



## Center for Academic Resources in Engineering (CARE) Peer Exam Review Session

Phys 212 – University Physics: Electricity and Magnetism

### Midterm 1 Worksheet Solutions

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*The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.*

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Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Feb. 12, 5-7pm Ray, Devansh, Krish

Session 2: Feb. 14, 6:30-8:30 pm Jung Ki, Sahilkrishna, Ray

Can't make it to a session? Here's our schedule by course:

<https://care.grainger.illinois.edu/tutoring/schedule-by-subject>

Solutions will be available on our website after the last review session that we host.

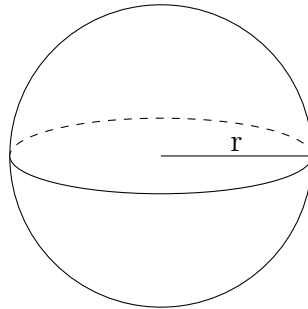
Step-by-step login for exam review session:

1. Log into Queue @ Illinois: <https://queue.illinois.edu/q/queue/848>
2. Click “New Question”
3. Add your NetID and Name
4. Press “Add to Queue”

**Please be sure to follow the above steps to add yourself to the Queue.**

Good luck with your exam!

1. A solid spherical conductor centered at the origin has radius  $r = 90$  cm and carries a total positive charge  $Q = 9 \mu\text{C}$



- (i) What is the magnitude of the electric field,  $|E|$ , at a radius of 1.7 m from the origin
- $|E| = 28000$  N/C
  - $|E| = 1.75 \times 10^5$  N/C
  - $|E| = 1 \times 10^5$  N/C
- (ii) If we define the electric potential to be zero at infinity, what is the potential  $V$  at a radius of 1.7 m from the origin? (Note: this is outside of the conducting sphere)
- $V = -47650$  Volts
  - $V = 90000$  Volts
  - $V = 47650$  Volts
  - $V = -90000$  Volts
  - $V = 0$  Volts
- (iii) If we define the electric potential  $V$  to be zero at infinity, what is the potential at a radius of 0.15 m from the origin? (Note: this is inside the conducting sphere)
- 540000 Volts
  - 540000 Volts
  - 90000 Volts
  - 90000 Volts
  - 0 Volts
- (iv) What would the answer to problem (iii) be if the sphere were an insulator instead of a conductor
- 90000 Volts
  - 90000 Volts
  - 164000 Volts
  - 164000 Volts
  - 134000 Volts
  - 134000 Volts

(i) Using Gauss' Law

$$\oint E \cdot dl = \frac{Q_{enc}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Plugging in all the values we get that  $|E|$  is around  $\boxed{28000 \text{ N/C}}$ . Note: don't forget to convert from centimeters to meters.

(ii)

$$\Delta V = - \int E \cdot dl$$

$$V(1.7) - V(\infty) = - \int_{\infty}^{1.7} \frac{Q}{4\pi\epsilon_0 r^2} dr$$

$$= - \frac{Q}{4\pi\epsilon_0} \left( - \frac{1}{r} \right)_{\infty}^{1.7}$$

$$= + \frac{Q}{4\pi\epsilon_0 r} \Big|_{\infty}^{1.7}$$

$$= \frac{Q}{4\pi\epsilon_0 * 1.7}$$

Plugging in the values we get that V is around  $\boxed{47650 \text{ Volts}}$ .

(iii) Since this is a conducting sphere, the electric field inside of it is zero. This means that the potential inside of it is *constant* ( $\Delta V = 0$  inside the sphere). Therefore, the potential at any point in the sphere is equal to the potential at the surface of the sphere.

$$\Delta V = - \int E \cdot dl$$

$$V(0.9) - v(\infty) = - \int_{\infty}^{0.9} E_{outside} \cdot dl$$

$$= - \frac{Q}{4\pi\epsilon_0} \left( - \frac{1}{r} \right)_{\infty}^{0.9}$$

$$= + \frac{Q}{4\pi\epsilon_0 0.9}$$

Plugging in the values we get that V is around  $\boxed{90000 \text{ Volts}}$ .

