EMB Consider light propagating through a dielectric material in the presence of a magnetic field $\mathbf{B}_0$ pointing along the z axis. Model the material as a collection of harmonically-bound electrons of mass m and number density ρ. Each electron satisfies Newton's equations with a force

$$\mathbf{F}(t) = -m\omega^2 \mathbf{x}(t) - e\mathbf{E} - e\frac{d\mathbf{x}(t)}{dt} \times \mathbf{B}_0.$$

Assume that the the electromagnetic plane wave propagates in the z direction and is circularly polarized with complex form

$$\mathbf{E} = E\frac{1}{\sqrt{2}}(\mathbf{e}_x \pm i\mathbf{e}_y)e^{ikz-i\omega t},$$

where $\mathbf{e}_x$, $\mathbf{e}_y$ are unit vectors in the x and y directions respectively.

a) Show that the dipole moment of a single electron located at the origin as a function of time is of the form $\mathbf{d} \equiv e\mathbf{x}(t) = d(\mathbf{e}_x \pm i\mathbf{e}_y)e^{-i\omega t}$ and find the constant d. (You may assume that the electron has been exposed to the wave for a long time)

b) Use this dipole moment to find the polarization $\mathbf{P}$ (the average dipole moment per unit volume) and the electric displacement $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$.

c) Use Maxwell's equations in a dielectric medium to find the wave number $k = \omega/v_\pm$ for the two polarizations ($\pm$) in $\mathbf{E}(t)$. Note that the propagation speeds $v_\pm$ will depend on ω.

d) Linearly polarized light, with the electric field pointing in the x direction, and frequency ω, is incident on a slab of this material of thickness w in the z-direction. Find an expression for w as a function of v_+, v_- such that the light emerges as linearly polarized light pointing in the y direction on the other side of the slab. You need not have solved parts (a-c) to do this part of the problem.