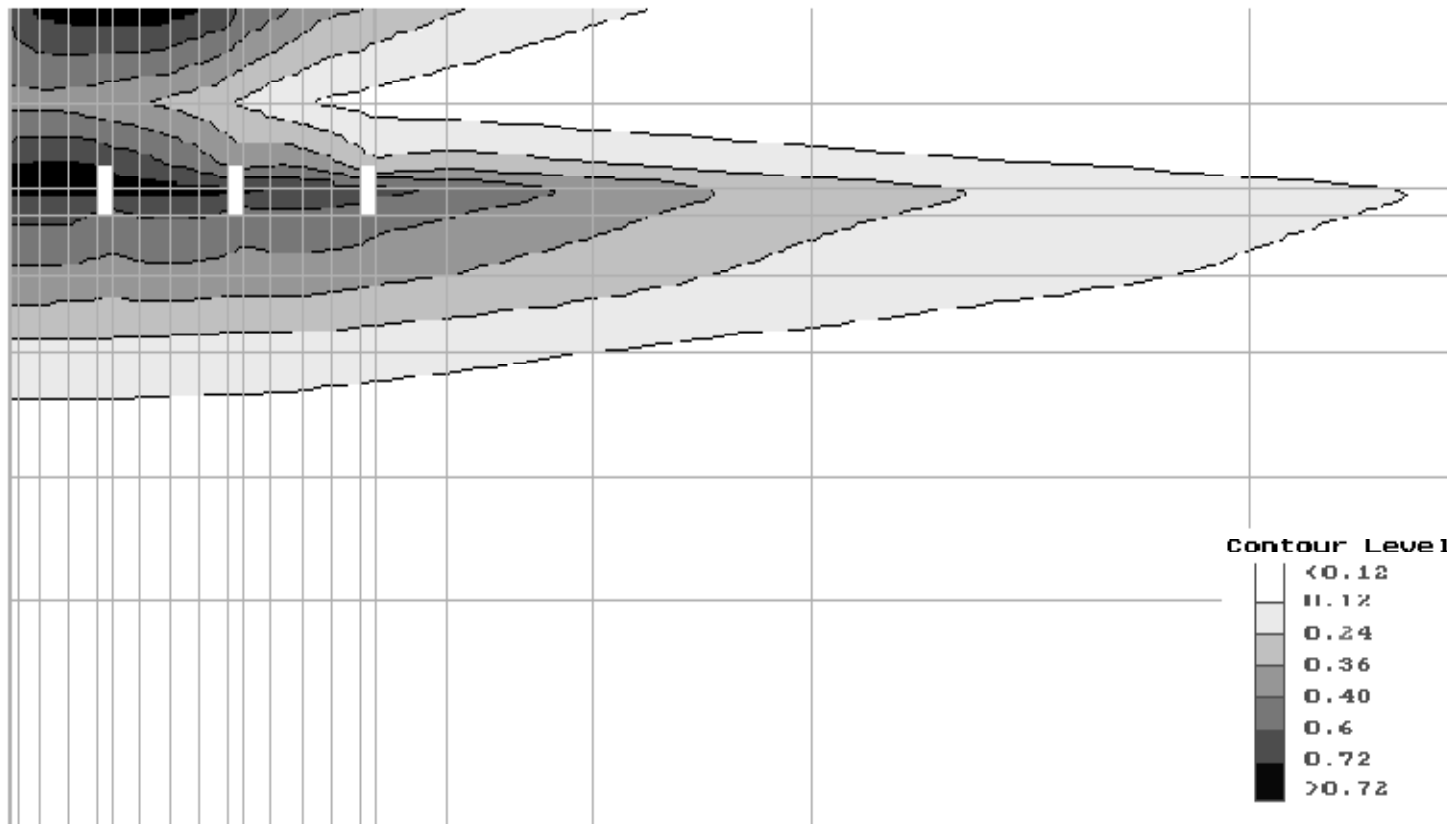
(c) N = 20,000 Cycles

(inside white curve)



