Center for Academic Resources in Engineering (CARE)
Mid-Semester Review Session

Phys211 – University Physics: Mechanics

Midterm 1 Worksheet Solutions

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Feb. 11th, 2:30-4:30pm Wyatt, Josh, Matthew

Session 2: Feb. 12th, 5:00-7:00pm Jon, Lucy, Matthew

Can't make it to a session? Here's our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

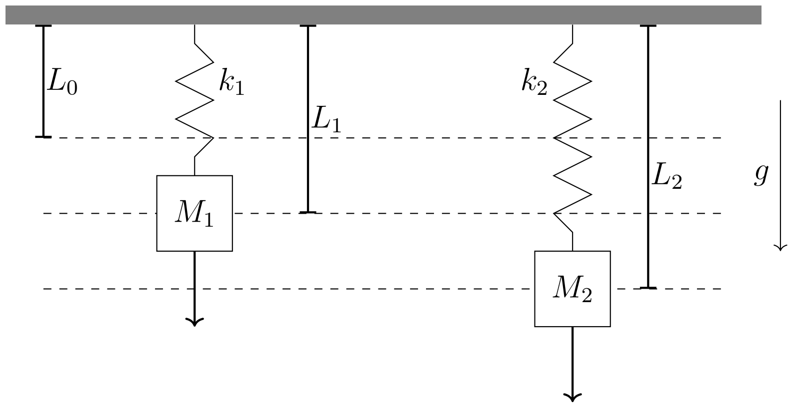
Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/847
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. Two massless springs have the same natural length (the length when there is no elongation or compression), L_0. The spring constant of each spring is k_1 and k_2, respectively. Mass M_1 hangs from spring 1 and it reaches equilibrium at length L_1. Mass M_2 hangs from spring 2 and it reaches equilibrium at length L_2.



(a) If $k_2 = \frac{1}{2}k_1$ and $M_2 = 3M_1$, what is the relationship between L_1 and L_2?

(b) Suppose we displace spring 1 by some additional $\Delta x = 0.5$, how much potential energy was added to the spring? Assume $L_0 = 0.5$, $L_1 = 1$, and $k_1 = 5$.

(a) The problem asks us to find a relationship between the stretched lengths of the two springs. To do this, we will use Hooke's Law in the form $F = k(L - L_0)$ where L is the length of the spring when an external force, F, is applied.

Spring 1	Spring 2
$\Delta x = L_1 - L_0$	$\Delta x = L_2 - L_0$
$F = M_1 g$	$F = M_2 g$
$M_1 g = k_1(L_1 - L_0)$	$M_2 g = k_2(L_2 - L_0)$

At this point, we can make the substitutions given in the problem statement. We will rewrite the equation for spring 1 in terms of spring 2, but you'd get the same answer if you decided to do this the other way. The new equation for spring 1 then becomes

$$\frac{1}{3}M_2 g = 2k_2(L_1 - L_0)$$

We can now equate the equations for springs 1 and 2 by eliminating the external force term. Then simplifying, we end up with

$$k_2(L_2 - L_0) = 6k_2(L_1 - L_0)$$
$$L_2 - L_0 = 6L_1 - 6L_0$$

$$\boxed{L_2 = 6L_1 - 5L_0}$$

(b) The potential energy at length L of any spring obeying Hooke's Law is $U = \frac{1}{2}k(L - L_0)^2$. The question, however, does not want the total potential energy at length $L_1 + \Delta x$, it only asks for the *additional* energy stored as a result of the added length. Therefore, we calculate ΔU, the difference between the energy at $L_1 + \Delta x$ and at L_1.

$$\begin{aligned}\Delta U &= U(L_1 + \Delta x) - U(L_1) \\ &= \frac{1}{2}k_1(L_1 + \Delta x - L_0)^2 - \frac{1}{2}k_1(L_1 - L_0)^2\end{aligned}$$

