PHYS 214 Final Exam Review

2/26/23

Tutors: Matthew, Wesley, Vedha

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Welcome to the Center for Academic Resour

Final Exam Review Schedule:

- Saturday, 10/1 5:30 - 7:30 pm in CIF 20

Worksheet

Solutions

Slides

Worksheets and solutions for past exams car … site.

Additionally, here is a Jupyter Notebook file … you on your test. If you don't have Jupyter on your computer, you may be able to open

Jupyter Notebook file

Good luck on your exam!

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Units for the Exam (from past midterms)

- Waves
- Interference
- Diffraction
- Photons & The Photoelectric Effect
- Probability and Complex Numbers
- The Wave Function
- Momentum and Position
- Energy Eigenstates

Units for the Exam (New Material)

- Harmonic Oscillator
- Multiple Electrons
- Band Structure
- Polarization and Spin

Units 1-4

Wave Equation

General wave propagation: $y(x,t) = A\cos(kx - \omega t + \phi)$

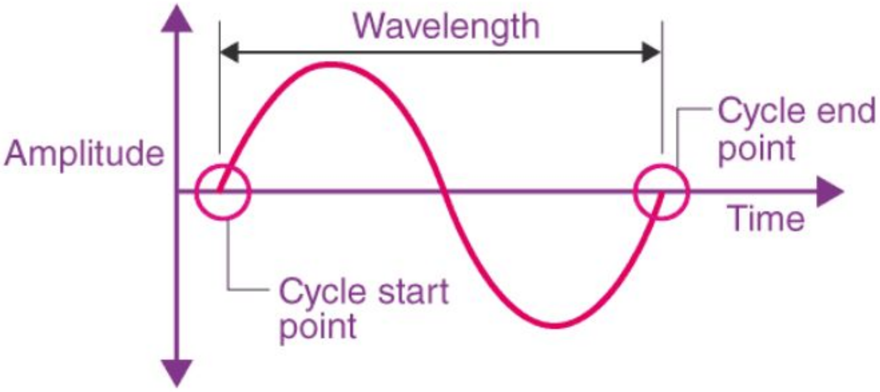
k = wave number (how the wave repeats in space)

ω = angular frequency (how the wave repeats in time)

ϕ = phase shift (the starting phase of the wave)

Properties of Waves

- $\lambda = 2\pi/k$; $f = \omega/\,2\pi$
- $v = \omega/k$
- Intensity: $I(x,t) = |y(x,t)|^2$
- $I_{average} = |A|^2/2$



Interference

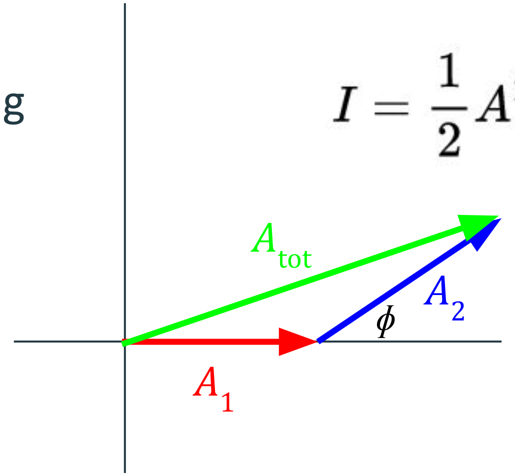
- Superposition (adding): A fancy way of saying that when two waves interact, the resulting wave is the sum of the two individual waves
- $y_1(x, t) = A_1\cos(kx - \omega t + \varphi_1)$
- $y_2(x, t) = A_2\cos(kx - \omega t + \varphi_2)$
- $y_{tot}(x,t) = y_1(x,t)+y_2(x,t) = A_1\cos(kx - \omega t + \varphi_1) + A_2\cos(kx - \omega t + \varphi_2)$
- If $\varphi_1 = \varphi_2$, the frequencies (ω) are the same, and distance is the same,then they are IN PHASE.

Phasor Diagrams

Add amplitudes, not intensities!

- Phasor diagrams present a graphical method for adding waves of any amplitude.
- The method:
    - 1. Find the phase difference
    - 2. Draw one vector with length A_1 at the horizontal
    - 3. Draw the second vector with length A_2 at an angle ϕ from the first vector
    - 4. Use trigonometry (i.e. Law of Cosines) to find the resulting length

$$I = \frac{1}{2}A^2$$



$$A_1^2 + A_2^2 + 2A_1A_2\cos\phi = A_{tot}^2$$

In your equation sheet:

$$A^2 + B^2 + 2AB\cos\phi = C^2$$

Add amplitudes, not intensities!

