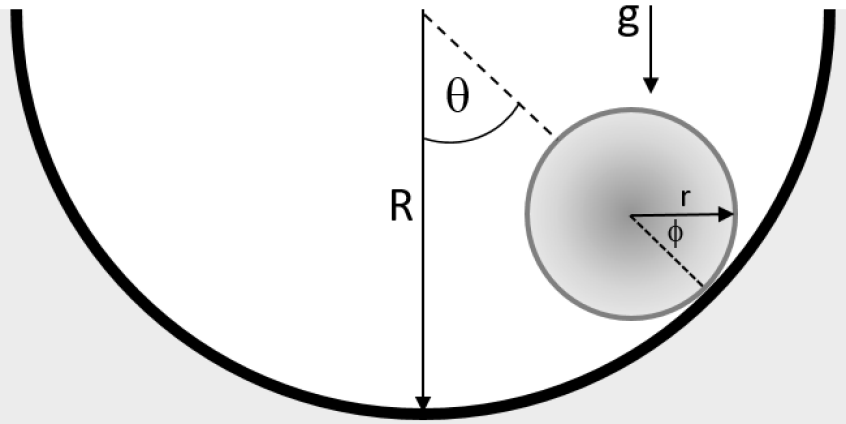
CMB



A solid sphere of mass m, moment of inertia $I = Kmr^2$, and radius r rolls without slipping inside a fixed cylinder of radius $R > r$. Initially the ball is held with its point of contact P with the cylinder at an angle $\theta_{\max}$ measured from the lowest point of the cylinder. It is then released and starts to roll.

a) Write down the rolling condition that relates $\dot{\theta}$ to the angular velocity $\dot{\phi}$ of the ball about its center. Use this relation to find expressions for the kinetic energy of the sphere in terms of the angular velocity $\dot{\theta}$ and the coordinate θ.

b) Now find the gravitational potential energy of the sphere in terms of θ and use it to compute the speed v_{CofM} of the centre of mass of the sphere at the moment when it reaches the lowest point ($\theta = 0$) on the cylinder.

c) What is the minimum coefficient of friction $\mu_{\min}$ for which the sphere starts rolling, rather than slipping, at the moment of its release?

d) Either by using a Lagrangian, or by an application of Newton's laws, find the equation of motion in terms of the coordinate θ.

e) Find the frequency of small oscillations of the sphere about $\theta = 0$ when $\theta_{\max}$ is small.