Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

Phys 213 – University Physics: Thermal Physics

Quiz 2 Worksheet Solutions

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Saturday, November 12, from 3-5 pm Jonah, Wesley, and Jay

Can’t make it to a session? Here’s our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/844
2. Click “New Question”
3. Add your NetID and Name
4. Press “Add to Queue”

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!
0.1 Entropy, Energy, and Heat

1. Consider 5 coins, each initially starting on heads.

(a) What is the entropy, \( S \), of this system in its current configuration?

(b) List all the macrostates available to this system.

(c) Identify the most probable macrostates. Hint: there are two.

(d) How many microstates would lead to the macrostates identified above?

(e) Calculate the change in entropy, \( \Delta S \), if the system changed to either of its most probable macrostates.

(a) The first thing to note here is that this system is in a 5H 0T macrostate (in this scenario, macrostates are the number of heads up (or tails) in a given configuration). There’s only one way to attain 5 heads — all of them are heads! So there is one microstate corresponding to this macrostate. We can now use the statistical definition of entropy, \( S = k_b \ln(\Omega) \), with the microstates in this configuration, \( \Omega \), equalling one. Thus,

\[
S = k_b \ln(1) = 0
\]

Which is a reasonable answer because this system is highly uniform.

(b) The macrostates of this system are all the combinations of heads (H) and tails (T) available to the 5 coins.

- 5H 0T
- 4H 1T
- 3H 2T
- 2H 3T
- 1H 4T
- 0H 5T

(c) The most probable macrostates have the highest number of associated microstates.

- 3H 2T
- 2H 3T

(d) To calculate the number of microstates in either of these macrostates, we must use the combination formula, \( \binom{n}{r} \), where \( n \) is the total number of items in the set (5), and \( r \) is the number of items we’re choosing (hence \( n \) choose \( r \)).

\[
\binom{5}{3} = \binom{5}{2} = 10 \text{ microstates}
\]

(e) The change in entropy is found by taking the difference of the final state entropy, \( S_{\text{final}} \), and initial state entropy, \( S_{\text{initial}} \).

\[
\Delta S = S_{\text{final}} - S_{\text{initial}} = k_b \ln(10) - k_b \ln(1) = k_b \ln(10)
\]

Note that the entropy increased, as we would expect by the second law of thermodynamics.
2. Let’s investigate the classic “cook a whole chicken by slapping it“ experiment. The average whole raw chicken has a mass of 1.4 kg with a specific heat capacity of 3350 J/kg K. Your hand (along with a heat-insulating glove you’re wearing) weighs 0.7 kg. Let’s assume the chicken is in an insulated environment and is held in place, so the chicken cannot transfer heat to its surroundings and cannot move. The chicken starts out at room temperature, $T_0 = 298.15$ K.

(a) Determine how much energy must be added to the chicken to fully cook it, i.e. bring it to $T_f = 350$ K.

(b) If you wanted to cook the chicken in one slap, determine how fast your hand must be moving during the slap.

(c) If you wanted to cook the chicken with multiple normal slaps ($\approx 7$ m/s), determine how many slaps you would need.

(a) 

$$U = C\Delta T = (1.4)(3350)(350 - 298.15) \text{ J}$$

$$= 243.177 \text{ kJ}$$

(b) We want the kinetic energy of the slap to be equivalent to energy needed to cook the chicken.

$$\frac{1}{2}mv^2 = 243177 \text{ J}$$

$$v = \sqrt{\frac{(2)(243177)}{0.7}} \text{ m/s}$$

$$= 833.541 \text{ m/s}$$

For reference, the speed of sound in air is 343 m/s.

(c) We’ll need $N$ normal slaps of kinetic energy $K$:

$$(N)\frac{1}{2}mv^2 = 243177 \text{ J}$$

$$N = \frac{(2)(243177)/(0.7)(7)^2}{1}$$

$$= 14179.417$$

So you’d need at least 14180 normal slaps to cook the chicken.

0.2 Heat Capacity

3. Two blocks, A and B, come in to contact. Block A starts out at $T_A = 150$ K, while block B starts at $T_B = 400$ K. The heat capacity of block A is 15 J/K, and that of block B is 5 J/K.