Phasor Diagrams

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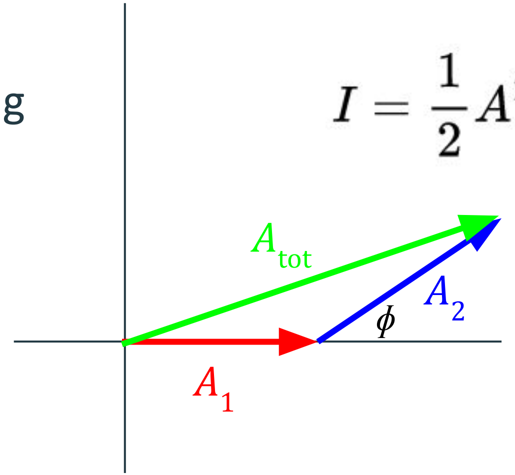

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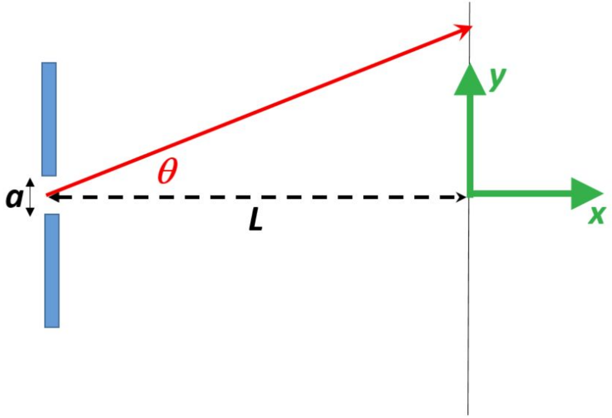
In your equation sheet:

$$A^2 + B^2 + 2AB\cos\phi = C^2$$

Diffraction

- Single slit diffraction
  - a = slit width
  - θ_0 = angle of first minimum
  - λ = wavelength
- Small a $\longrightarrow$ Large θ_0
- Small angles
  - $\theta \cong \sin(\theta) \cong \tan(\theta) \cong y_0/L$

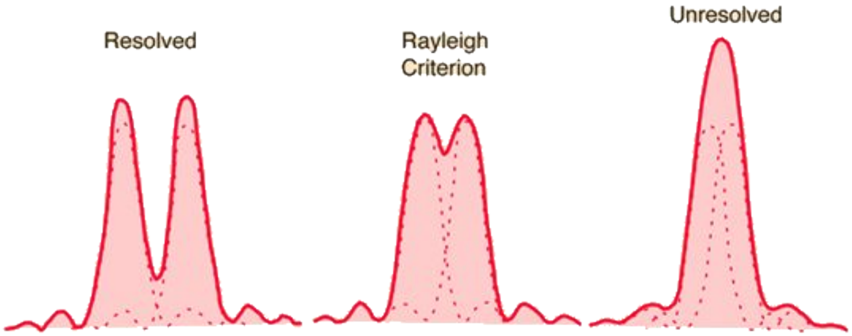
$$a \sin(\theta_\circ) = \lambda$$



Diffraction

- Circular aperture diffraction
  - Similar to single slit; 1.22 factor
- Rayleigh criterion
  - Center of one bright spot cannot overlap with the other bright spot
  - i.e. $\theta_0 \leq \theta$
    - θ_0 = angle of first minimum of central bright spot
    - θ = angle between two bright spots

$$D \sin(\theta_\circ) = 1.22\lambda$$

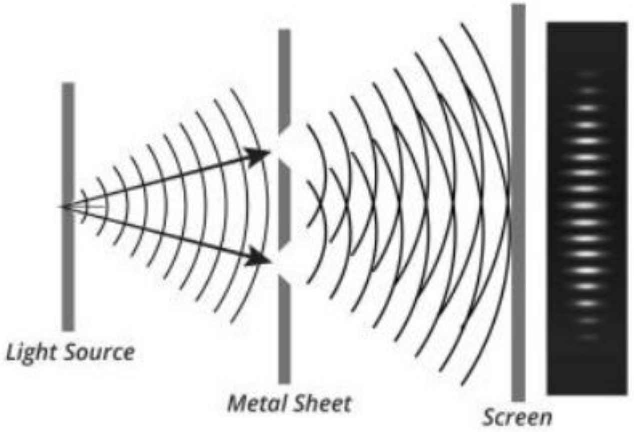


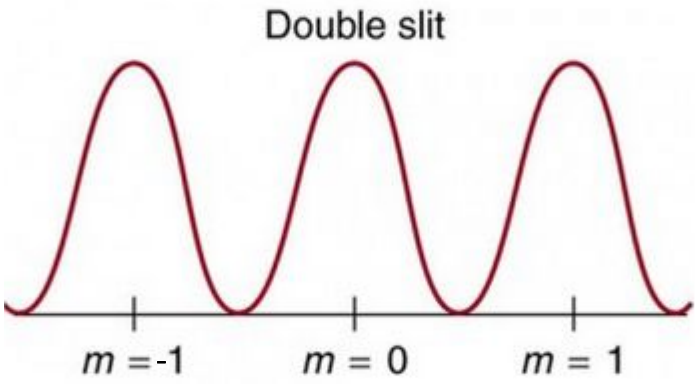
Diffraction

- Double slit diffraction:

$$d \sin\theta = m\lambda$$

- d = distance between slits
- If m is an integer $(0, \pm 1, \pm 2 \ldots)$ then theta shows the location of a **maximum**
- If m is a half-integer $(\pm 1/2, \pm 3/2 \ldots)$ then theta shows the location of a **minimum**