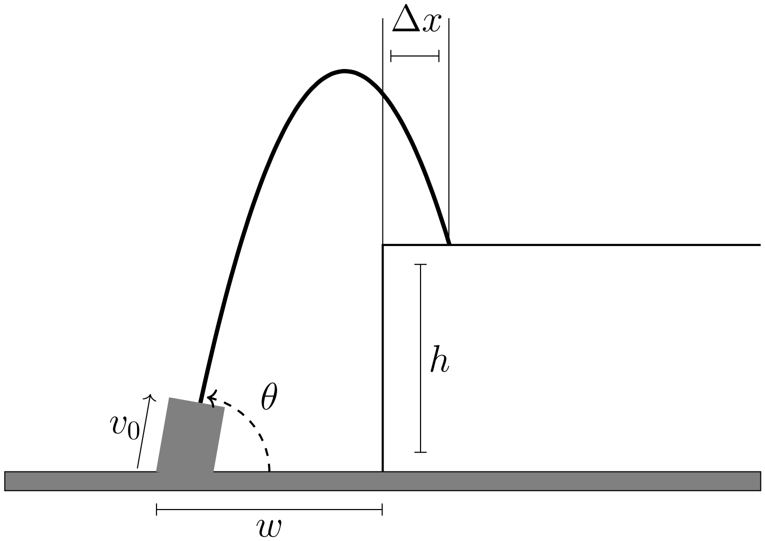
After expanding the squared terms and simplifying, you get

$$\Delta U = \frac{1}{2}k_1 \Delta x(2L_1 - 2L_0 + \Delta x)$$

Inputting the given values, the final answer is

$$\boxed{1.875\ \mathrm{J}}$$

2. A fountain has several water jets with the geometry shown in the figure. The distance from the jet to the cliff is $w = 2$ m and the height of the cliff is $h = 1.5$ m. If the initial speed of the water coming out of the jet is $v_0 = 7$ m/s and the jets are firing at an angle $\theta = 45°$ does the water reach the top of the cliff? If so, what is the value of Δx?



A) 0.81 m

B) 0 m

C) Doesn't make it

D) 1.62 m

E) 2 m

Let's first establish that $+\hat{\mathbf{x}}$ is to the right, and $+\hat{\mathbf{y}}$ is upward. Let's start with the x component to solve for the time it takes to get to the cliff. Solving

$$2 = 7\cos(45°)t$$

$$\text{gives the time } t = \frac{4}{7\sqrt{2}} \approx 0.404$$

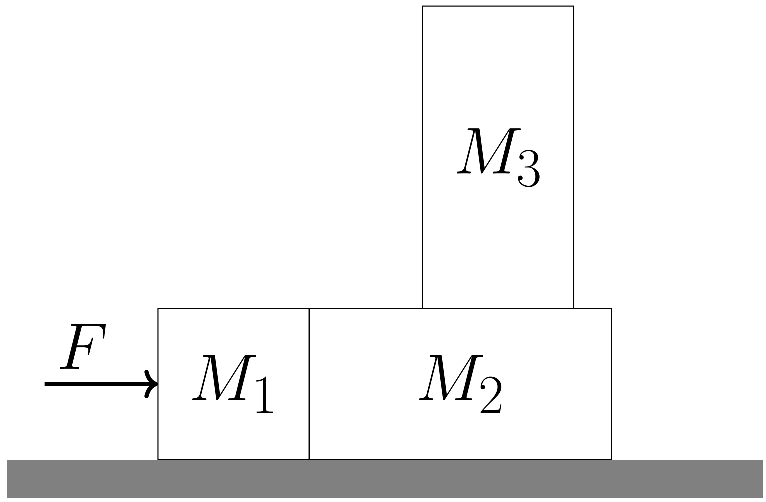
If we substitute this time into the equation for the water's height, we get

$$y\left(\frac{4}{7\sqrt{2}}\right) \to h = 7\sin(45°)(t) - \frac{1}{2}at^2 = 1.2 < 1.5$$

The water does not reach the full 1.5 meters at the expected value of x

$$\boxed{\text{(C) The water does not reach the upper pool}}$$

3. Three blocks are placed in contact on a horizontal frictionless surface. A constant force of magnitude $F = 30$ N is applied to the box of mass $M_1 = 8$kg. There is friction between the surfaces of blocks $M_2 = 2M_1$ and $M_3 = 3M_1$ ($\mu_s = 0.5$, $\mu_k = 0.3$) so the three blocks accelerated together to the right.