(a) Suppose Block A has a mass of 5 kg, for Block B, 1 kg. What would be the specific heat capacity for each?
(b) Determine the equilibrium temperature, $T_f$.
(c) Determine the net change in entropy. Which block lowered in entropy, and which block rose in entropy? Hint:

$$\frac{1}{T} = SU, \ C = UT$$

(a) Remember that $C = mc$, where $C$ is the heat capacity and $c$ is the specific heat capacity. Thus $c_A = 3 \text{ J/kg K}$ and $c_B = 5 \text{ J/kg K}$.
(b) We can use the weighted average formula to do this quickly:

$$T_f = \frac{(15)(150) + (5)(400)}{15 + 5} \text{ K} = 212.5 \text{ K}$$

(c) To find the change of entropy, we perform the integral

$$\Delta S = \int_{T_i}^{T_f} \frac{dU}{T} = \int_{T_i}^{T_f} \frac{C \, dT}{T}$$

for each block.

$$\Delta S_A = \int_{150}^{212.5} \frac{(15) \, dT}{T} \text{ J/K} = 15 \ln(212.5/150) \text{ J/K} = 5.225 \text{ J/K}$$

$$\Delta S_B = 5 \ln(212.5/400) \text{ J/K} = -3.163 \text{ J/K}$$

Thus block A increases in entropy while block B decreases in entropy. The net entropy change is

$$\Delta S = 5.225 \text{ J/K} - 3.163 \text{ J/K} = 2.062 \text{ J/K}$$

which, as expected, is positive.

4. Timmy buys an ice cream cone on a hot summer day, but he gets distracted and leaves it on a park bench. The specific latent heat of fusion of ice cream is $2.34 \times 10^5 \text{ J/kg}$, and his scoop has a mass of 75 g.

(a) If the sun is adding energy to his ice cream at a rate of 5 W, estimate how long it will take for his ice cream to completely melt, assuming it’s already at its melting point.
(b) Now, let’s say the specific heat capacity of melted ice cream is 2400 J/kg K. Assuming that all of the ice cream has to melt before the liquid ice cream starts to increase in temperature, and that the melting point of ice cream is about 273.15 K (which is also its initial temperature), determine the total time for the ice cream to go from solid to a room temperature liquid (room temp. = 298.15 K).

(a) The amount of energy needed to melt the ice cream is simply \((2.34 \times 10^5)(0.075)\) J = 17550 J. The rate of energy being added to the scoop is 5 J/s, so the time \(t\) required to melt the scoop is \(17550/5\) s = 3500 s, a little less than an hour.

(b) We already found the time needed to melt the ice cream, so all we need is the time to heat up the melted ice cream. The energy needed to get the ice cream to room temperature is \((2400)(0.075)(298.15 - 273.15)\) J = 4500 J. The time it take to do this is \(4500/5\) s = 900 s. Thus the entire melting and warming process takes \(3500 + 900 = 4400\) s.

### 0.3 Equipartition and Ideal Gases

5. Determine the specific heat capacity of solid aluminum via equipartition. Use the value of molar mass in the equation sheet.

Solid aluminum has 6 DOFs, so

\[
U = \frac{6}{2}NkT
\]

\[
C = \frac{dU}{dT} = 3Nk
\]

for \(N\) particles of aluminum. There are \(N_A\) particles in a mole (Avogadro’s number), so

\[
c_m = 3N_Ak
\]

is the molar heat capacity. Finally, using the molar mass of aluminum (0.028 kg),

\[
c = \frac{3N_Ak}{M} = 890.4 \text{ J/K}
\]

6. We have helium gas at temperature of 400 K. The molar mass of helium is 4.003 g/mol.

(a) Determine the RMS velocity of the helium particles.

(b) How does the RMS velocity of these particles compare to the RMS velocity of neon gas at the same temperature? The molar mass of neon is 20.180 g/mol.

(a) The internal energy of a mole of helium, being a monatomic gas:

\[
U = \frac{3}{2}N_AkT
\]
This internal energy should correspond to the kinetic energy of the mole of helium. We are given the molar mass of helium, $M$:

\[
\frac{1}{2} M v_{\text{rms}}^2 = \frac{3}{2} N_A kT
\]

\[
v_{\text{rms}} = \sqrt{\frac{3 N_A kT}{M}} \approx 1580 \text{ m/s}
\]

(b) We would expect the neon particles to have a smaller RMS velocity, as it has a higher molar mass.

### 0.4 Boltzmann Factors

7. We have a particle with two possible energy states $\pm \varepsilon$, where $\varepsilon = 6 \times 10^{-22}$ J. It is in contact with a thermal reservoir at temperature $T$.

