



Center for Academic Resources in Engineering (CARE) Mid-Semester Review Session

Phys211 – University Physics: Mechanics

Midterm 2 Worksheet Solutions

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Sun. October 15th, from 4-6pm Josh, Lucy, Matthew

Session 2: Mon. October 16th, from 4-6pm Jon, Wyatt, Josh

Can't make it to a session? Here's our schedule by course:

<https://care.grainger.illinois.edu/tutoring/schedule-by-subject>

Solutions will be available on our website after the last review session that we host.

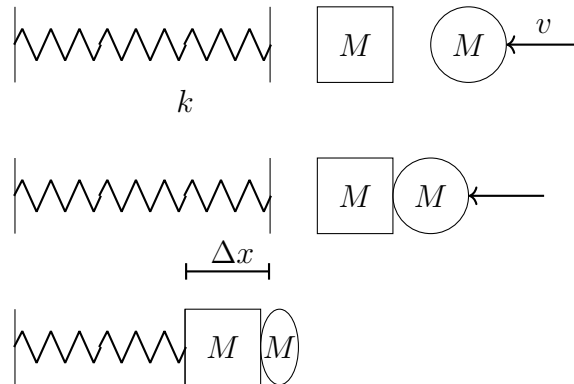
Step-by-step login for exam review session:

1. Log into Queue @ Illinois: <https://queue.illinois.edu/q/queue/847>
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. A putty ball of mass $M = 4$ kg is traveling horizontally at $v = 7$ m/s. (Ignore the effects of gravity). It strikes a block of the same mass, which is adjacent to a relaxed ideal spring attached to an infinitely massive wall with a spring constant of $k = 8$ N/m. The putty ball sticks to the block. After the collision, the spring is compressed.



What is the maximum compression of the spring?

Conservation of Momentum:

$$P_i = P_f$$

$$m_p v_p = m_{b+p} v_f$$

$$v_f = 3.5 \text{ m/s to the left}$$

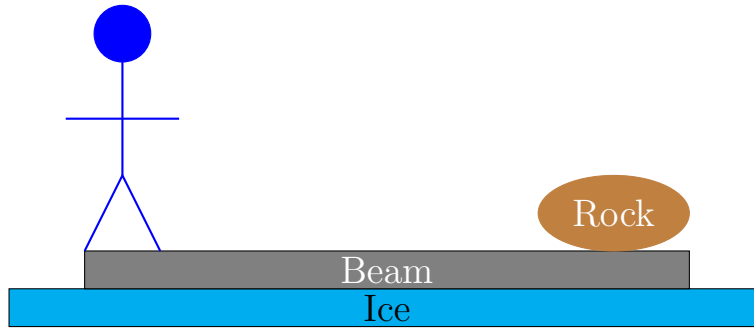
One thing to note is that conservation of momentum is only valid in cases where there's not an external force (otherwise momentum changes). So we use momentum *during* the collision, but once the spring starts compressing, we must resort to energy conservation. Energy conservation here is justified because the spring force is conservative.

$$KE = PE$$

$$\frac{1}{2}(m_p + m_b)v_f^2 = \frac{1}{2}kx^2$$

$$x = 3.5 \text{ m}$$

2. A very strong man is standing at one end of a beam of length $L = 23$ m. The man has mass $M_{man} = 120$ kg and the beam has mass $M_{beam} = 59$ kg and the beam is atop a frictionless sheet of ice. At the other end of the beam sits a large rock of mass $M_{rock} = 251$ kg.



- (a) The man walks to the other end of the beam and sits down on the rock. How far did the beam move along the ice?
- (b) True or False? As the man walks, the momentum of the beam+man+rock system is not conserved because the man is exerting a force on the beam.
- (c) Suppose the man throws the rock off the beam with some velocity $v = 2$ m/s in the $+\hat{x}$ direction to the right. What is the final velocity of the man who is still standing on the beam?