(a) Which block has the smallest magnitude of net force acting on it?

(b) What is the acceleration of the blocks? (You may assume block M_3 does not slide or fall off block M_2)

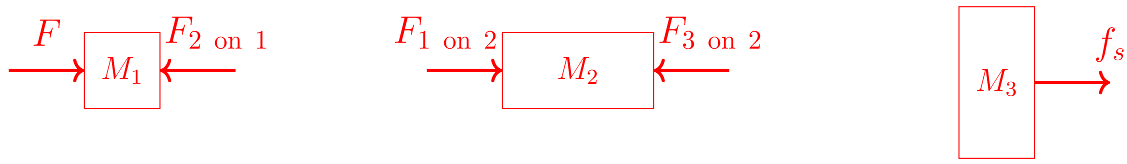
(c) What is the maximum force F that can be applied, before the M_3 block slides off?

(a) Since the blocks are accelerating together, we can treat a as a constant. Using Newton's second law, $F = ma$, we can see that, given constant a, the block with the smallest net force is the box with the smallest mass. In this case, that would be block M.

(b) Conveniently, we can treat the three blocks as a single system with one applied external force. Thus, only one iteration of Newton's second law is necessary.

$$F = (6M)a$$
$$30 = 48a$$
$$\boxed{a = 0.625 \text{ m/s}^2}$$

(c) In the previous problem, we treated the three blocks as a single system, but this time, we will write a Newton's second law equation for each block. Since we are working in magnitudes, the following facts hold: $f_s \leq mg\mu_s$ and $F_{1,2} = F_{2,1}$.



Block 1

$F - F_{2 \text{ on } 1} = Ma$

Block 2

$F_{1 \text{ on } 2} - f_s = 2Ma$

Block 3

$f_s = 3Ma$

Now, using these three equations, we essentially want to find a function for F in terms of f_s and maximize it. We also want to eliminate the term a.

Combining the first two equations gives $F = 3Ma + f_s$. Taking this and combining it with the third equation, we get $F = 2f_s$.

It's easy to see that the maximum value of F occurs at the maximum value of f_s since they're directly related. Therefore, we can say (using the inequality of static friction) that

