QMB Consider a qubit (represented by a spin 1/2 particle) with Hamiltonian

$$H_0 = \omega(S_x + S_y + \sqrt{2}S_z)/2$$

where $\omega > 0$, and the spin operators are $S_a = \hbar\sigma_a/2, a = x, y, z$. The σ's are the Pauli matrices which are displayed at the bottom of the page.

a) Find the eigenvectors and corresponding eigenvalues of the Hamiltonian H_0.

b) Assume that the system is in the eigenstate with largest energy. What is the probability of finding the system in the eigenstate of S_z with eigenvalue $+\hbar/2$?

c) At $t = 0$ the system is in the eigenstate of S_z with eigenvalue $+\hbar/2$. What is the probability of finding the same eigenvalue in a subsequent measurement of S_z after some small amount of time $\tau > 0$ has passed? Work to lowest non-zero order in τ.

d) Consider adding to the system's Hamiltonian a perturbation $H_1 = \lambda S_z$ for some real parameter λ. Use perturbation theory in λ to compute the change in the energy splitting between the two eigenstates of H_0 due to the addition of H_1. Work to the lowest order that gives a non-zero change in the splitting. Verify your answer by computing the exact eigenvalues of H. Is it possible to tune λ so that an energy eigenvalue of this physical system becomes zero?

The Pauli matrices:

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$