Photons

Photons: the quantized bits of light (particles of light)

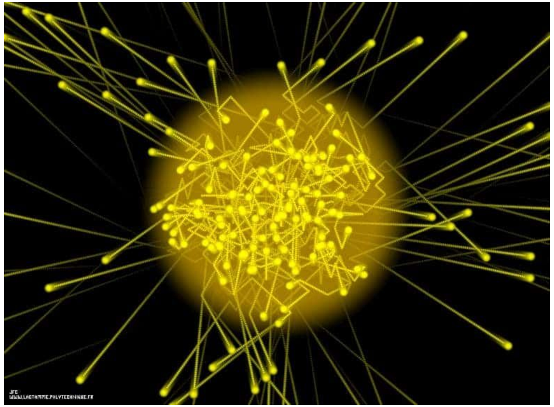
- Energy of single photon with frequency f :

$$E = hf = \hbar\omega = \frac{1240\ \text{eV nm}}{\lambda}$$

- Momentum of single photon with wavelength λ :

$$p = \hbar k = h/\lambda$$

- $h = 6.626 \times 10-34\ \mathrm{J \cdot s}$
- $\hbar$ (“h-bar”) $= h/2\pi$ $k = 2\pi/\lambda$ $\omega = 2\pi f$



Photoelectric Effect

This experiment proves the existence of photons and that light can be both a particle and a wave

$$KE_{\text{electron}} = eV_{\text{stop}}$$

Stopping Potential: Voltage applied to stop electrons from flowing between the two plates

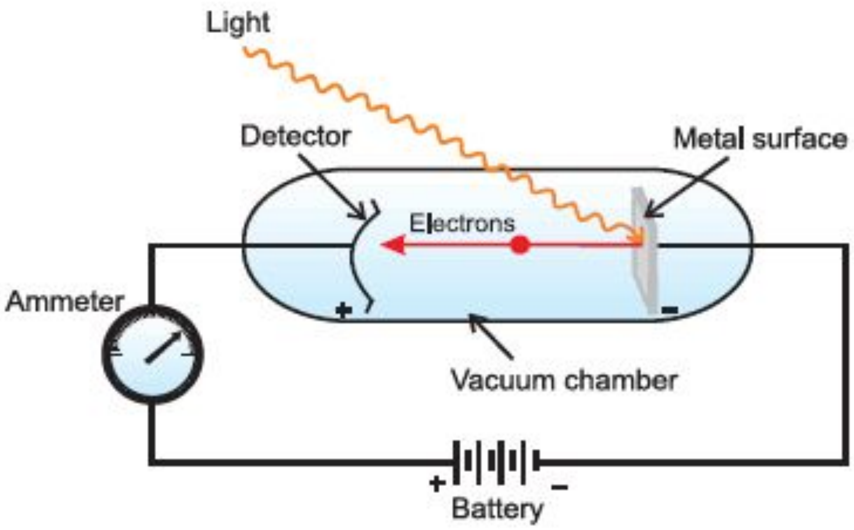
Work Function (property of the material the light is shining on)

$$KE_{\text{electron}} = hf - \Phi$$

Maximum Kinetic Energy of an ejected electron

Planck's constant times frequency of incoming photons (light)

Photoelectric Effect Setup



$$\frac{\#\text{ photons}}{\text{sec}} = \frac{P\text{ Joules}}{\text{sec}} \times \frac{1\text{ photon}}{X\text{ Joules}}$$
$$\text{where } X = hf = hc/\lambda$$

Increasing the power of a photon source will not increase photon energy! It will only increase photon flux. Frequency/wavelength is what determines photon energy.

Units 5-8

Probability and Complex Numbers

Probability

- Probability distribution:

$$\rho(x)$$

- Probability between two points:

$$P(a \leq x \leq b) = \int_a^b \rho(x) \mathrm{d}x$$

- Normalization:

$$P(-\infty \leq x \leq \infty) = \int_{-\infty}^{\infty} \rho(x) \mathrm{d}x = 1$$

Complex Numbers

- Two forms:

$$z = a + bi = |z| e^{i\theta}$$

- Conversion:

$$\theta = \arctan(b/a) \quad z = |z|(\cos\theta + i \sin\theta)$$

- Magnitude:

$$|z|^2 = (z^*)(z)$$

- Identities:

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \quad \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

The Wave Function

- The wavefunction, denoted Ψ(x), contains ALL information about the properties of a quantum particle

Properties:

- Probability Density: $\rho(x) = (\Psi^*)(\Psi) = |\Psi|^2$
  - Ψ* is the complex conjugate
- Probability of finding a particle between points a, b:

$$P(a \leq x \leq b) = \int_a^b (\Psi^*)(\Psi)\,\mathrm{d}x$$

- Normalization: $P(-\infty \leq x \leq \infty) = 1$
  - (We want there to be a 100% probability of finding the particle *somewhere*...)