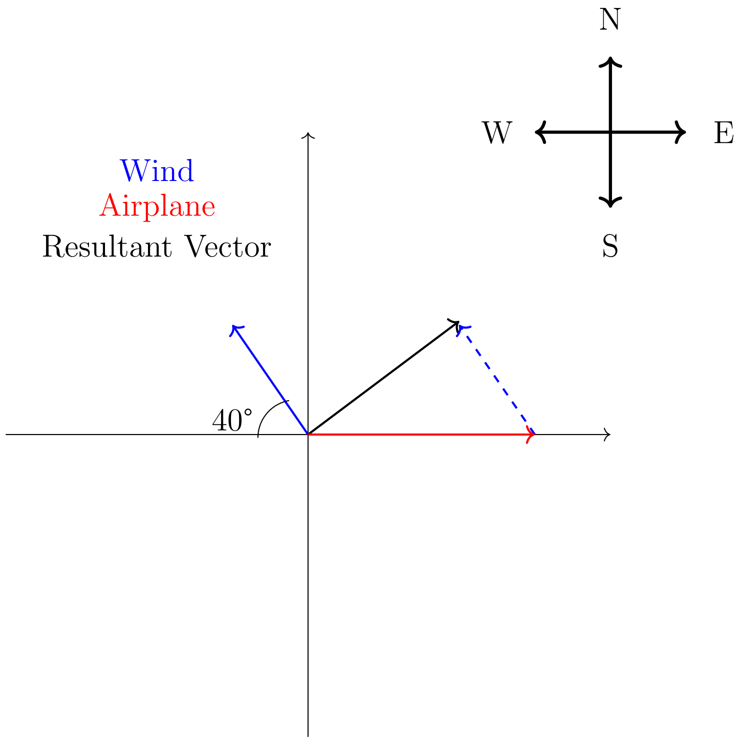
$$F_{max} = 2f_{s,max} = 6Mg\mu_s$$

And the maximum force occurs at $F_{max} = 6Mg\mu_s$

Plugging in the given values gives the final answer of

$$\boxed{F = 235.2 \text{ N}}$$

4. An airplane travels with a velocity of 115 m/s due east with respect to the air. The air is moving at a speed of 25 m/s with respect to the ground at an angle of 40° north of west. What is the speed of the plane with respect to the ground?



This problem is a simple application of the relation for relative motion:

$$\mathbf{v}_{p,g} = \mathbf{v}_{p,a} + \mathbf{v}_{a,g}$$

where the subscripts are plane, ground, and air respectively.

$\mathbf{v}_{p,a}$ is given in the problem as 115 m/s due east, which is $+\hat{\mathbf{x}}$ according to the coordinate system above. This means $\mathbf{v}_{p,a} = 115\hat{\mathbf{x}}$.

Next, we have to find $\mathbf{v}_{a,g}$, which will involve some trig:

$$\mathbf{v}_{a,g} = -25\cos(40°)\hat{\mathbf{x}} + 25\sin(40°)\hat{\mathbf{y}}$$

The reason for the negative sign is that the air velocity is in the second quadrant, and the angle we're using doesn't give us the negative that we need, so we add it from context.

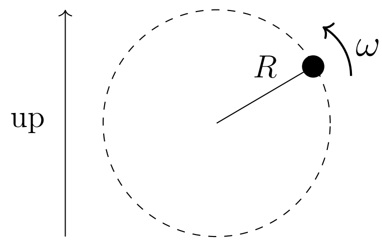
The sum of these two velocities is what we need to find the speed of the plane.

$$\mathbf{v}_{p,a} + \mathbf{v}_{a,g} = 115\hat{\mathbf{x}} - 25\cos(40°)\hat{\mathbf{x}} + 25\sin(40°)\hat{\mathbf{y}}$$

The result is $\mathbf{v}_{p,g} = 95.85\hat{\mathbf{x}} + 16.07\hat{\mathbf{y}}$. This is the velocity of the plane with respect to the ground. The question asks for the speed, however, so we must take the magnitude of this velocity vector.

$$\boxed{|\mathbf{v}_{p,g}| = \sqrt{v_x^2 + v_y^2} = \sqrt{95.85^2 + 16.07^2} = 97.19 \text{ m/s}}$$

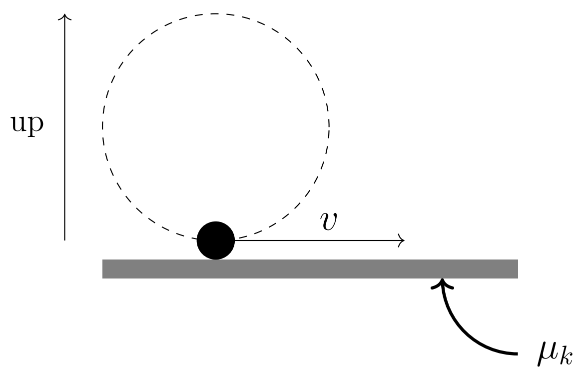
5. A mass, $m = 2.3$ kg, is tied to a string of length $R = 0.9$ m and set in uniform circular motion in the vertical plane, as shown in the lower figure. The angular velocity of the rotating mass is $\omega = 26$ rad/s.



(i) The string tension is largest at:

A) The top of the circle
B) The bottom of the circle
C) It is the same at all points on the circle

(ii) When the same mass is at the bottom of the circle, the string suddenly breaks so that the mass slides (without rolling) on a rough, horizontal surface with $\mu_k = 0.82$, as shown. What is the magnitude of the work done by friction as it comes to rest?



