Q3 Consider a hydrogen atom whose electron is subjected to an additional three-dimensional spherically symmetric harmonic oscillator potential so that the total potential is $V(r) = -e^2/4\pi\epsilon_0 r + \frac{1}{2}kr^2$.

a) Compute the first-order in k shift of the hydrogen *ground state* energy due to the potential V. Express your answer in terms of k and the Bohr radius a_0.
b) Find the first-order in k and leading order in n (the principal quantum number) shift for the *highly exited* states $n \gg 1$. Assume that the angular momentum quantum number l takes its maximum allowed value of l for each n.

In certain circumstances the diamagnetic effect of an extremely large magnetic field on the atom can be approximated by taking $V(r) = (e^2B^2/8m_e)r^2$.

c) Estimate (to nearest power of 10) the ratio R of magnetic to Coulomb energy of the ground state in a hydrogen atom on the surface of a neutron star where $B = 10^9\,\mathrm{T}$.
d) For a laboratory magnetic field with $B = 10\,\mathrm{T}$ estimate the n values (again taking the largest l) above which the diamagnetic energy is greater that the Coulomb energy.
e) Does a high magnetic field tend to strip the electron from the proton or encourage them to merge to form a neutron? (Hint: You may use the properties of the one dimensional harmonic oscillator)

Useful Information:

$$\text{Bohr radius} \equiv a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = 5.3\times10^{-11}\mathrm{m}$$

$$\psi_{n,l,m}(r.\theta,\phi) = \langle r,\theta,\phi|n,l,m\rangle, \quad \psi_{1,0,0} = \frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_o}$$

$$\begin{aligned}\langle n,l,m|(r/a_0)|n,l,m\rangle &= \frac{3n^2 - l(l+1)}{2}\\ \langle n,l,m|(r/a_0)^2|n,l,m\rangle &= \frac{n^2(5n^2-3l(l+1)+1)}{2}\end{aligned}$$