Momentum and Position

- For particles with momentum p = ℏk, they are described by the wave function

$$\psi(x) = Ae^{ikx}$$

Recognize this momentum eigenstate!

- Some particles' wave function are superposition of these

First eigenstate Second eigenstate

$$\psi(x) = \boxed{ae^{ik_1x}} + \boxed{be^{ik_2x}}$$

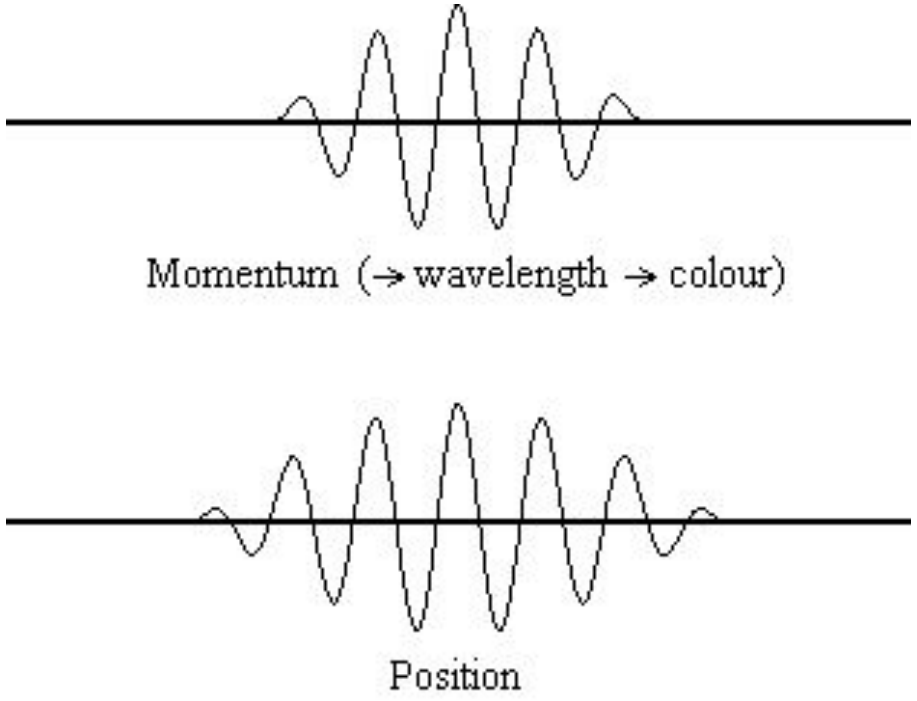
- In this case, when measured, it will collapse into one of these eigenstate and the momentum will be corresponding to that eigenstate, the possibility of collapsing to the second eigenstate is given by:

$$\frac{|b|^2}{|a|^2+|b|^2}.$$

- The **fewer** momentum eigenstates, the more certain we are for momentum
- The **more** momentum eigenstates, the more certain we are for position
- The overall uncertainty is given by Heisenberg uncertainty principle:

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

Momentum and Position (cont)



$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

Energy Eigenstates

- Quantum particles can sometimes be in two or more energy states at once!
- If we are in an energy eigenstate, we know with certainty that we are only in 1 energy state.
- This means that if we measure the energy, we will get a definite value.

$$\hat{H}\Psi = E\Psi$$

Hamiltonian Operator (Energy operator)

Energy eigenvalue

Energy Eigenstate Example

Infinite Square Well: (here "a" is the length of the well)

$$\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right), \quad E_n = \frac{n^2\pi^2\hbar^2}{2ma^2} = \frac{n^2h^2}{8ma^2}$$

Our wave function can ONLY be observed to have an energy which matches the above formula (for integer values of n)

Energy difference:

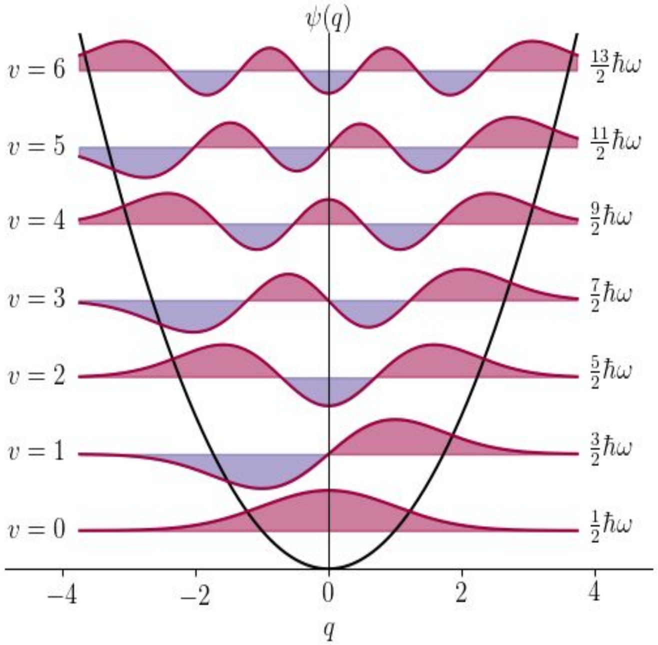
$$E_{n_1} - E_{n_2} = \frac{\hbar^2\pi^2}{2ma^2}\left(n_1^2 - n_2^2\right) = E_1\left(n_1^2 - n_2^2\right)$$

Units 9-12

(the focus of the final!)

Harmonic oscillator

- Potential in the form: $U(x) = \frac{1}{2} kx^2$
- Leads to a ground state wave function:
  - $\Psi_0(x) = Ae^{-ax^2}$
- Energy Levels: $E_n = \hbar\omega(n + \frac{1}{2})$ $n = 0, 1, 2 \ldots$
- Notice that these levels start at n = 0 and are evenly spaced!
- Similarity to PHYS 211: $\omega = \sqrt{k/m}$
  - You can think of atoms as having their own spring constant k



Photon Emission

- When a multi-state system **drops to a lower energy level**, the system **emits a photon**
- Likewise, when a multi-state system **absorbs a photon**, the system **raises to a higher energy level**
- Energy lost/gained by system is equal to the energy of the emitted/absorbed photon

Infinite Square Well:

$$E_{n_1} - E_{n_2} = \frac{\hbar^2\pi^2}{2ma^2}\left(n_1^2 - n_2^2\right) = E_1\left(n_1^2 - n_2^2\right)$$

Quantum Harmonic Oscillator:

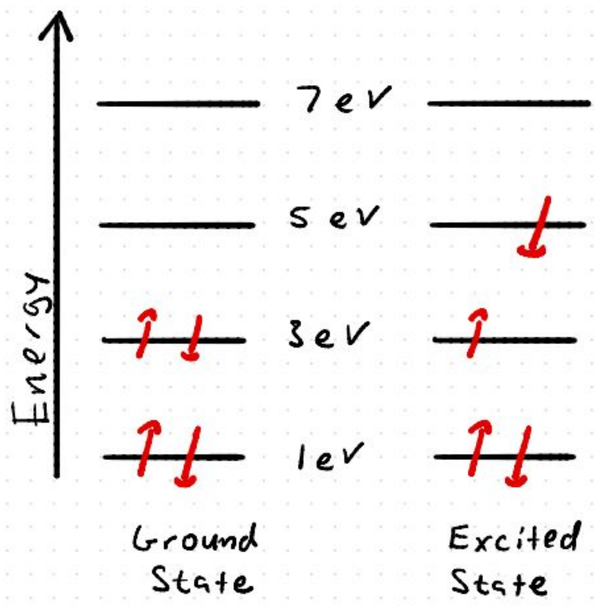
$$E_{n_1} - E_{n_2} = \hbar\omega(n_1 - n_2)$$

Photon Energy:

$$E = hf = \frac{hc}{\lambda}$$

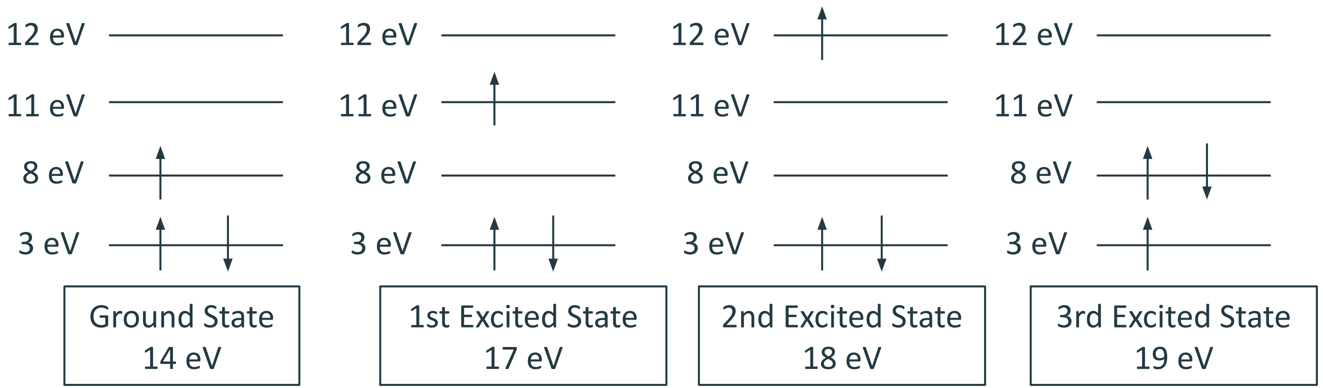
Multi-electron system

- **Aufbau Principle:** We fill energy states from bottom to top
- **Pauli-Exclusion Principle:** No two electrons can have the same spin in the same state
- Each excited state has the **next smallest possible energy**



