# MATH 285 Midterm 1 Review

CARE

# Disclaimer

- These slides were prepared by tutors that have taken Math 285. We believe that the concepts covered in these slides could be covered in your exam.
- HOWEVER, these slides are NOT comprehensive and may not include all topics covered in your exam. These slides should not be the only material you study.
- While the slides cover general steps and procedures for how to solve certain types of problems, there will be exceptions to these steps. Use the steps as a general guide for how to start a problem but they may not work in all cases.

# Topics

I. Classifying Differential Equations
II. Slope Fields
III. Existence and Uniqueness
IV. Autonomous Equations
V. Solving Methods:
   I. Separable
   II. Exact
   III. Integrating Factor

# Differential Equations

- "A differential equation is any relationship between a function (usually denoted y(t)) and its derivatives up to some order."
- **Slope Fields:** Help visually model a differential equation
  - Lines parallel to the derivative at each point
  - Can show overall direction and shape of the solution, as well as equilibrium values

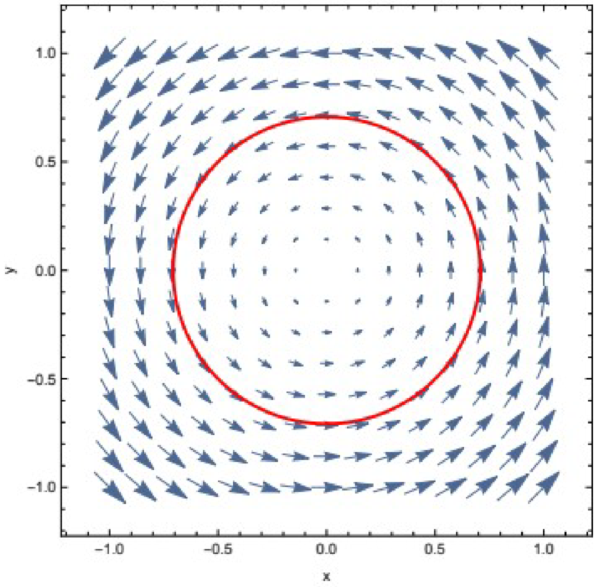


Figure 1.4: A slope field for $\frac{dy}{dt} = -\frac{t}{y}$ (blue) together with a solution curve (red).

*Differential Equations*. Bronski J., Manfroi A., Figure 1.4

# Classifications

Linear

2nd order

Ordinary

$$\frac{d^2y}{dt^2} + \sin(t)\frac{dy}{dt} + 15y = e^t$$

| **Ordinary vs Partial** | **Linear vs Nonlinear** | **Order** |
|---|---|---|
| • ODE's involve only standard derivatives<br>• PDE's involve partial derivatives | • Linear differential equations only have linear terms of the function and its derivatives<br>• Nonlinear equations are everything else | • The order of a differential equation is the degree of the highest derivative it contains |

# Existence and Uniqueness Theorem

$$\frac{dy}{dt} = f(y,t) \qquad\qquad y(t_0) = y_0$$

- A solution to the differential equation is **guaranteed to exist** in the **interval** in which the **first derivative is continuous around the initial value**
- That solution is **guaranteed to be** unique if $\frac{\partial f(y,t)}{\partial y}$ is also **continuous around the initial value**

# Autonomous Equations

- Autonomous equation: **does not explicitly involve independent variable**

$$\frac{dy}{dt} = f(y)$$

- Draw a **phase line**, identify points where the **derivative is 0**, and then **identify equilibria**
- Types of equilibria:
  - **Stable**: nearby points converge to the equilibrium
  - **Semi-stable**: points converge from one direction
  - **Unstable**: points diverge away from the equilibrium

# Separable Equations

- Separable Equations can be written as:

$$\frac{dy}{dt} = f(y)g(t)$$

- If your equation is separable, it can be solved **directly through integration**:

$$\int \frac{dy}{f(y)} = \int g(t)dt + C$$

# Exact Equations

- Exact equations have the form of:

$$\boldsymbol{N(x,y)\cdot y' + M(x,y) = 0}$$

- An equation is exact if the **partial derivatives of the two coefficient terms are equal**:

$$\boldsymbol{\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}}$$

# Solving Exact Equations

1. **Partially integrate** either *N* or *M*: $\int \boldsymbol{N \partial y}$ **or** $\int \boldsymbol{M \partial x}$

2. **Set equal to** $\Psi$ + a constant of integration function: $\boldsymbol{\Psi = \int N \partial y + f(x)}$ **or** $\boldsymbol{\Psi = \int M \partial x + f(y)}$

3. Take the **derivative** with respect to the **opposite variable:** $\boldsymbol{\frac{d\Psi}{dx}}$ **or** $\boldsymbol{\frac{d\Psi}{dy}}$

4. **Set equal** to the **other term** you didn't integrate: $\boldsymbol{\frac{d\Psi}{dx} = M}$ **or** $\boldsymbol{\frac{d\Psi}{dy} = N}$

5. **Integrate** to solve for f(x) or f(y) and plug back into step 2 $\int \boldsymbol{f'(x)dx}$ **or** $\int \boldsymbol{f'(y)dy}$

# Exact Equation Example

Solve the following differential equation:

$$\left(5x^2y + 2x + 4\right)\frac{dy}{dt} + \left(5xy^2 + 2y + 7\right) = 0$$

# Integrating Factor Method

1. Make sure your equation looks like: $$\frac{dy}{dt} + p(t)y = q(t)$$

2. Calculate the **integrating factor:** $$\mu(t) = e^{\int p(t)dt}$$

3. **Multiply the entire equation** by the integrating factor: $$\mu(t)\frac{dy}{dt} + p(t)\mu(t)y = \mu(t)q(t)$$

4. Re-write the left-hand side as the **result of product rule:** $$\frac{d}{dt}(\mu(t)y) = \mu(t)q(t)$$

5. **Integrate both sides** and rearrange to solve for *y(t)* $$\mu(t)y = \int \mu(t)q(t)dt$$

# Integrating Factor Example

Solve the following differential equation:

$$\boldsymbol{y' + 3y = 2}$$

# Thanks for Coming!