(iv) First we have to find the expression for the E-field inside the sphere:

$$\rho = \frac{Q}{\frac{4}{3}\pi(0.9^3)} = 2.94 \times 10^{-6} \frac{\text{C}}{\text{m}^3}$$

$$\oint E \cdot dA = \frac{Q_{enc}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{\rho(\frac{4}{3}\pi r^3)}{\epsilon_0}$$

$$E_{inside} = \frac{\rho r}{3\epsilon_0} = \frac{Qr}{4\epsilon_0 R^3 \pi}$$

Where  $R$  is the radius of the sphere.

Now we can find the potential difference

$$\Delta V = - \int E \cdot dl$$

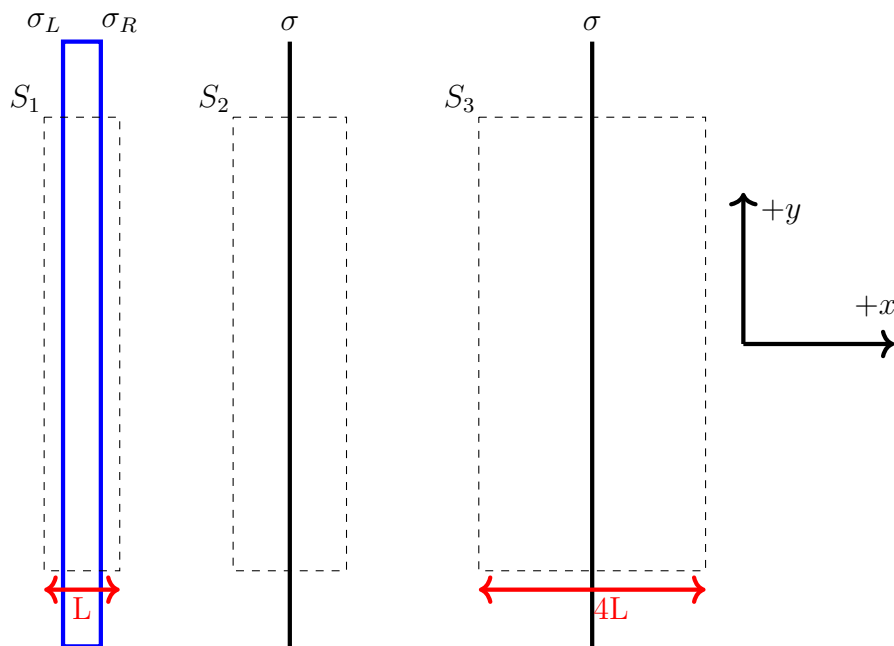
$$V(0.15) - v(\infty) = - \int_{\infty}^{0.9} E_{outside} \cdot dl - \int_{0.9}^{0.15} E_{inside} \cdot dl$$

$$= - \frac{Q}{4\pi\epsilon_0} \left( -\frac{1}{r} \right)_{\infty}^{0.9} - \frac{\rho}{6\epsilon_0} (r^2)_{0.9}^{0.15}$$

$$= + \frac{Q}{4\pi\epsilon_0(0.9)} - \frac{\rho}{6\epsilon_0} (0.15^2 - 0.9^2)$$

Plugging in the values the expression yields 134000 Volts.

2. The figure shows three infinite planes. The right two planes are insulating with uniform charge and density  $\sigma = 7 \text{ C/m}^2$ . The left plane is uncharged ( $\sigma_L + \sigma_R = 0$ ) and conducting. Also shown in the figure are three Gaussian surfaces labeled  $S_1$ ,  $S_2$  and  $S_3$ . All three Gaussian surfaces have identical dimensions in the  $yz$  plane, but surface  $S_3$  is 4 times as wide in the  $x$ -direction.



