

**EMA** An infinitely long, thin, massless, and non-conducting cylindrical shell of radius  $R$  has a uniform surface charge density  $\sigma$ . The cylinder rotates around its axis at angular velocity  $\omega$ .

- a) Initially assume that  $\omega$  is constant. Find the electric field  $\mathbf{E}$  and the magnetic field  $\mathbf{B}$  at all points inside and outside the cylinder.

*For all remaining parts of this problem the angular velocity increases steadily, so  $\omega = \alpha t$  where  $\alpha$  is a constant.*

- b) Find the electric field  $\mathbf{E}$  at all points inside and outside the cylinder.
- c) Compute the Poynting vector  $\mathbf{S} = \mathbf{E} \times \mathbf{H}$  just outside and just inside the cylinder. Use the Poynting vector to find the direction and rate of the energy flow per unit cylinder length.
- d) Find the time rate of change of the energy density

$$U \equiv \frac{1}{2} \mathbf{B} \cdot \mathbf{H} + \frac{1}{2} \mathbf{E} \cdot \mathbf{D}$$

both inside and outside the cylinder.

- e) Is your answer to part (d) consistent with your result in part (c)? If not explain why it is not.
- f) Because the energy stored in the fields is increasing, external work is being done on the cylinder. What is the torque  $\tau$  per unit length that must be applied to do the work? Show that  $\tau$  is time independent.