A) 2.78 J
B) 6.29 J
C) 277.5 J
D) 629.7 J
E) 3.14 J

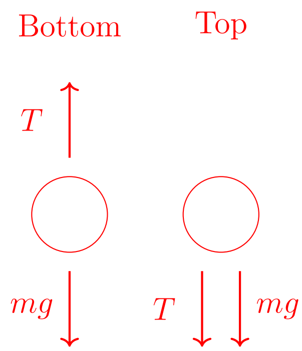
(i) Define the positive direction as up on the diagram

Bottom	Top
$T - mg = m\omega^2 r$	$-T - mg = -m\omega^2 r$
$T = mg + m\omega^2 r$	$T = m\omega^2 r - mg$

We can see that T is larger when the mass is at the bottom since that's when it equals the sum, instead of the difference, of the centripetal force and gravity.

Let's think about why this is conceptually right. We are told that the rotating mass moves at a constant angular velocity at a constant radius, and this means that the centripetal force is unchanging. Remember that the centripetal force in this case is the sum of the tension and the radial component of gravity (constantly changing with the position of the mass). So to find where the tension is greatest, we must find where the radial component of gravity is smallest. Mathematically, this means that if $T + F_{g,radial} = C$ (C is a constant), a decreasing $F_{g,radial}$ means an increasing T. Remember these are signed numbers.

The minimum of $F_{g,radial}$ is at the lowest possible point since that's when it is least aligned with the radial vector. This is also where it's fighting tension the most.



(ii) The fundamental idea here is energy conservation (more specifically the work-energy theorem). That is, all the kinetic energy initially within the mass will be removed as a result of the negative work that friction will do as it slides across the floor. In this case, it can be said that

$$|\Delta K| = |W_f|$$

since we're only looking for the magnitude of the frictional force (but bear in mind, it would be negative if we wanted the sign included).

So we must find the kinetic energy of the mass. We can do this by using the relation $v = \omega r$.

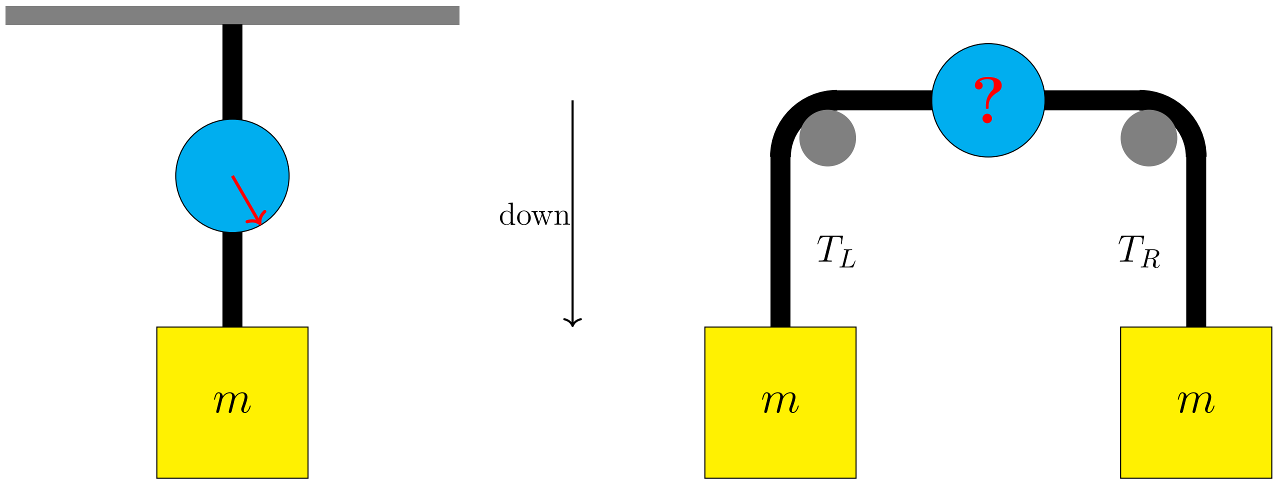
$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 r^2$$

Now substitute in the given values, getting a final answer of

$$\boxed{629.7 \text{ J}}$$

You could have also done this problem using Newton's laws, but this is the most straightforward method.