- (i) What is the induced charge on the right side of the conducting slab?
- a)  $\sigma_R = -7 \text{ C/m}^2$

- b)  $\sigma_R = -14 \text{ C/m}^2$   
 c)  $\sigma_R = -3.5 \text{ C/m}^2$

(ii) Compare the total flux through Gaussian surface  $S_1$  with the total flux through surface  $S_3$

- a)  $\Phi_1 < \Phi_3$   
 b)  $\Phi_1 = \Phi_3$   
 c)  $\Phi_1 > \Phi_3$

(iii) Compare the total flux through Gaussian surface  $S_2$  with the total flux through the surface  $S_3$

- a)  $\Phi_2 = \Phi_3$   
 b)  $\Phi_2 < \Phi_3$   
 c)  $\Phi_2 > \Phi_3$

(i) Treat  $\sigma_L$  and  $\sigma_R$  to be their own infinite planes

$$E = \frac{\sigma}{2\epsilon_0}$$

Inside conductor  $E = 0$

$$E = 0 = \frac{1}{2\epsilon_0}(-\sigma_L + \sigma_R + \sigma + \sigma)$$

Using  $\sigma_L = -\sigma_R$

$$2\sigma_R = -2\sigma$$

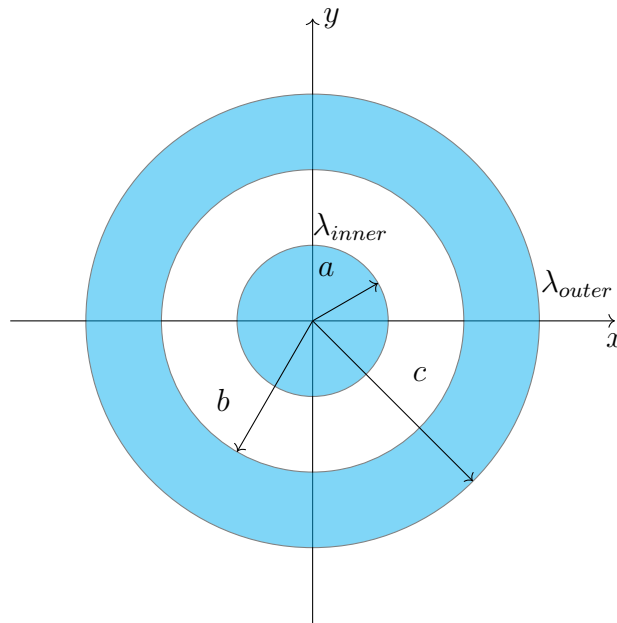
$$\sigma_R = -\sigma = \boxed{-7 \text{ C/m}^2}$$

(ii) The flux through  $S_1$  is zero. This is because every field line that enters it, also exits it (and there is no net charge within  $S_1$  for electric field lines to leave it). This is because  $|\sigma_L| = |\sigma_R|$ . Also all the field lines from the other planes enter and exit the surface, netting zero flux.

$S_3$  on the other hand has many field lines leave its right side, but zero flux out of its left side, there the flux through  $S_3$  is greater than that of  $S_1$ . The answer is therefore **(a)**.

(iii) The flux through  $S_2$  and  $S_3$  is zero at the inner edges (right edge of  $S_2$  and left edge of  $S_3$  respectively). Therefore, it's only the outer edges of those two surfaces that contribute to the total flux, and those two contributions are equal since they have the same magnitude of electric fields passing through them. The answer is **(a)**.

3. A solid, infinite metal cylinder of radius  $a = 2 \text{ cm}$  is centered on the origin, and has charge density  $\lambda_{inner} = -10 \text{ nC/cm}$ . Surrounding this cylinder is a cylindrical metal shell of inner radius  $b = 5 \text{ cm}$  and outer radius  $c = 7.5 \text{ cm}$ . This shell is also centered on the origin, and has total charge density  $\lambda_{shell} = +5 \text{ nC/cm}$ .