- (a) Let the left end of the beam denote the origin of the system. First, we find the center of mass:

$$X_{CM} = \frac{\sum mx}{\sum m} = \frac{120 * 0 + 59 * 11.5 + 251 * 23}{120 + 59 + 251} = 15.003 \text{ m}$$

After the man moves, the new center of mass relative to the left end of the beam is:

$$X_{CM} = \frac{120 * 23 + 59 * 11 + 251 * 23}{120 + 59 + 251} = 21.422 \text{ m}$$

This means that the beam has moved $21.422\text{m} - 15.003\text{m} = 6.419\text{m}$ to the left, since we know that the center of mass relative to the ice must remain the same.

$$\Delta x = 6.419 \text{ m}$$

- (b) There are no external forces acting on the system, so even though the man is exerting a force on the beam, this is an internal force that is coupled by another internal force of the beam on the man.

False

- (c)

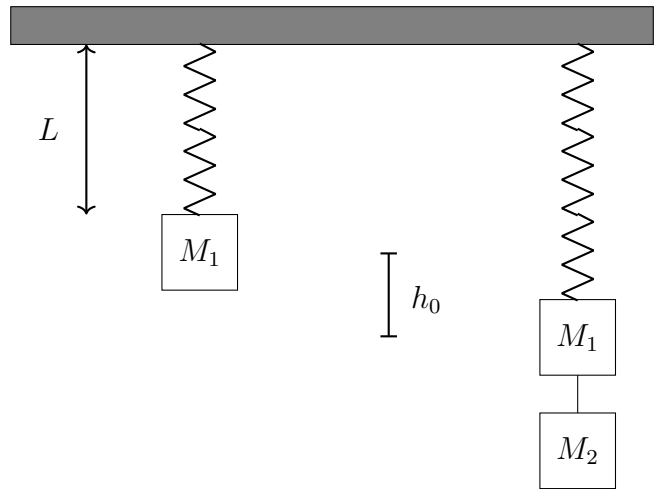
Conservation of Momentum:

$$M_{rock}V_{rock} = (M_{man} + M_{beam})V_{man+beam}$$

$$251 * 2 = (59 + 120)V_{man+beam}$$

$$V_{man+beam} = -2.8 \text{ m/s}$$

3. A block of mass $m_1 = 15$ kg hangs from the ceiling on an ideal, massless spring with spring constant $k = 60$ N/m. With the block hanging on the spring, the total length of the spring is $L = 4.5$ m. When a second block with an identical mass of $m_2 = 15$ kg is tied to the first with a massless string, the spring stretches an additional $h_0 = 2.45$ m. The string is cut so that mass m_2 falls away. What is the maximum velocity of mass m_1 ?



Since all the forces acting on the blocks are conservative (spring and gravity), we can use energy conservation. The total potential energy stored in the spring initially is given by

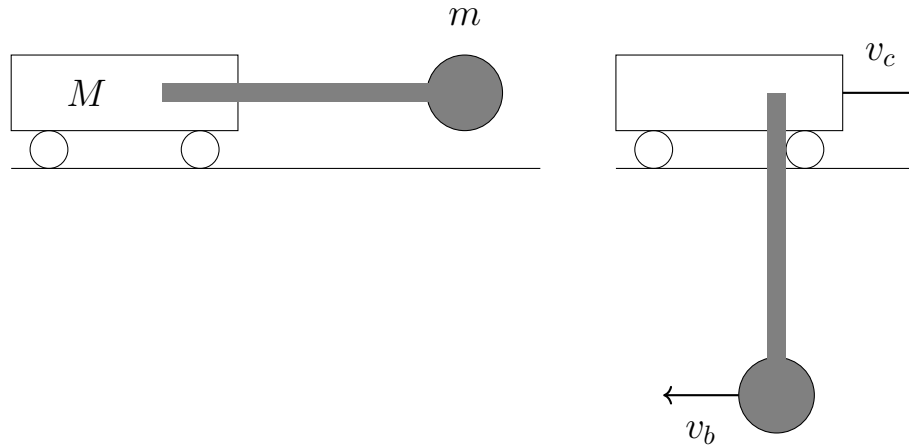
$$PE = \frac{1}{2}kx^2$$

This potential energy, when completely converted to kinetic, will be the point where the mass is moving with the maximum speed.

