Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

Math 241 – Calculus III

Mid-Semester Review Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Mar. 23 4:00-6:30pm John, Matthew, Ribhav, Rose

Can't make it to a session? Here's our schedule by course:

`https://care.grainger.illinois.edu/tutoring/schedule-by-subject`

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/845
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!

1. **(Dot and Cross Product)** Consider the following vectors: $\vec{A} = \langle 2, 4, 12 \rangle$, $\vec{B} = \langle 1, 5, 3 \rangle$, $\vec{C} = \langle 3, 4, 5 \rangle$, and $\vec{D} = \langle 2, 2, -1 \rangle$

   (a) Find the angle between $\vec{A}$ and $\vec{B}$

   (b) Which two vectors are orthogonal?

   (c) Find the projection of $\vec{A}$ onto $\vec{C}$

   (d) What is the volume of the parallelopiped formed by $\vec{B}$, $\vec{C}$, and $\vec{D}$?

2. **(Equations for Lines and Planes)** There is a line $\vec{L}$ that contains two points: $(5, 3, 1)$ and $(6, 2, 2)$, and there is a plane with normal vector $\langle 3, -4, 1\rangle$ that contains a point $(0, 0, -12)$. Find the intersection of the line $\vec{L}$ and the plane.

3. **(Identifying Vector Functions)** Match the vector functions below to its corresponding curve.

(a) $\vec{r}_1(t) = \langle t\cos t, t\sin t, t\rangle$

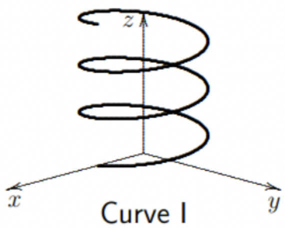
(b) $\vec{r}_2(t) = \langle 2\cos t, 1 + 4\cos t, 3\cos t\rangle$

(c) $\vec{r}_3(t) = \langle \cos t^3, \sin t^3, t^3\rangle$

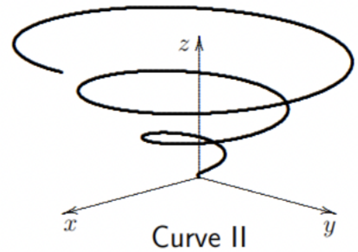
(d) $\vec{r}_4(t) = \langle \cos t, \sin t, 1 + 4\sin t\rangle$

(e) $\vec{r}_5(t) = \langle \cos t, \sin t, t^3\rangle$

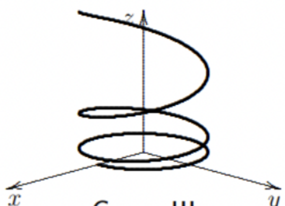
(f) $\vec{r}_6(t) = \langle \cos t, \sin t, 1 + \sin 4t\rangle$



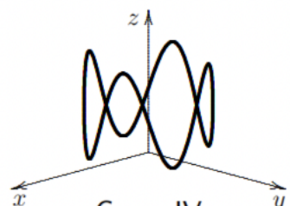
Curve I



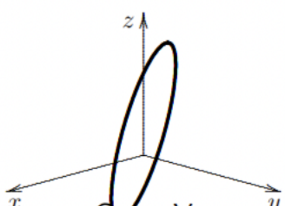
Curve II



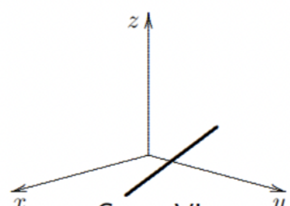
Curve III



Curve IV



Curve V



Curve VI

4. **(Vector Function Parameterization)** Find the curve of intersection between the cylinder $y^2 + z^2 = 16$ and the plane $x - z = 8$.

5. **(Chain Rule)** Suppose that there is a rectangle where the length of its sides changes with time. The first side is increasing at a rate of 5cm/s and the second side is decreasing at a rate of 1cm/s. How fast is the diagonal of the rectangle changing if the rectangle has a dimension of 3cm as the first side and 4cm as the second side? Is it increasing or decreasing?

6. **(Limits)** Evaluate the following functions with $\lim_{(x,y)\to(0,0)}$

$$\text{(a)} f(x,y) = \frac{x^3 - y^6}{xy^4}$$

$$\text{(b)} f(x,y) = \frac{3xy^2}{x^2 + y^2}$$

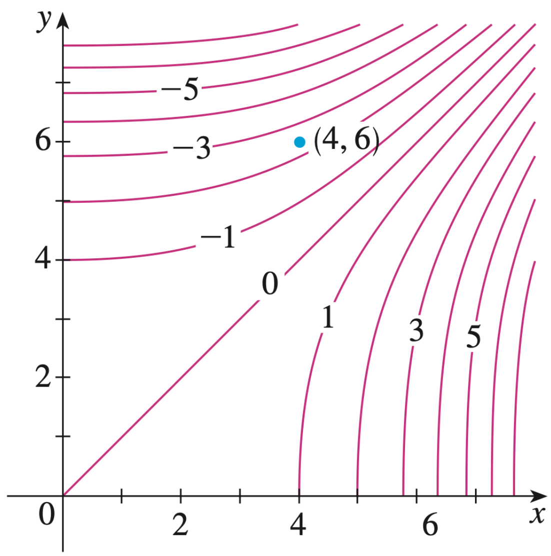
7. **(Directional Derivatives and Partial Derivatives)** Consider the contour map below for the arbitrary function $f(x, y)$, and answer the following questions.

   (a) Is the directional derivative at $(4, 6)$ in the direction of $\langle -1, 2 \rangle$ positive, negative, or zero?

   (b) What is the sign of f_{xx} at (4,6)?

   (c) What is the sign of f_{xy} at (4,6)?

   (d) Is the partial derivative $\partial f/\partial y$ at $(4, 0)$ positive, negative, or zero?



8. **(Critical Points)** Find the critical points of the function $f(x, y) = 2x^2 + 4xy - \frac{8}{3}y^3$, and classify the critical points as local maximum, local minimum, or saddle point.

9. **(Lagrange Multiplier)** In an imaginary world, the gravitational force experienced by an object at a certain position is modeled by $f(x, y, z) = x^2 + x + 2y^2 + 3z^2$. For a spaceship that is traveling on the unit sphere $x^2 + y^2 + z^2 = 1$, what is the maximum and minimum force experienced by it?