Excited Energy States

Example: System with energy levels 3 eV, 8 eV, 11 eV, 12 eV; 3 electrons



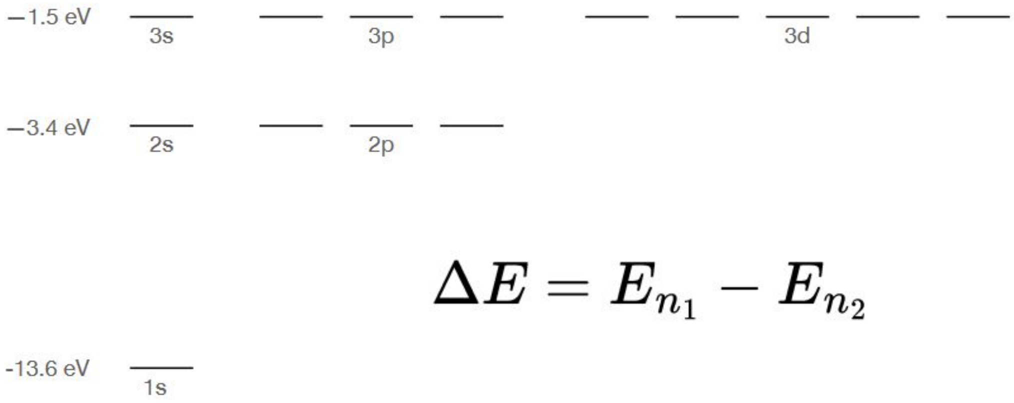
Successive excited states are always given by the <u>smallest change in energy!</u>

If the system has strange spacing (like in this example), then try different combinations to be sure.

Hydrogen: Another Multi-Level System

- Energy levels are filled from bottom to top, and from left to right
- As usual, a maximum of two electrons per state
- Note: there are many repeats of energy levels!
  - -13.6 eV has 1 levels
  - -3.4 eV has **4 levels**

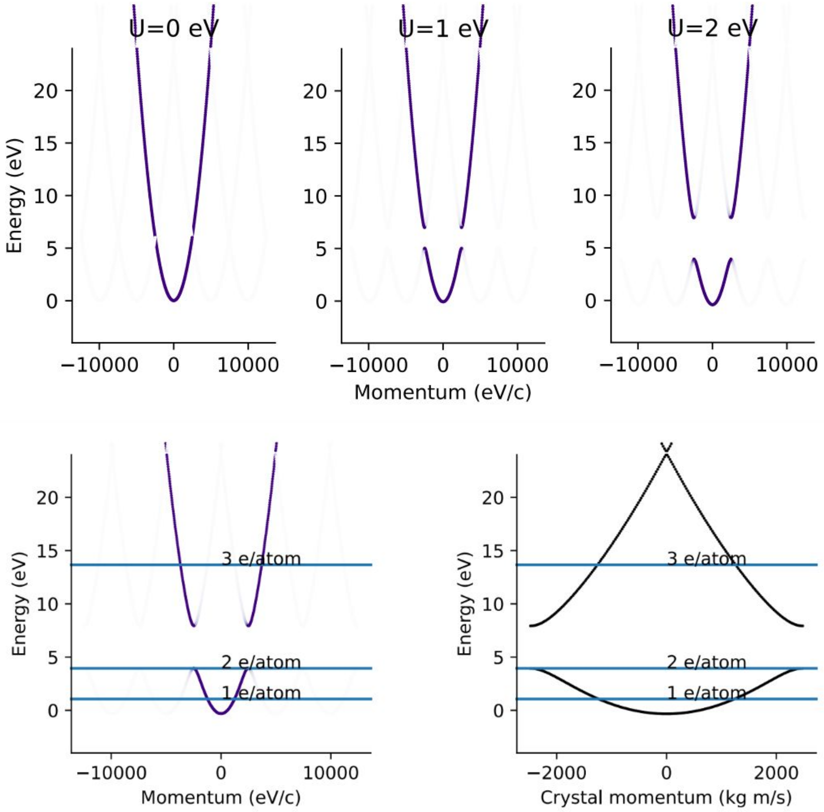
$$E_n = \frac{-13.6Z^2}{n^2}\ \mathrm{eV}$$