$$\begin{aligned} PE_i &= KE_f \\ \frac{1}{2}kx^2 &= \frac{1}{2}mv^2 \\ 60 * (2.45)^2 &= 15 * v^2 \end{aligned}$$

$$v = 4.9 \text{ m/s}$$

4. A cart of mass $M = 8 \text{ kg}$ rolls without friction on a horizontal surface. It is attached at the cart's center of mass to a freely pivoting initially-horizontal massless rod of length L to a ball of mass $m = 4 \text{ kg}$. The system is initially at rest when the ball is released. The pendulum swings down and to the left, and at the bottom of its swing the ball is observed to have a $v_b = 3 \text{ m/s}$ to the left.



- (i) Which one of the following remains constant as the pendulum swings down?
- A) horizontal component of the momentum of the ball
 - B) horizontal component of the momentum of the cart
 - C) horizontal component of the momentum of the ball + cart
- (ii) What is the speed of the cart when the ball is at the bottom?
- (iii) What is the length L of the pendulum?
- (iv) How far to the right has the cart moved, when the ball is at the bottom (in terms of L)?
- (i) Momentum separately is not conserved because they both start at rest but end with some non-zero velocity. As a system, however, momentum is conserved because the momentum of the ball will be the same magnitude but opposite direction of the momentum of the cart. The answer is (C).
- (ii)

$$P_i = P_f = 0 = P_{ball} + P_{cart}$$

$$\vec{P}_{ball} = -\vec{P}_{cart}$$

$$m_b \vec{v}_b = -M_c \vec{v}_c$$

$$v_c = 1.5 \text{ m/s}$$

- (iii) The only force acting on this cart-ball system is gravity, and that acts vertically. So we can use energy conservation here since gravity is conservative.

$$PE_{ball} = KE_{ball} + KE_{cart}$$

$$m_b g L = \frac{1}{2} m_b v_b^2 + \frac{1}{2} m_c v_c^2$$

$$(4)9.81L = \frac{1}{2}(4)(-3)^2 + \frac{1}{2}(8)(1.5)^2$$

$$L = 0.68 \text{ m}$$

(iv) Either of the following are valid solutions:

Reference Frame of Ball

$$\begin{aligned}\frac{m_b x_b + m_c x_c}{m_b + m_c} &= X_i \\ \frac{4L + 8 * 0}{4 + 8} &= X_i \\ X_i &= \frac{L}{3}\end{aligned}$$

$$\begin{aligned}\frac{m_b x_b + m_c x_c}{m_b + m_c} &= X_f \\ \frac{4 * 0 + 8 * 0}{4 + 8} &= X_f \\ X_f &= 0\end{aligned}$$

$$\Delta X = \frac{L}{3}$$

Reference Frame of Cart

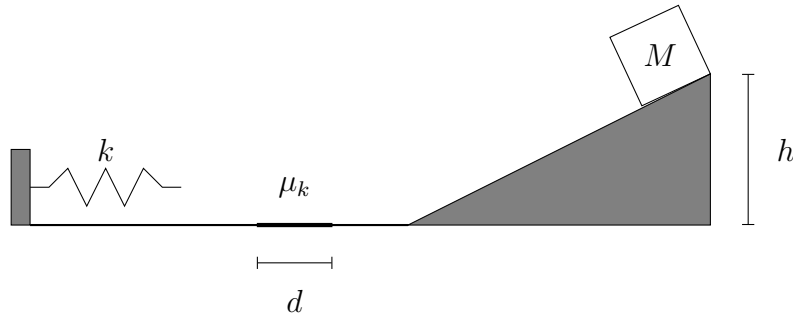
$$\begin{aligned}\frac{m_b x_b + m_c x_c}{m_b + m_c} &= X_i \\ \frac{4 * 0 + 8 * 0}{4 + 8} &= X_i \\ X_i &= 0\end{aligned}$$

$$\begin{aligned}\frac{m_b x_b + m_c x_c}{m_b + m_c} &= X_f \\ \frac{4L + 8 * 0}{4 + 8} &= X_f \\ X_f &= \frac{L}{3}\end{aligned}$$

$$\Delta X = \frac{L}{3}$$

The idea behind this solution is the same as the man and the rock question. The center of mass must remain in the same physical location.

