EMB A plasma is modeled as a gas of charge $-e$, mass m, mobile electrons with number density $n(\mathbf{r},t) = n_0 + \delta n(\mathbf{r},t)$ together with an immobile positive background charge density $\rho = +en_0$. In the presence of electric and magnetic fields, the electrons move according to

$$m\frac{d^2\mathbf{r}}{dt^2} = -e\left(\mathbf{E}(\mathbf{r},t) + \frac{d\mathbf{r}}{dt} \times \mathbf{B}_{\text{ext}}\right),$$

where the constant magnetic field $\mathbf{B}_{\text{ext}}$ points in the $+z$ direction. Consider the special case of right (+) or left (-) circularly polarized waves

$$\mathbf{E}(\mathbf{r},t) = \begin{bmatrix} E_x \\ E_y \\ 0 \end{bmatrix} = E_0 \begin{bmatrix} \cos(k_\pm z - \omega t) \\ \pm\sin(k_\pm z - \omega t) \\ 0 \end{bmatrix}$$

of frequency ω propagating through the plasma.

a) Assuming that such a wave exists, show that a possible solution of the equation of motion results in the given $\mathbf{E}$ field creating a current $\mathbf{j}(\mathbf{r},t)$ that obeys

$$\frac{d\mathbf{j}}{dt} = \alpha_\pm \mathbf{E}(x,t).$$

You should find the coefficients $\alpha_\pm$ in terms of the wave frequency ω, the plasma frequency ω_p which is defined by $\omega_p^2 \equiv n_0 e^2/(m\epsilon_0)$, and the cyclotron frequency $\omega_c \equiv e|\mathbf{B}_{\text{ext}}|/m$. (You may assume that E_0 is small enough that you can ignore the effect of the wave's own magnetic field on the motion of the electron.)

b) Use Maxwell's equations to derive the wave equation obeyed by $\mathbf{E}$, taking into account the effect of the current you found in (a).

c) From your wave equation derive an expression for the wavenumber $k_\pm$ in terms of ω, ω_p, and ω_c.

d) Now consider a wave that at $z = 0$ is *linearly* polarized in the x direction. Through what angle will the polarization have rotated at the point $z = z_0 > 0$? Write your answer in terms of δk defined by $\delta k = k_+ - k_-$.

Hint: If you did not find the explicit expression for $\alpha_\pm$ in part (a), or for $k_\pm$ in part (c), you can still obtain credit for subsequent parts by expressing their answers in terms of "$\alpha_\pm$" or "δk".