- (i) Find the potential difference  $V_a - V_c$  between the surface of metal cylinder ( $r = a$ ) and the outer surface of the metal shell ( $r = c$ ).
- a) -16.5 kV
  - b) -8.7 kV
  - c) 0 kV
  - d) 8.7 kV
  - e) 16.5 kV
- (ii) What is the linear charge density,  $\lambda_{shell-outer}$ , on the outer surface of the cylinder shell?
- a) 5 nC/cm
  - b) 3 nC/cm
  - c) -3 nC/cm
  - d) 0 nC/cm
  - e) -5 nC/cm
- (iii) What is the surface charge density,  $\sigma$ , on the inner solid metallic cylinder which has radius  $a$ ?
- a) 0
  - b) -0.80 nC/cm<sup>2</sup>
  - c) -4.44 nC/cm<sup>2</sup>
  - d) 0.80 nC/cm<sup>2</sup>
  - e) 4.44 nC/cm<sup>2</sup>
- (iv) If the inner cylinder is connected to ground, the charge density on the inner surface of the outer shell will
- a) Remain unchanged

- b) Decrease in magnitude  
 c) Increase in magnitude  
 d) Be zero
- (i) The potential difference between  $a$  and  $c$  is the same as the potential difference between  $a$  and  $b$  because the electric field inside the conductor is zero. Therefore, to solve the problem all we need is

$$\Delta V = - \int_a^b E \cdot dl$$

We know that

$$\oint E \cdot dA = \frac{q_{enc}}{\epsilon_0}$$

$$q_{enc} = \lambda_{inner} * L$$

$$E(2\pi r L) = \frac{\lambda_{inner} L}{\epsilon_0}$$

$$E = \frac{\lambda_{inner}}{2\pi\epsilon_0 r}$$

from the electric field equation outside of a cylinder. Putting all of the values into the integral gives an answer of  $\boxed{-16.5 \text{ kV}}$  (be careful with units, nC and cm).

- (ii) Using Gauss' Law we draw a Gaussian surface through the conductor. Since the E-field has to be 0 along the surface

$$0 = \lambda_{inner} + \lambda_{shell-inner}$$

This means that the inner shell has a charge density of 10 nC/cm. Now to get the outer charge density we know that

$$5 \text{ nC/cm} = \lambda_{inner} + \lambda_{shell-inner}$$

which gives us an answer of  $\boxed{-5 \text{ nC/cm}}$ .

- (iii)

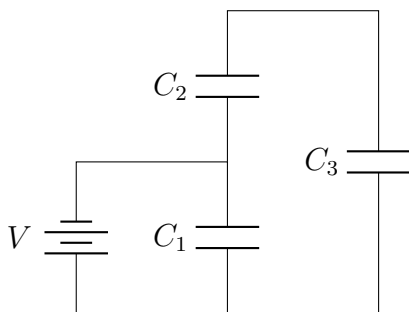
$$Q = \lambda_{inner} * L$$

$$\sigma = \frac{Q}{A} = \frac{\lambda_{inner} * L}{(2\pi a L)} = \frac{\lambda_{inner}}{(2\pi a)}$$

This gives an answer of  $\boxed{-0.8 \text{ nC/cm}^2}$

- (iv) When the inner cylinder is connected to ground, it has a net charge of zero, so drawing a Gaussian surface through the outer cylinder has to be 0 as well to get an E-field of 0 along the Gaussian surface. Therefore, the charge density of the inner surface of the outer shell becomes 0. The answer is **(d)**.

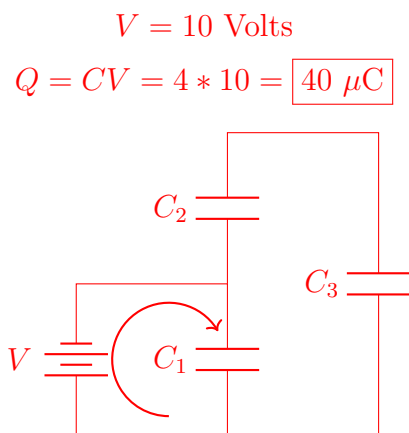
4. The circuit below contains 3 capacitors, all of initial value  $C = 4 \mu\text{F}$ . The voltage source is  $10 \text{ V}$ .