5. A block of mass $M = 2.0$ kg is released from rest $h = 1.5$ meters above the ground and slides down a frictionless ramp. It slides across a floor that is frictionless, except for a small section with width $d = 0.5$ meters that has a coefficient of kinetic friction $\mu_k = 0.5$. At the left end, is a spring with spring constant $k = 300$ N/m. The box compresses the spring, and is accelerated back to the right.



- (a) What is the speed of the box at the bottom of the ramp?
 (b) What is the maximum distance the spring is compressed by the box?
 (c) What is the maximum height to which the box returns on the ramp?

(a)

$$PE = KE$$

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

$$v = 5.42 \text{ m/s}$$

- (b) Here we need to use the Work-Kinetic Energy Theorem:

$$W = \Delta KE$$

$$\frac{1}{2}mv_i^2 - W_f = \frac{1}{2}mv_f^2$$

$$\frac{1}{2}mv_i^2 - mg\mu_k d = \frac{1}{2}mv_f^2$$

$$v_f^2 = v_i^2 - 2g\mu_k d$$

$$v_f^2 = 24.5 \text{ m/s}$$

This gives us the velocity of the box after it has lost some energy due to friction. We can now use energy conservation again with the spring.

$$KE = PE$$

$$\frac{1}{2}mv_f^2 = \frac{1}{2}k\Delta x^2$$

$$\Delta x = \sqrt{\frac{mv_f^2}{k}}$$

$$\Delta x = 0.404 \text{ m}$$

(c) Note: v_i in this part is v_f in the previous part

$$W = \Delta KE$$

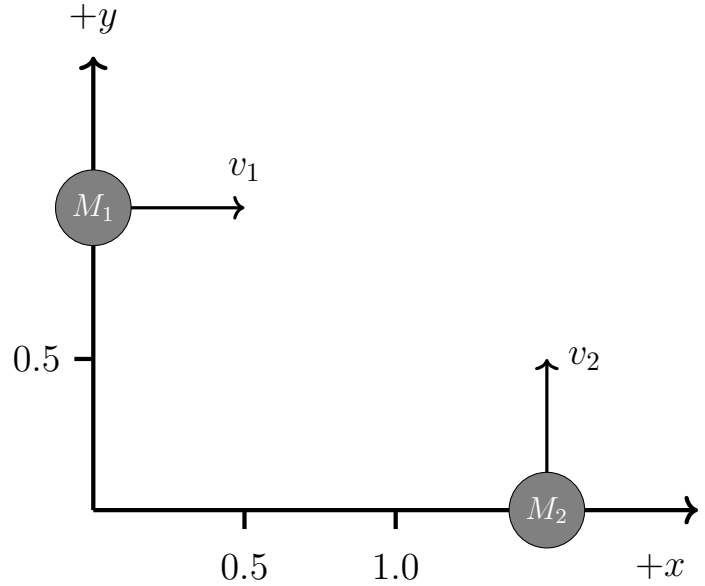
$$\begin{aligned}\frac{1}{2}mv_i^2 - mg\mu_k d &= \frac{1}{2}mv_f^2 \\ v_f^2 &= v_i^2 - 2g\mu_k d \\ v_f^2 &= 19.6 \text{ m/s}\end{aligned}$$

This is the velocity of the box right before it begins climbing back up the ramp (taking into account the lost energy from friction again). We can use energy conservation to find that its maximum height is at the point where all of its initial kinetic energy is converted into potential.

