Consider light propagating through a dielectric material in the presence of a magnetic field $B_0$ pointing along the $z$ axis. Model the material as a collection of harmonically-bound electrons of mass $m$ and number density $\rho$. Each electron satisfies Newton’s equations with a force

$$F(t) = -m\omega^2 x(t) - eE - e\frac{dx(t)}{dt} \times B_0.$$ 

Assume that the electromagnetic plane wave propagates in the $z$ direction and is circularly polarized with complex form

$$E = E \frac{1}{\sqrt{2}} (e_x \pm ie_y) e^{ikz-i\omega t},$$

where $e_x, e_y$ are unit vectors in the $x$ and $y$ directions respectively.

a) Show that the dipole moment of a single electron located at the origin as a function of time is of the form $d \equiv \epsilon x(t) = d(e_x \pm ie_y) e^{-i\omega t}$ and find the constant $d$. (You may assume that the electron has been exposed to the wave for a long time)

b) Use this dipole moment to find the polarization $P$ (the average dipole moment per unit volume) and the electric displacement $D = \epsilon_0 E + P$.

c) Use Maxwell’s equations in a dielectric medium to find the wave number $k = \omega/v_\pm$ for the two polarizations ($\pm$) in $E(t)$. Note that the propagation speeds $v_\pm$ will depend on $\omega$.

d) Linearly polarized light, with the electric field pointing in the $x$ direction, and frequency $\omega$, is incident on a slab of this material of thickness $w$ in the $z$-direction. Find an expression for $w$ as a function of $v_+, v_-$ such that the light emerges as linearly polarized light pointing in the $y$ direction on the other side of the slab. You need not have solved parts (a-c) to do this part of the problem.