$$\Delta E = E_{n_1} - E_{n_2}$$

Band structure

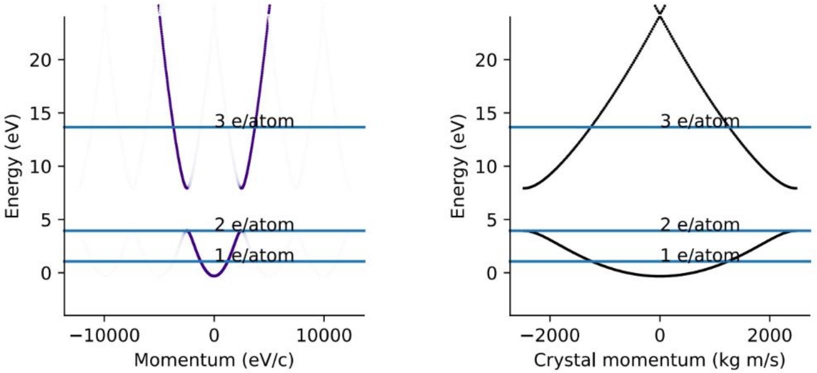
- Band structure describes the range of **energy levels** that electrons may have within it, as well as the **ranges of energy that they may not have**
- Gap: energy it takes to get from the ground state to the first excited state
- **Metals/Conductors:** Zero gap
- **Insulators:** Non-zero, large gap
- **Semiconductor:** Non-zero, smaller gap
    - Cooling down a semiconductor increase the size of the gap



Which electron densities are metals (conductor) and which ones are insulators?

Band Structure

- In this diagram:
  - Blue line = highest electron occupancy
  - **1 e/atom: conductor** (available states above highest electron occupancy)
  - **2 e/atom: insulator** (no available states past the highest electron occupancy)

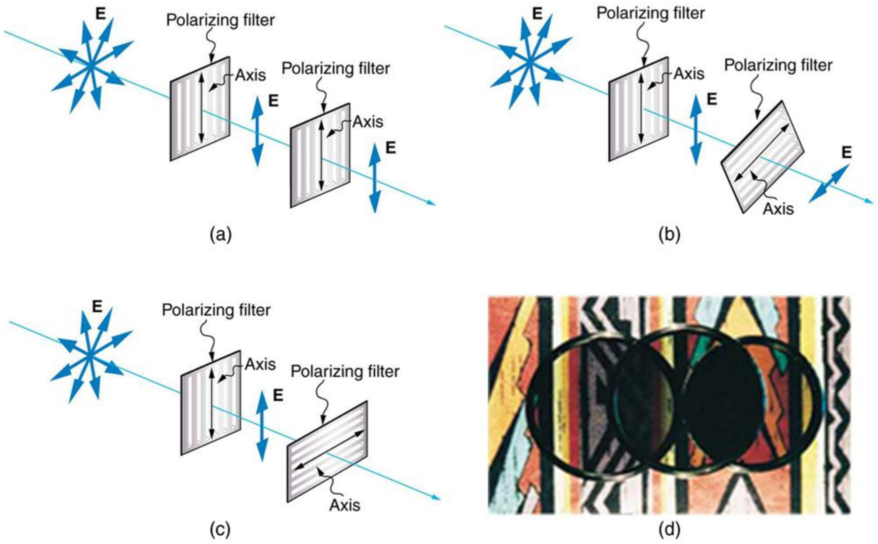


- For a material with a band gap, if we send a photon with frequency f:
  - $hf <$ band gap: photon passes through material
  - $hf >$ band gap: photon is absorbed and electron is excited (overcomes gap)

Polarization

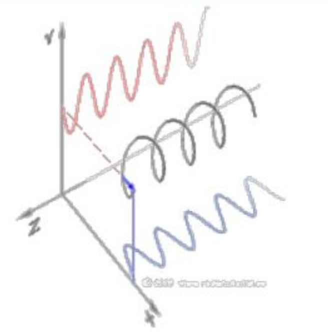
- Recall from PHYS 212…
  - "Polarization" refers to the direction of the electric field vector in an EM wave
  - Polarizers enforce a polarization direction
    - e.g. a horizontal polarizer will extract only the horizontal component of the light
    - Similar to taking the dot product to extract a horizontal component
- **In quantum physics , polarization is represented by polarization *states*:**

$$\Psi = a\Psi_v + b\Psi_h$$



Polarization

Polarization direction	State
Vertical	Ψ_v
Horizontal	Ψ_h
Diagonal (45 degrees)	$\frac{1}{\sqrt{2}}(\Psi_h + \Psi_v)$
Diagonal (-45 degrees)	$\frac{1}{\sqrt{2}}(\Psi_h - \Psi_v)$
Circular (right-handed)	$\frac{1}{\sqrt{2}}(\Psi_h + i\Psi_v)$
Circular (left-handed)	$\frac{1}{\sqrt{2}}(\Psi_h - i\Psi_v)$



