SMA Consider a gas of photons in a cavity of volume V and at equilibrium temperature T.

a) Consider first a single cavity mode with frequency ω. Use the fact that photons are bosons to write down the canonical partition function for this single mode and from it find the thermal average energy $\langle u \rangle = u(\omega, T)$ of this mode.

b) Given that the number of photon modes (including both polarizations) in the box with frequency between ω and $\omega + d\omega$ is

$$\text{number of modes} = \frac{V\omega^2 d\omega}{\pi^2 c^3},$$

write down the integral giving average energy density U/V in the cavity. Do not attempt to evaluate the integral.

c) Show that one can change variables in your integral so as to express the result in the form

$$U/V = \alpha T^n$$

where you should find n and express α in terms of $\hbar$, the speed of light c and Boltzmann's constant k_{B}, together with an integral (that you should write down but not evaluate) over a dimensionless variable x.

d) Use your result from part (c) and the differential form of the first law to compute the entropy density $s = S/V$ of the radiation, expressing your answer in terms of T and α.

e) Compute the Helmholtz free energy $F(V, T)$ as function of α, T and V, and use a thermodynamic relation to compute the pressure P on the walls of the cavity as a function of α and T.