The Grainger College of Engineering
Center for Academic Resources in Engineering

MATH 241

Midterm 1 Review

Keep in mind that this presentation was created by CARE tutors, and while it is thorough, it is not comprehensive.

QR Code to the Queue

The queue contains the worksheet and the solution to this review session

Dot Product

Dot product from components. #rvv-es

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Dot product from length/angle. #rvv-ed

$$\vec{a} \cdot \vec{b} = ab \cos\theta$$

Length and angle from dot product. #rvv-el

$$a = \sqrt{\vec{a} \cdot \vec{a}}$$

$$\cos\theta = \frac{\vec{b} \cdot \vec{a}}{ba}$$

Cross Product

Cross product in components. #rvv-ex

$$\vec{a} \times \vec{b} = (a_2 b_3 - a_3 b_2)\,\hat{\imath} + (a_3 b_1 - a_1 b_3)\,\hat{\jmath} + (a_1 b_2 - a_2 b_1)\,\hat{k}$$

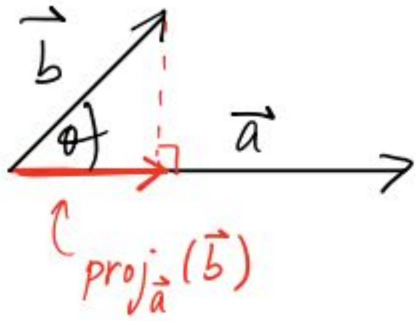
Cross products of basis vectors. #rvv-eo

$$\hat{\imath} \times \hat{\jmath} = \hat{k} \qquad \hat{\jmath} \times \hat{k} = \hat{\imath} \qquad \hat{k} \times \hat{\imath} = \hat{\jmath}$$

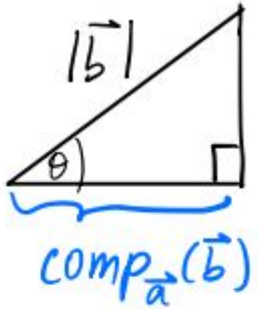
$$\hat{\jmath} \times \hat{\imath} = -\hat{k} \qquad \hat{k} \times \hat{\jmath} = -\hat{\imath} \qquad \hat{\imath} \times \hat{k} = -\hat{\jmath}$$

Projection and Components

$$\text{proj}_{\vec{a}}(\vec{b}) = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)\frac{\vec{a}}{|\vec{a}|}$$



$$\text{comp}_{\vec{a}}(\vec{b}) = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$$

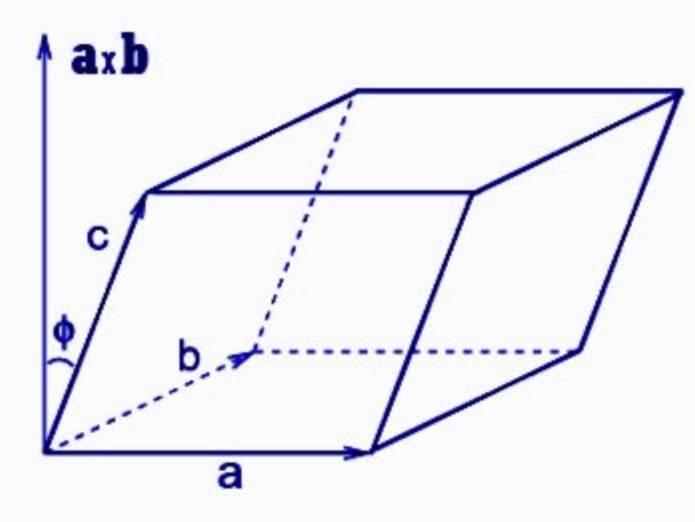


Scalar Triple Product

- A · (B x C)
- Represents the parallelepiped volume enclosed by the three vectors

$$\vec{A} = \langle a_1, a_2, a_3 \rangle, \;\; \vec{B} = \langle b_1, b_2, b_3 \rangle, \;\; \vec{C} = \langle c_1, c_2, c_3 \rangle$$