6. A box of mass m is hung from a spring scale as shown on the left side of the figure. The tension displayed on the spring scale is 30 N. A student now places the same spring scale between ropes that run over frictionless pulleys and support two identical boxes having the same mass m, as shown on the right side of the figure. Assume the spring-scale itself has no mass.



(a) What is the tension in the rightmost rope in the figure on the right side, T_R?

(b) What is the reading on the spring scale in the figure on the right side?

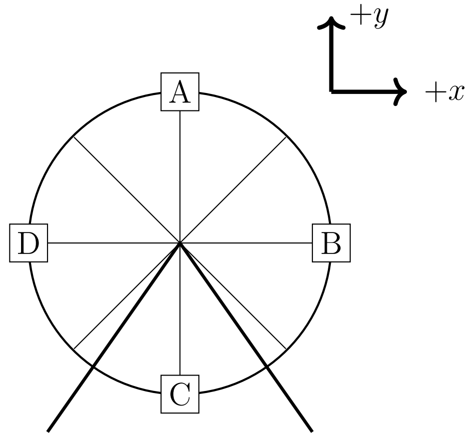
(a) This question is another conceptually tricky one, but it's answer relies solely on Newton's third law. Think about the left figure. As gravity pulls on the weight, there is a tension in the lower rope equal to 30 N. This tension pulls on the scale, which pulls on the upper rope with equal force, 30 N. Thus, the tension in the upper rope must also be 30 N, and the upper rope pulls on the ceiling with force 30 N also.

The right figure is no different from the left one physically. The setup appears different, but the physics is exactly the same. T_R pulls with a force of 30 N, pulling the scale to the right, and as a result, the scale pulls on the left rope with force 30 N. The leftmost weight acts as the ceiling in this case. Therefore, both tensions are $T_L = T_R = mg$.

The other fact to confirm this is that the scale is at rest, so the net force on it must be zero. The only way this can be is if both tensions equal 30 N.

(b) The scale on the right side will read exactly the same as the scale on the left side. Both read 30 Newtons because, as established above, the tensions of both ropes must be 30 N to maintain the scale at rest. The trick is to realize that these two rope tensions do NOT add to 60 N! That's a very common mistake at first.

7. A Ferris wheel has a radius of 9 meters, and spins counterclockwise with a constant angular velocity $\omega = 0.4$ radians/second. (Cart B is going up at the instant shown.)



(a) What is the speed of the cart A at the instant shown?

(b) What is the x-component of the acceleration of cart B at the instant shown?

(c) What is the y-component of the acceleration of cart B at the instant shown?

(d) What is the magnitude of the **maximum** force the Ferris wheel will exert on a 65 kg person as they go around the ride?

(a) Using the relationship $v = \omega r$ for rotations, we end up with $\boxed{3.6 \text{ m/s}}$

(b) The only component of acceleration along the x-axis for cart B is the centripetal acceleration responsible for keeping the cart in orbit. Using the normal formula for centripetal acceleration, we get

$$\boxed{\vec{a_c} = -\frac{v^2}{r}\hat{r} = -1.44 \text{ m/s}^2\hat{x}}$$

(c) There's only one force acting along the y-axis at point B, and that's gravity. $\boxed{\vec{a_y} = -9.8 \text{ m/s}^2\hat{y}}$

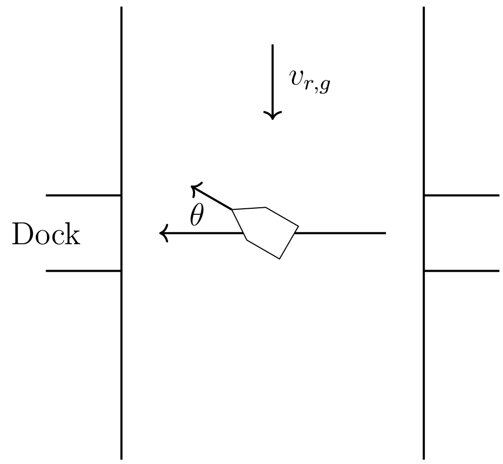
(d) As was established in problem 5, the force that a system (be it tension, a Ferris wheel, etc...) exerts on a rotating object is maximized when it is most strongly opposing other forces (to keep centripetal force constant). In this case, that is again the bottom of the circular path (position C). At C, Newton's second law is