$$\begin{aligned}KE &= PE \\ \frac{1}{2}mv_f^2 &= mgh \\ h &= \frac{v_f^2}{2g} \\ h &= \frac{v_i^2 - 2g\mu_k d}{2g}\end{aligned}$$

$$\boxed{h = 1 \text{ m}}$$

6. Two discs are free to move without friction on a horizontal table. Disc $M_1 = 10.4$ kg and is initially at the position $(x = 0, y = 1.0)$ m, moving with velocity $v_1 = (v_x = 3.0, v_y = 0)$ m/s. Disc $M_2 = 10.6$ kg and is initially at $(x = 1.5, y = 0)$ m, moving with velocity $v_2 = (v_x = 0, v_y = 2.0)$ m/s. The figure below displays the initial conditions for the two discs in the x-, y- coordinates.



- (i) The initial velocity of the center of mass of the two-disc system is:
- A) $(v_x, v_y) = (3.12, 2.12)$ m/s
 - B) $(v_x, v_y) = (2.12, 3.12)$ m/s
 - C) $(v_x, v_y) = (1.51, 0.99)$ m/s
 - D) $(v_x, v_y) = (2.50, 2.50)$ m/s
 - E) $(v_x, v_y) = (1.48, 1.01)$ m/s
- (ii) The two discs are allowed to collide elastically. Given that the velocity of the 10.6 kg disc is $(v_x, v_y) = (1.4, 1.2)$ m/s after the collision, what is the velocity of the 10.4 kg disc?
- A) $(2.3, 1.2)$ m/s
 - B) $(1.2, 1.4)$ m/s
 - C) $(1.6, 0.8)$ m/s
 - D) $(1.4, 1.2)$ m/s
 - E) $(0.8, 1.6)$ m/s

(i)

$$v_{cm} = \frac{\Sigma mv}{\Sigma m}$$

$$v_{cm,x} = \frac{3 * 10.4 + 0 * 10.6}{10.4 + 10.6} = 1.48 \text{ m/s}$$

$$v_{cm,y} = \frac{0 * 10.4 + 2 * 10.6}{10.4 + 10.6} = 1.01 \text{ m/s}$$

The answer is therefore **(E)**.

(ii)

$$\begin{aligned}
 p_i &= p_f \\
 p_{10.4,i} + p_{10.6,i} &= p_{10.4,f} + p_{10.6,f} \\
 p_{cm,i} &= p_{10.4,f} + p_{10.6,f} \\
 p_{10.6,f} &= 10.6 * (1.4, 1.2) \\
 p_{cm,i} &= 21 * (1.48, 1.01) \\
 p_{10.4,f} &= p_{cm,i} + p_{10.6,f} \\
 &= (16.64, 8.32) \\
 v_{10.4,f} &= \frac{p_{10.4,f}}{10.4}
 \end{aligned}$$

$$v_{10.4,f} = (1.6, 0.8) \text{ m/s}$$

7. Show that for a system of particles with total mass M and total momentum \vec{P}_{total} , the center of mass follows

$$\vec{P}_{total} = M\vec{V}_{CM}$$

The velocity of the center of mass for N particles is, by definition,

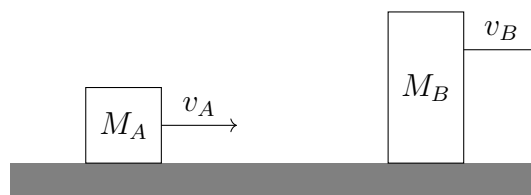
$$\vec{V}_{CM} = \frac{\sum_{n=1}^N m_n \vec{v}_n}{M}$$

If we now multiply each side by the total mass M , we get

$$M\vec{V}_{CM} = \sum_{n=1}^N m_n \vec{v}_n = \vec{P}_{total}$$

Where the middle term is the total momentum of the system.

8. Two blocks A and B are on a path to collide with each other. $M_A = 10 \text{ kg}$ and $v_A = 30 \text{ m/s}$ to the right while $M_B = 30 \text{ kg}$ and $v_B = 10 \text{ m/s}$ to the right. What is the velocity of each in the center of mass frame?



