CMA a mass $M$ is connected to two massless springs with the same spring constant $k$, as in the figures below. Originally the springs are not stretched and have length $L$.
a) First assume that the mass executes small oscillations along the $x$-axis. Write down the equation of motion and find the angular frequency $\omega$ for the oscillation.

b) Now both springs are stretched by the same amount $S$ and the mass $M$ undergoes small oscillations in the $x-y$ plane. Find the potential energy of the system $V(x, y)-V(0,0)$ to second order in $x$ and $y$. Calculate the normal frequencies and normal modes. (Hint: Use symmetry to argue that there is no $x y$ term, so you can compute the $y^{2}$ term by assuming that $x=0$, and similarly for the $x^{2}$ term.)

c) When both springs are stretched by the same amount $S$ as in (b), the mass undergoes circular motion with a small radius $\rho$ in the $y-z$ plane perpendicular to the $x$ axis. Determine the rotational frequency $\omega$ around the x -axis. Compare your $\omega$ with the result of (b) by relating the circular motion to your solutions to part (b).


