SMB Consider a system of N quantum spins, each with with two possible energy levels. One of the levels has zero energy and the other has energy $\epsilon > 0$.

Initially work in the *microcanonical ensemble* in which the system as a whole is thermally isolated and has fixed total energy E, but the individual spins can exchange energy with each other.

a) Compute $\Omega(E, N)$, the number of microscopic states accessible to this system at a given E and N, and from it write down an expression for the system's entropy per spin S/N. In the limit that E and N both become large while $u = E/N$ remains fixed, use Stirling's approximation $n! = n \ln n - n$ to find s as a function of u, and sketch a graph of $s(u)$.

b) Recall that if the system is large enough, we can assign to it a temperature T. Use this definition of T in terms of S and E to compute $u(T)$ and hence the specific heat $C = dE/dT$ as a function of ϵ and T. Sketch a graph of $u(T)$ and discuss the physics corresponding to the case where $u > \epsilon/2$.

Consider now the situation where the system is no longer isolated, but is instead in contact with a reservoir at temperature T.

c) Compute the canonical partition function Z for the system by computing the Z_{spin}'s for each individual spin and by suitably combining them. Show that the Z of the full system can also be obtained using the same $\Omega(E, N)$ as in part (a).

d) From Z write down the Helmholtz free energy per particle as a function of ϵ and T and use it to compute $u(T)$. Compare your result with that from part (b). Are they compatible?