



Center for Academic Resources in Engineering (CARE) Peer Exam Review Session

Math 231 – Calculus II

Mid-Semester Review Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Mar. 23 7:00-9:20pm Greta, Shreya, Sofi, Trusha

Can't make it to a session? Here's our schedule by course:

<https://care.grainger.illinois.edu/tutoring/schedule-by-subject>

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: <https://queue.illinois.edu/q/queue/844>
2. Click “New Question”
3. Add your NetID and Name
4. Press “Add to Queue”

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1 Improper Integrals and Riemann Sums

1. Determine if the integral converges or diverges:

$$\int_{2\pi}^{\infty} \frac{\sin(x) + 4x}{x^2} dx$$

2. Determine if the following integral converges or diverges

$$\int_1^{\infty} \frac{x^3 + 12x - 2}{x^6 + 5x^5 + 3x^2 - 1} dx$$

3. Evaluate the following integral using a Trapezoidal Riemann sum with 4 equal intervals:

$$\int_0^4 \frac{x^{0.5}}{2} + 1$$

2 Partial Fractions and Integration by Parts

4. Evaluate the following integral:

$$\int_1^{e^3} 4x^3 \ln(x)$$

5. Evaluate the following integral

$$\int \frac{5x - 1}{(x - 2)^2(x + 1)}$$

6. Evaluate the following integral

$$\int \frac{2x^2 - x + 4}{x^3 + 4x} dx$$

- (a) $\ln|x| + \ln(x^2 + 4) - \frac{1}{4} \arctan\left(\frac{x}{2}\right) + C$
- (b) $\ln|x| + \frac{1}{2} \ln(x^2 + 4) - \frac{1}{2} \arctan\left(\frac{x}{2}\right) + C$
- (c) $\ln|x| + \frac{1}{2} \ln(x^2 + 8) - \frac{1}{2} \arctan\left(\frac{x}{4}\right) + C$
- (d) $\frac{1}{2} \ln|x| + \frac{1}{2} \ln(x^2 + 4) - \frac{1}{3} \arctan\left(\frac{x}{2}\right) + C$
- (e) $\frac{1}{2} \ln|x| + \frac{1}{2} \ln(x^2 + 2) - \frac{1}{3} \arctan\left(\frac{x}{4}\right) + C$

3 Trig Integrals and Substitutions

7. What is the best substitution to make in order to solve the following integral?

$$\int y^2 \sqrt{y^2 + 4} dy$$

- (a) $y = \ln(\theta)$
- (b) $y = 2 \sec(\theta)$
- (c) $y = \sin(\theta)$
- (d) $y = 2 \tan(\theta)$
- (e) $y = e^t + 1$

8. Evaluate the following integral

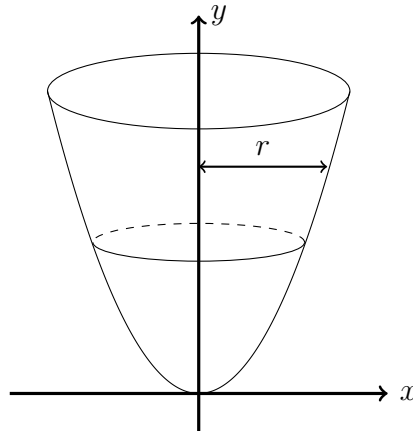
$$\int_0^4 \frac{dx}{(x^2 + 16)^{\frac{3}{2}}}$$

9. Evaluate the following Integral:

$$\int_0^{\pi/4} \tan^5(x) \sec^4(x) dx$$

4 Physics Applications, Arc Length, Solids of Revolution

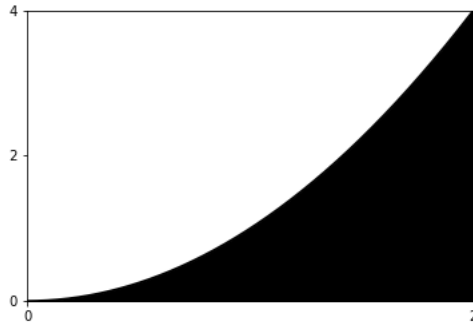
10. The figure below represents the calculation of the volume of solid of revolution. Which of the following integrals most closely represents the calculation of the volume as represented in the figure?



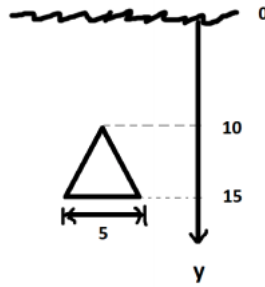
- (a) $\int_0^1 \pi y^2 dx$
(b) $\int_0^1 \pi x^2 dy$
(c) $\int_0^1 2\pi xy dx$
(d) $\int_0^1 2\pi xy dy$
(e) None of the above

11. Compute the arc length of the function $y = 1 + 2x^{\frac{3}{2}}$ between $x = 0$ and $x = 1$

12. Find the M_x , M_y , and the centroid of $y = x^2$ with density λ on $x \in [0, 2]$.



13. Determine the hydrostatic force on the triangle given the density of water $\rho = 1000\text{kg/m}^3$ with a depth y and $g = 9.8\text{m/s}^2$.



14. Which of the following expressions best represents the volume of the solid of revolution found by rotating the area between the curve $y = 1 + x^2 - 2x^4$ and the x -axis for x on the interval $(0,1)$ around the y -axis?
- (a) $2\pi \int_0^1 x \sqrt{1 + (2x - 8x^3)^2} dx$
 - (b) $2\pi \int_0^1 \sqrt{1 + (2x - 8x^3)^2} dx$
 - (c) $2\pi \int_0^1 (x + x^3 - 2x^5) dx$
 - (d) $2\pi \int_0^1 (1 + x^2 - 2x^4) dx$
 - (e) $2\pi \int_0^1 y \sqrt{1 + (2x - 8x^3)^2} dx$