Let's establish that rightward is the $+\hat{x}$ direction since both blocks are moving that way. We must calculate the velocity of the center of mass in the lab frame. We can do that by

$$\vec{V}_{CM,lab} = \frac{\sum_{n=1}^N m_n \vec{v}_n}{M} = \frac{10 \cdot 30 + 30 \cdot 10}{10 + 30} = 15 \text{ m/s}$$

Now we use the relative velocity relation: $v_{A,B} = v_{A,C} + v_{C,B}$

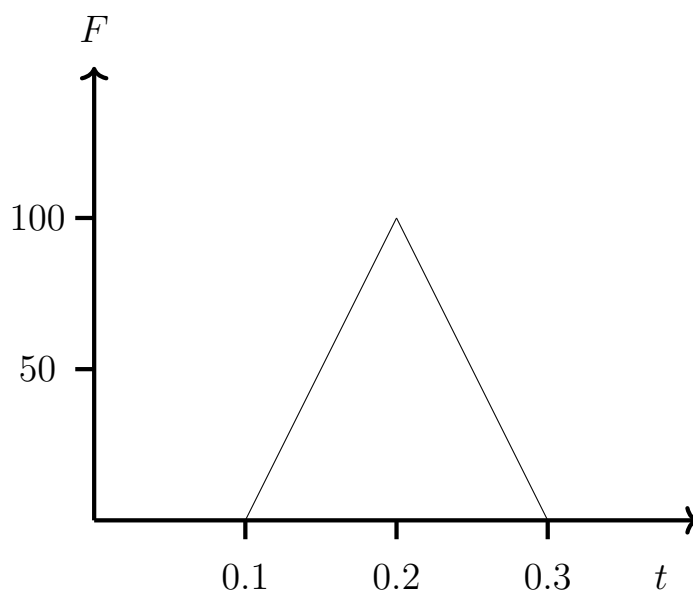
For Block A:

$$\begin{aligned} v_{A,CM} &= v_{A,lab} + v_{lab,CM} \\ v_{A,CM} &= 30 - 15 = 15 \text{ m/s} \end{aligned}$$

For Block B:

$$\begin{aligned} v_{B,CM} &= v_{B,lab} + v_{lab,CM} \\ v_{B,CM} &= 10 - 15 = -5 \text{ m/s} \end{aligned}$$

9. A ball of mass $M = 2 \text{ kg}$ is moving left at constant velocity $v = -3 \text{ m/s}$ when it hits a wall. The force applied by the wall onto the ball is plotted against time as shown below. What is velocity of the ball after hitting the wall?



Impulse is given by $\Delta P = F_{ave} \Delta t$. We find F_{ave} by taking the integral of the Force function and dividing by the change in time.

$$F_{ave} = \frac{Area}{\Delta t} = \frac{\frac{1}{2} * (.3 - .1) * 100}{0.2} = 50N$$

Plugging into the impulse formula,

$$\begin{aligned} \Delta P &= F_{ave} \Delta t \\ \Delta P &= 50 * .2 = 10 \end{aligned}$$

Finally,

$$\begin{aligned} \Delta P &= mv_f - mv_i \\ 10 &= (2)v_f - (2)(-3) \end{aligned}$$

$$v_f = 2 \text{ m/s}$$

10. During an explosion, a single particle at rest splits into two smaller particles both of mass $m = 5$ kg. The total chemical energy released is 270 J. What is the final speed each of particle and the total momentum of the the whole system?

The total mass of the single particle is 10 kg. Also, since it starts at rest, and no external forces are involved, its final must momentum must also be zero. This means that $\vec{v}_1 = -\vec{v}_2$ since they have equal masses.

To find the speed of the "half particles", we use energy conservation. Let K_{chem} be the chemical energy released, then

$$K_{half} = \frac{1}{2}mv^2 = \frac{1}{2}K_{chem}$$

We can solve for final velocity by

$$v = \sqrt{\frac{K_{chem}}{m}}$$