$$F_{FW} + F_g = F_c$$

where F_{FW} is the force from the Ferris wheel and $F_g = -mg$. So we solve for F_{FW} and substitute in values (F_c comes from (a), don't forget to multiply by mass) to get

$$\boxed{F_{FW} = \frac{mv^2}{r} + mg = 730.6 \text{ N}}$$

8. A boat is traveling directly across a river (as seen by an observer standing on the shore) that flows at a uniform rate of $v_{r,g} = 10$ ft/s, as shown in the figure. To compensate for the flow of the river, the boat must head upstream as it travels. The speed of the boat is 18 ft/s with respect to the water. What is the angle between the direction the boat points and the direction it is traveling with respect to the shore?



These problems are often tricky because there's usually some caution that's needed when converting the words into vectors. Nonetheless, it's a relative motion problem, so it's best to start with the relation for relative motion:

$$\mathbf{v}_{b,s} = \mathbf{v}_{b,w} + \mathbf{v}_{w,s}$$

where the subscripts are boat, shore, and water respectively. Let's also say that $+\hat{\mathbf{x}}$ is leftward (in the direction of motion) and $+\hat{\mathbf{y}}$ is upward.

We see from the diagram that the water flows in the $-\hat{\mathbf{y}}$ direction at a speed of 10 ft/s. In vectors, that is $\mathbf{v}_{w,s} = -10\hat{\mathbf{y}}$. We also know the *speed* of the boat relative to the water, but not its direction. So in vectors, that is

$$\mathbf{v}_{b,w} = 18\cos(\theta)\hat{\mathbf{x}} + 18\sin(\theta)\hat{\mathbf{y}}.$$

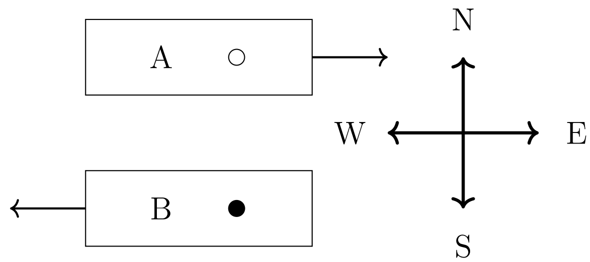
Now here's a tricky part: the problem tells us that the boat moves directly across the river as viewed from the shore. This means that the y-component is zero, and $\mathbf{v}_{b,s} = v_{b,s_x}\hat{\mathbf{x}} + 0\hat{\mathbf{y}}$. So we only know one of the components. But we can solve for the angle without knowing the other one fortunately because all the y-components are known.

$$\mathbf{v}_{b,s} = \mathbf{v}_{b,w} + \mathbf{v}_{w,s}$$
$$v_{b,s_x}\hat{\mathbf{x}} + 0\hat{\mathbf{y}} = 18\cos(\theta)\hat{\mathbf{x}} + 18\sin(\theta)\hat{\mathbf{y}} - 10\hat{\mathbf{y}}$$

Taking the y-components as one equation, we can solve

$$0 = 18\sin(\theta) - 10$$
$$\theta = \arcsin\left(\frac{10}{18}\right) \approx \boxed{33.7^\circ}$$

9. You are sitting on train A, which is moving East at a speed of 30 m/s with respect to the ground. You get up from your seat and move West at a speed of 2 m/s relative to the train. The positive direction is defined as being to the East.



(i) What is your velocity with respect to the ground?

A) 32 m/s
B) -32 m/s
C) 28 m/s
D) -28 m/s
E) 15 m/s

(ii) Train B begins moving West at a speed of 25 m/s with respect to the ground. Your friend on train B begins to move East at a speed of 3 m/s relative to the train. What is your speed with respect to your friend?