- Light can have two polarization states:
  - Horizontal polarization ψ_h
  - Vertical polarization ψ_v
- Wave function can be the superposition of horizontal polarization and vertical polarization:

$$\Psi = a\Psi_v + b\Psi_h$$

- In this case, the probability of passing through a vertical filter is : –> $P(\text{vertical}) = \frac{|a|^2}{|a|^2 + |b|^2}$
- There are also circular polarization (when polarized this way, 50% chance to go through linear filter (horizontal or vertical)

Spin

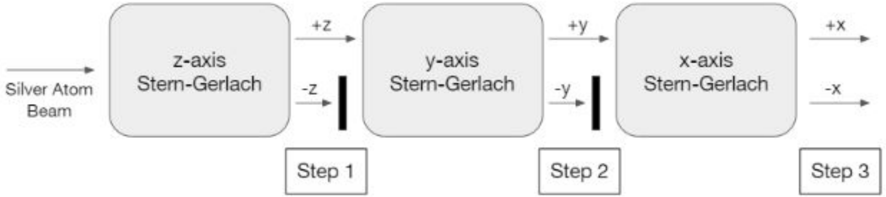
- Spin is similar in idea, just like horizontal and vertical in light, we have up and down spin for electrons:
- Spin up: $\uparrow$
- Spin down: $\downarrow$
- Any electron can be the superposition of these two spins:

Spin direction	State
$\hat{z}$	$\uparrow$
$-\hat{z}$	$\downarrow$
$\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow + \downarrow)$
$-\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow - \downarrow)$
$\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow + i \downarrow)$
$-\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow - i \downarrow)$


Spin up


Spin down

Stern-Gerlach setup: the “polarizer” for spin states:



Analogy to Vector Dot Products

- If a system is in the Ψ state and we want to find the probability of observing the S state:

$$\mathrm{P}(S) = |S^* \cdot \Psi|^2$$

dot product

- Recall **vector dot products:**

$$\hat{x} \cdot \hat{x} = 1, \quad \hat{y} \cdot \hat{y} = 1$$
$$\hat{x} \cdot \hat{y} = 0$$
$$\vec{a} = a_x \hat{x} + a_y \hat{y}, \quad \vec{b} = b_x \hat{x} + b_y \hat{y}$$
$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$$

- This dot product method works for **any S or Ψ**

In quantum:

$$\Psi_h, \Psi_v \text{ are like } \hat{x}, \hat{y}$$
$$\uparrow, \downarrow \text{ are like } \hat{x}, \hat{y}$$

$$\Psi_h^* \cdot \Psi_h = \Psi_v^* \cdot \Psi_v = 1$$
$$\uparrow^* \cdot \uparrow \;=\; \downarrow^* \cdot \downarrow \;= 1$$
$$\Psi_h^* \cdot \Psi_v = \Psi_v^* \cdot \Psi_h = 0$$
$$\uparrow^* \cdot \downarrow \;=\; \downarrow^* \cdot \uparrow \;= 0$$

Polarization and Spin

Polarization:

- Unit states: Ψ_h, Ψ_v
- State filter: **polarizers**
- Probability of observing state S from state Ψ: $\mathrm{P}(S) = |S^* \cdot \Psi|^2$
- Probability of observing opposite states:

$$\Psi_h^* \cdot \Psi_v = \Psi_v^* \cdot \Psi_h = 0$$

- Probability of observing a "table" state from unpolarized photons: 1/2

Spin:

- Unit states: $\uparrow, \downarrow$
- State filter: **Stern-Gerlach experiment**
- Probability of observing state S from state Ψ: $\mathrm{P}(S) = |S^* \cdot \Psi|^2$
- Probability of observing opposite states:

$$\uparrow^* \cdot \downarrow \;=\; \downarrow^* \cdot \uparrow \;= 0$$

- Probability of observing a "table" state from "unpolarized" particle beam: 1/2

"Table" States

Polarization direction	State
Vertical	Ψ_v
Horizontal	Ψ_h
Diagonal (45 degrees)	$\frac{1}{\sqrt{2}}(\Psi_h + \Psi_v)$
Diagonal (-45 degrees)	$\frac{1}{\sqrt{2}}(\Psi_h - \Psi_v)$
Circular (right-handed)	$\frac{1}{\sqrt{2}}(\Psi_h + i\Psi_v)$
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Spin direction	State
$\hat{z}$	$\uparrow$
$-\hat{z}$	$\downarrow$
$\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow + \downarrow)$
$-\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow - \downarrow)$
$\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow + i \downarrow)$
$-\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow - i \downarrow)$

Good Luck!!!

Let us know if you have any questions on homework, our worksheet, or practice exams!