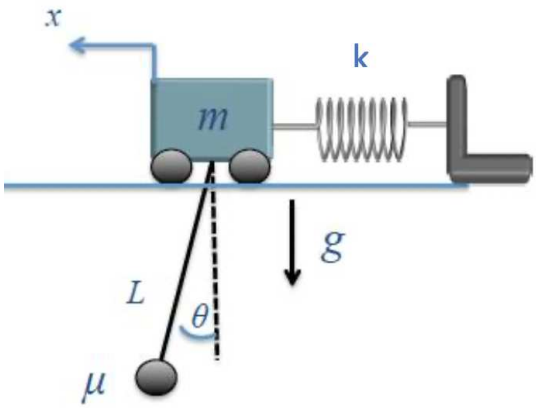
CMA Consider a small bob of mass μ at the end of a massless rod of length L which is attached to the bottom of a rolling cart. The cart is fixed to a wall via a spring with spring constant k.



a) Find a Lagrangian which describes the motion of this system in terms of the linear displacement x of the cart and the angular displacement θ of the rod from the vertical.

b) Use your Lagrangian to find the equations of motion for small oscillations about the equilibrium point $x = \theta = 0$. Express your answer in the form of the matrix equation

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = 0, \quad \mathbf{q} = \begin{bmatrix} x \\ \theta \end{bmatrix},$$

where the matrices $\mathbf{M}$ and $\mathbf{K}$ should be specified in terms of μ, m, k, L, and g.

c) Now consider the particular case where the matrices become

$$\mathbf{M} = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}.$$

What are the normal mode frequencies for this system?

d) If the system from (c) starts from rest at

$$\mathbf{q} = \begin{bmatrix} 0 \\ \theta_0 \end{bmatrix}$$

with θ_0 small, what are the solutions $x(t)$, $\theta(t)$?