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$



Equations for Lines and Planes

$$Ax + By + Cz = D$$

- Describes a plane in which *A*, *B*, and *C* are the components of the normal vector
- To find *D*, you need a point on the plane:

$$< x_0, y_0, z_0 >$$

$$D = Ax_0 + By_0 + Cz_0$$

- The equation for a line *L* on a plane can be parametrized:
  - Here, r_0 is a vector between the origin and a point on the plane
  - And *v* is a line on the plane

$$L = \vec{r_0} + t\vec{v}$$

$$\vec{r_0} = < x_0, y_0, z_0 >$$

$$\vec{v} = < v_1, v_2, v_3 >$$

$$\left\{ \begin{aligned} x(t) &= x_0 + tv_1 \\ y(t) &= y_0 + tv_2 \\ z(t) &= z_0 + tv_3 \end{aligned} \right\}$$

Example Question #1

Let **P** be the plane with equation $x + 2z = 0$. Find the distance from the point $(-1, 3, 0)$ to the plane **P**.

Example Solution #1

$$The\ plane\ passes\ through\ (0,0,0)\ and\ the\ normal\ vector\ \vec{N}\ is\ \langle 1,0,2\rangle$$

$$Create\ a\ vector\ \vec{V}\ from\ (0,0,0)\ to\ the\ point\ (-1,3,0) \rightarrow \langle -1,3,0\rangle$$

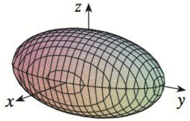
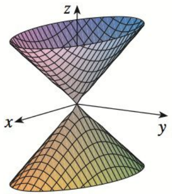
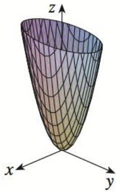
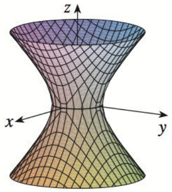
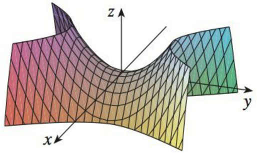
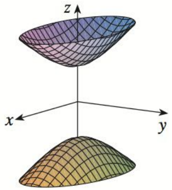
The magnitude of the projection of $\vec{\mathrm{V}}$ onto $\vec{\mathrm{N}}$ will be the distance from the point to the plane

$$proj_{\vec{N}}\vec{V} = \frac{\vec{V}\cdot\vec{N}}{|\vec{N}|^2}\vec{N} = \langle {}^{-1}/_{5}, 0, {}^{-2}/_{5}\rangle$$

$$|proj_{\vec{N}}\vec{V}| = \sqrt{\left({}^{-1}/_{5}\right)^2 + ({}^{-2}/_{5})^2} = {}^{1}/_{\sqrt{5}}$$

