QMA Suppose we have a large number of independent two-level quantum systems (qubits) labelled by n, each of which has an energy splitting $E = \hbar\delta_n$ between its ground state $|0\rangle$ and its excited state $|1\rangle$. The δ_n are randomly distributed and all $|\delta_n| < \delta_{\max} \ll \Omega$ for some Ω.

Initially all qubits are in the state $|0\rangle$. At time $t = 0$ we briefly apply a "$\pi/2$-pulse" by switching on a magnetic field so that the qubits evolve under Hamiltonian $H = -\frac{1}{2}\hbar\Omega\sigma_y$ for a time $t = \pi/(2\Omega)$, after which H is switched off. Recall that $|\delta_n| \ll \Omega$ so that we can ignore the effect of the δ_n during this interval.

a) What is the state of qubit n following this process?

b) What are the expectation values of the σ_x and σ_z operators averaged over all qubits?

After switching off the field we wait a time $\tau \gg 1/\delta_{\max}$.

c) What is the state of qubit n after this wait?

d) What now are the expectation values of the σ_x, σ_z operators averaged over all qubits?

We then apply a brief "π-pulse" so that the qubits evolve under Hamiltonian $H = -\frac{1}{2}\hbar\Omega\sigma_y$, but now for a time $t = \pi/\Omega$.

e) What is the state of qubit n immediately after the π-pulse?

f) Now we wait yet another time τ, taking us to $t = 2\tau$. What are the expectation values of the σ_x, σ_z operators averaged over all qubits at time $t = 2\tau$?

The Pauli matrices are

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and we are using the basis

$$|0\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad |1\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$