A) 53 m/s
B) -53 m/s
C) 56 m/s
D) -50 m/s
E) 50 m/s

(i) Take east to be $+\hat{\mathbf{x}}$. Applying the relative motion equation, $\mathbf{v}_{p,g} = \mathbf{v}_{p,t} + \mathbf{v}_{t,g}$, where the subscripts are person, ground, and train respectively, we get $\mathbf{v}_{p,g} = -2 + 30 = 28$

$$\boxed{\mathbf{v}_{p,g} = 28 \text{ m/s}}$$

(ii) We seek the velocity of the person on train A relative to the person on train B, so we will need to use the relative motion formula: $\mathbf{v}_{A,B} = \mathbf{v}_{A,g} + \mathbf{v}_{g,B}$ where the subscripts are the person on train A, the person on train B, and ground respectively.

Using the information in the last problem, we know that the velocity of the person on train A relative to the ground is $+28\hat{\mathbf{x}}$.

We now need the velocity of the person on train B relative to the ground to solve the formula above. That requires another relative motion equation: $\mathbf{v}_{B,g} = \mathbf{v}_{B,t_W} + \mathbf{v}_{t_W,g}$ where the subscripts represent the person on train B, ground, and the train going west respectively.

$$\mathbf{v}_{B,g} = 3 - 25 = -22$$

Now we use the fact that $\mathbf{v}_{B,g} = -\mathbf{v}_{g,B}$ to say $\mathbf{v}_{g,B} = 22$

We can now revisit our first equation

$$\mathbf{v}_{A,B} = \mathbf{v}_{A,g} + \mathbf{v}_{g,B} = 28 + 22 = \boxed{50 \text{ m/s}}$$

We did this in two steps. But it's possible to do this in one step if we use the equation

$$\mathbf{v}_{A,B} = \mathbf{v}_{A,t_E} + \mathbf{v}_{t_E,g} + \mathbf{v}_{g,t_W} + \mathbf{v}_{t_W,B}$$

where the new variable, t_E, represents the train going east. This equation would give us

$$\mathbf{v}_{A,B} = -2 + 30 + 25 - 3$$

Which gives the same answer as above. Take note of the fact that $\mathbf{v}_{1,2} = -\mathbf{v}_{2,1}$ if the signs are confusing you.

10. A circular exit ramp is covered with ice (so it can be considered frictionless).

(i) If the curve is banked (inclined at an angle, for example the right edge of the road is higher than the left edge), is it possible for the car to not slip off? Why?

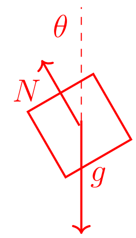
A) Yes

B) No

(ii) Assume your car is traveling into the ramp at a speed of 45 m/s. If the ramp has a radius of 95 m, what is the necessary angle for the car to not slip off the ramp?

A) 65°
B) 45°
C) 72°
D) 43°
E) 20°

(i) In order for the car to stay on the ramp, there must be a centripetal force keeping it in circular motion. Since the curve is banked, there is a component of the normal force that acts as the centripetal force and no friction is necessary. The answer is therefore **(A) Yes**.



(ii) NOTE: You'll need to break the Normal force vector into its components instead of gravity. We're doing this because we are NOT using a rotated coordinate system.

As stated in problem 12, the horizontal component of the normal force is what acts as the centripetal force. That is, $F_c = F_{N_x}$. This means

$$\frac{mv^2}{r} = N\sin(\theta)$$

The other component of the normal force must be equal to the weight of the car. That is, $W = F_{N,y}$. This means

$$mg = N\cos(\theta)$$

Now, we can solve for N in the first equation, and substitute it into the second one to get a theta in terms of known quantities.

$$\begin{aligned} mg &= \left(\frac{mv^2}{r\sin(\theta)}\right)\cos(\theta) \\ \cot(\theta) &= \frac{gr}{v^2} \\ \tan(\theta) &= \frac{v^2}{gr} \end{aligned}$$

When we substitute in the given values, we end up with

$$\boxed{\theta = \arctan\left(\frac{v^2}{gr}\right) \approx 65.3^\circ}$$