- (i) What is the charge on the capacitor  $C_1$ ?
- $18.3 \mu\text{C}$
  - $21.4 \mu\text{C}$
  - $40.0 \mu\text{C}$
  - $0 \mu\text{C}$
  - $12.0 \mu\text{C}$
- (ii) What is the charge on the capacitor  $C_2$ ?
- $20.0 \mu\text{C}$
  - $19.3 \mu\text{C}$
  - $7.9 \mu\text{C}$
  - $25.6 \mu\text{C}$
  - $25.0 \mu\text{C}$
- (iii) Now suppose  $C_3$  is removed and  $C_2$  is modified by filling it with a dielectric material with constant  $\kappa = 5$ . How does the charge  $Q_2$  change?
- $Q_2$  decreases
  - $Q_2$  stays the same
  - $Q_2$  increases
- (iv) Now add  $C_3$  back while the dielectric with constant  $\kappa = 5$  is kept in  $C_2$ . How does the charge  $Q_3$  change from when there was no dielectric?
- $Q_3$  decreases because the capacitance of  $C_3$  decreases relative to that of  $C_2$
  - $Q_3$  stays the same because the charge of capacitors in series is the same
  - $Q_3$  increases because the capacitance of  $C_2$  increases
- (v) How does the charge on  $C_1$  change when  $C_2$  is modified by filling it with a dielectric with constant  $\kappa = 5$ ?
- $Q_1$  decreases because the capacitance of  $C_1$  decreases relative to that of  $C_2$
  - $Q_1$  stays the same because the voltage across  $C_1$  is the same as it was originally
  - $Q_1$  increases because the effective capacitance of  $C_2$  and  $C_3$  part of the circuit increases



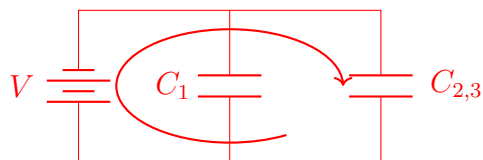
- (i) Based on the circuit schematic, we can see that the battery and  $C_1$  are in parallel (since a loop can be drawn only including those two elements). This means that the voltage across  $C_1$  must be  $V$  using KVL (loop drawn below). We then use the capacitor equation,  $Q = CV$ .



- (ii) Capacitor 2 and 3 are in series, so combine capacitors 2 and 3 in series using the equation

$$\frac{1}{C_{2,3}} = \frac{1}{C_2} + \frac{1}{C_3}$$

This gives that  $C_{2,3} = 2 \mu\text{F}$



$C_{2,3}$  is in parallel with the voltage source, thus the voltage across  $C_{2,3}$  is  $V$  using KVL (shown above).

$$Q_{2,3} = C_{2,3}V = 2 * 10 = 20 \mu\text{C}$$

$$Q_2 = Q_3 = Q_{2,3}$$

Therefore  $Q_2 = \boxed{20 \mu\text{C}}$

- (iii)

$$C = \frac{\kappa\epsilon_0 A}{d} = \frac{Q}{V}$$

From the equations above, we can see that adding a dielectric into the capacitor we increase the capacitance. This means that either the charge on it must increase or the voltage must decrease and since we have a constant voltage source in parallel with  $C_2$ , we know that the charge must increase. The answer is **(c)**.

- (iv) When the dielectric is added to  $C_2$  its capacitance must increase. Thus either its charge must increase, or its voltage must decrease, or both. Since  $C_2$  and  $C_3$  are in series, they must have the same charges. When  $C_3$  is added back, the voltage across  $C_2$  must decrease because we need the loop voltage to remain zero.

Therefore, when  $V_2$  goes down, it's a result of the third capacitor being added (since  $V_1$  is constant as it is directly attached to the battery).

Finally, the capacitance  $C_3$  does not change since the capacitor is not physically altered. So since its voltage goes up, its charge must go up by the same factor. The answer is **(c)**.

- (v) The charge  $C_1$  does not depend on any other capacitors **not** in series with it. The fact that it's connected to the battery forces its voltage (and therefore charge) to stay constant in this case. The answer is **(b)**.