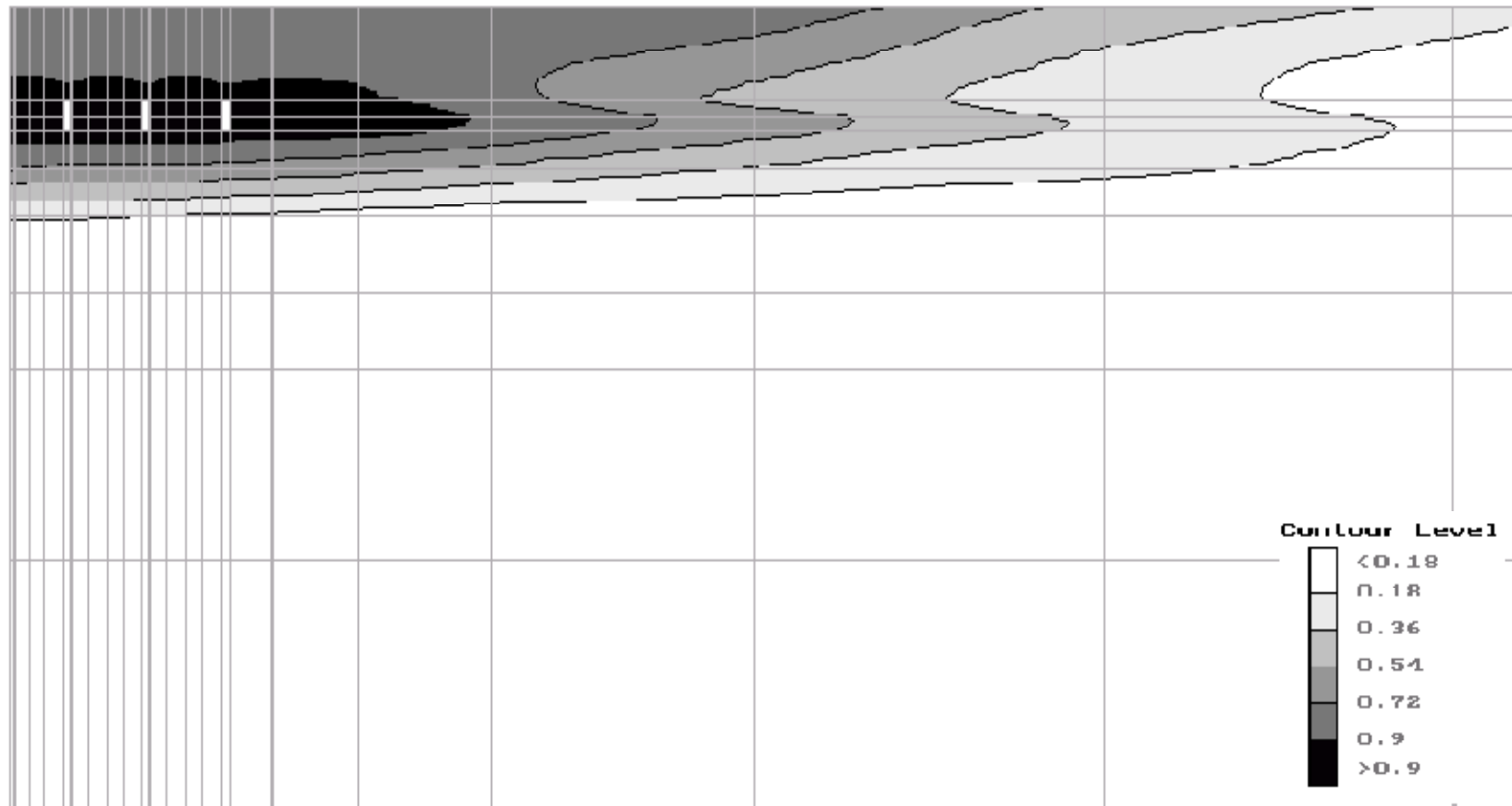
**Reflection
Cracking in
layered
pavements**

Figure 21. Reflection Cracking: Layered System with Three Existing Cracks in Asphalt



$N = 100$ cycles $D_c < 0.85$

Contours of Disturbance at Various Cycles:



$N = 10^6$ cycles

$D_c > 0.85$

Recent Publications:

- 1. Key Note Paper, 3rd Int. Conference on Pavements, Amsterdam, 2002.
- 2. Key Note Paper, 1st Int. Conf., on Transportation Geotechnics, Nottingham, 2008.
- 3. A Main Paper, Special Issue on Asphalt Pavements, Int. J. Of Geomechanics, ASCE, 2007.
- 4. Chapter 8, in Modeling of Asphalt Concrete, Y. Richard Kim (Editor), ASCE-McGraw Hill, 2009.
- Mechanics of Materials and Interfaces: The Disturbed State Concept, CRC Press, Boca Raton, FL, 2001

DSC Capabilities

Unified Constitutive model:

---Elasticity ----Elastoplasticity Creep: Elasto-viscoplasticity
---Microcracking to Fracture -----Softening or Degradation,
--- Healing or Stiffening --- Thermal effects -- Moisture

Loading: Static, Repetitive, Dynamic

Two- and Three-Dimensional Analyses with Dry and coupled(fluid flow through porous media)

Pavement Analysis and Design:

- Rutting or permanent deformation,
- Microcracking and Fracture
- Thermal Cracking
- Reflection Cracking

Hierarchical Options

$$d\sigma_{ij}^a = (1 - D) C_{ijkl}^i d\epsilon_{kl}^i + D C_{ijkl}^c \epsilon_{kl}^c + dD (\sigma_{ij}^c - \sigma_{ij}^i)$$

If $D=0$ Continuum Models, Elastic, Plastic or Creep etc.

If $D > 0$ Microcracking Leading to Fracture, Softening or Degradation

$D < 0$ Stiffening or Healing

Conclusions

- **DSC is considered to be unique and Unified model for pavement analysis, design and Maintenance**
- **Parameters have physical meanings and are equal to or lower than other models of comparable capabilities**
- **Parameters can be determined from standard triaxial or Multiaxial tests for asphalt or concrete and shear tests for interfaces and joints**
- **It has been implemented in 2- and 3-D nonlinear computer (finite element) procedures**

Conclusions (continued)

It is been validated at both specimen and practical boundary value problem levels for many engineering problems including geotechnical, structural, coupled porous media flow, mechanical, electronic packaging and Pavement engineering

Future Research on Unified DSC Models

- A number of factors affecting pavement materials have been considered; however, others such as thermal and moisture effects relevant to pavements are not fully developed.
- Laboratory testing under mechanical (two- and three-dimensional conditions) , thermal, moisture, etc. for pavement materials need to be investigated.
- Topics such as Healing, Reflection cracking, Creep and “static” yield surface, Rate Dependence and Compressive/Tensile yield, need to be investigated in details.
- **Validation** is a Vital aspect in which field and/or laboratory simulated practical problems should be tested and actual behavior be measured. Here, use of available test data could be considered.