$$The\ distance\ is\ 1/\sqrt{5}$$

Quadric Surface

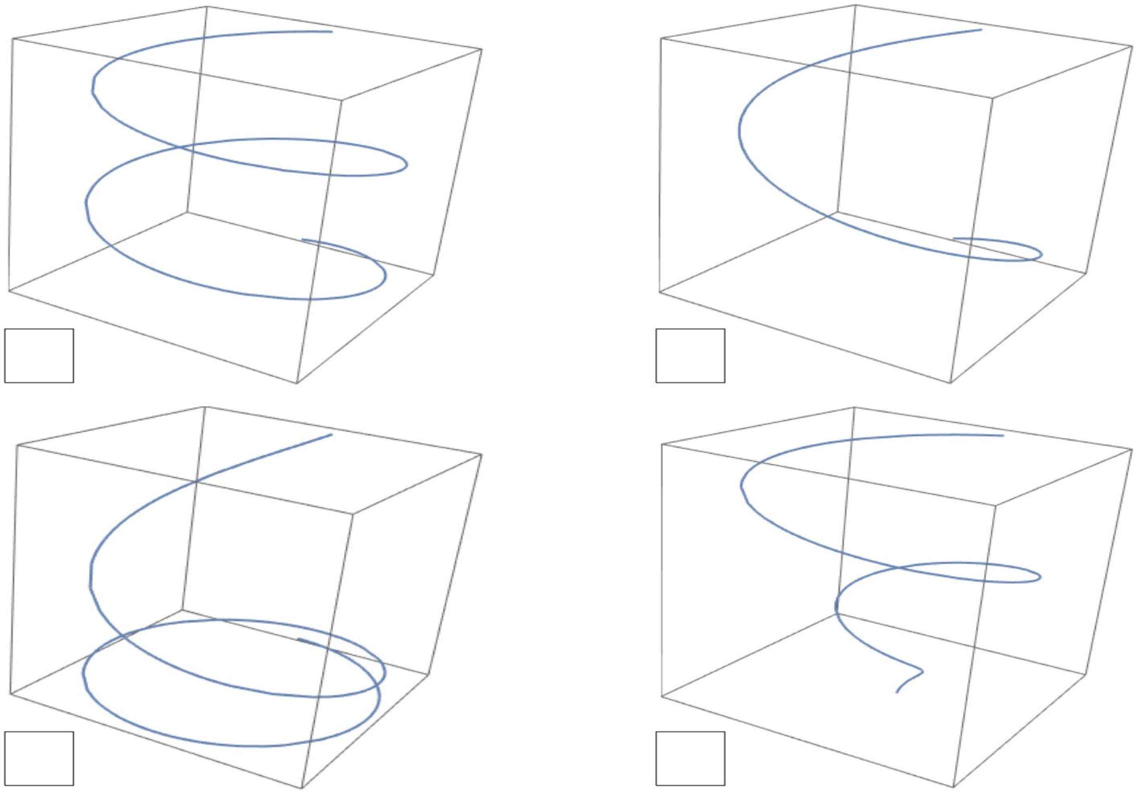
Surface	Equation	Surface	Equation
Ellipsoid 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ All traces are ellipses. If $a = b = c$, the ellipsoid is a sphere.	Cone 	$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces in the planes $x = k$ and $y = k$ are hyperbolas if $k \neq 0$ but are pairs of lines if $k = 0$.
Elliptic Paraboloid 	$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces are parabolas. The variable raised to the first power indicates the axis of the paraboloid.	Hyperboloid of One Sheet 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ Horizontal traces are ellipses. Vertical traces are hyperbolas. The axis of symmetry corresponds to the variable whose coefficient is negative.
Hyperbolic Paraboloid 	$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ Horizontal traces are hyperbolas. Vertical traces are parabolas. The case where $c < 0$ is illustrated.	Hyperboloid of Two Sheets 	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ Horizontal traces in $z = k$ are ellipses if $k > c$ or $k < -c$. Vertical traces are hyperbolas. The two minus signs indicate two sheets.

Example Question #2

Let C be the curve parameterized by **r**(t) = <sin(t^2), cos(t^2), t^2> for $0 \leq t \leq 2\sqrt{\pi}$. Check the corresponding picture of C.

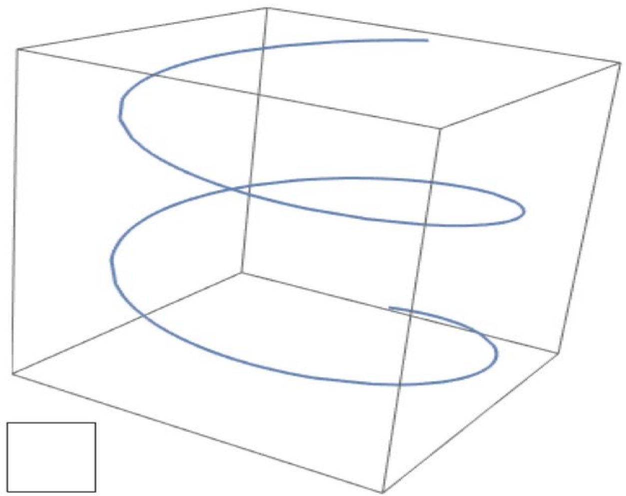
(Pictures are on the next slide)

Example Question #2



Example Solution #2

$\mathbf{r}(t) = \langle \sin(t^2), \cos(t^2), t^2 \rangle$ for $0 \le t \le 2\sqrt{\pi}$



Example Question #3

Find the vector function representing the curve of intersection between the circular cylinder of radius 4 centered on the z-axis and the surface $z = xy$.

Example Solution #3

Find the vector function representing the curve of intersection between the circular cylinder of radius 4 centered on the z-axis and the surface z = xy.

$$\overrightarrow{\mathrm{r}_{\mathrm{cyl}}} = \langle 4\mathrm{cost}, 4\mathrm{sint} \rangle$$

$$z = xy = 16cost \cdot sint$$

$$\vec{\mathrm{r}}(\mathrm{t}) = \langle 4\mathrm{cost}, 4\mathrm{sint}, 16\mathrm{cost} \cdot \mathrm{sint} \rangle$$