(a) Determine the temperature $T$ such that $P(-\varepsilon)/P(+\varepsilon) = 20$.

(b) What is the average energy at this temperature?

(c) What is the probability of measuring $+\varepsilon$ as $T$ becomes extremely large? What is the average energy at extremely large temperatures?

(a) We want $P(-\varepsilon)/P(+\varepsilon)$ to be 20, or

\[
\frac{e^{-(-\varepsilon)/kT}}{e^{-\varepsilon/kT}} = 20
\]

\[
e^{2\varepsilon/kT} = 20
\]

Taking the natural log of both sides:

\[
\frac{2\varepsilon}{kT} = \ln(20)
\]

\[
T = \frac{2\varepsilon}{k \ln(20)}
\]

\[
= 28.027 \text{ K}
\]

(b)

\[
E_{\text{avg}} = (-\varepsilon)P(-\varepsilon) + (+\varepsilon)P(+\varepsilon)
\]

\[
= -\varepsilon \frac{e^{\varepsilon/kT}}{e^{\varepsilon/kT} + e^{-\varepsilon/kT}} + \varepsilon \frac{e^{-\varepsilon/kT}}{e^{\varepsilon/kT} + e^{-\varepsilon/kT}}
\]

\[
= E_{\text{avg}} = -5.429 \times 10^{-22} \text{ J}
\]

(c) In the limit as $T$ goes to infinity, the Boltzmann factor goes to 1, regardless of energy. So

\[
\lim_{T \to \infty} P(+\varepsilon) = \frac{1}{1 + 1} = \frac{1}{2}
\]

$P(-\varepsilon)$ will also be 1/2 — each state will become equally likely. Thus the average energy is

\[
E_{\text{avg}} = -\varepsilon(1/2) + \varepsilon(1/2) = 0 \text{ J}
\]
8. We have a 10 km-tall cylinder filled with helium particles. The cylinder is heated to \( T = 500 \text{ K} \). The mass of a helium particle is \( m_{\text{He}} = 6.643 \times 10^{-24} \text{ kg} \).

(a) Write the potential energy of a single Helium particle as a function of height \( h \).
(b) Determine the ratio of probabilities between a particle at height \( h_1 \) and a particle at height \( h_2 \) as a function of \( h_1 \) and \( h_2 \).
(c) Locations of high probability in the cylinder correspond to higher-pressure areas, i.e.

\[
p \propto P(h)
\]

Determine the ratio of pressures between the top and bottom of the cylinder.

(a) This is simply the classical gravitational potential energy:

\[
U(h) = m_{\text{He}}gh
\]

(b) To find the ratio of probabilities, we divide the Boltzmann factors. The particle at \( h_1 \) has energy \( m_{\text{He}}gh_1 \), the one at \( h_2 \), \( m_{\text{He}}gh_2 \):

\[
\frac{P(h_1)}{P(h_2)} = \frac{e^{-m_{\text{He}}h_1/kT}}{e^{-m_{\text{He}}h_2/kT}} = e^{-m_{\text{He}}g(h_1-h_2)/kT}
\]

(c) Pressure is proportional to probability, so

\[
\frac{p_1}{p_2} = \frac{P(h_2)}{P(h_1)}
\]

which is just the ratio we wrote in (b). The difference in heights \( h_1 - h_2 \) is 10 km (the height of the cylinder), so

\[
\frac{p_{\text{top}}}{p_{\text{bottom}}} = e^{-m_{\text{He}}g(h_1-h_2)/kT} = 9.607 \times 10^{-42}
\]

9. An astrophysics major is trying to measure the temperature of a distant star. The star is primarily comprised of hydrogen. For simplicity, we’ll describe the energy states of hydrogen as the following:

\[
\begin{align*}
E_1 & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \\
E_0 & \quad \quad \quad \quad \quad \quad \quad \quad 
\end{align*}
\]

Here, \( E_0 = 0 \text{ J} \) and \( E_1 = 1.634 \times 10^{-18} \text{ J} \). Through science-y magic, the student determines that 99.9999993% of the hydrogen in the star is in the \( E_0 \) energy state.

(a) Calculate the ratio of hydrogen atoms in the \( E_0 \) state to those in the \( E_1 \) state.
(b) Determine the temperature. Remember that the energy \( E_1 \) has 4 available microstates.
(a) If 99.9999993% of the hydrogen atoms are in the $E_0$ state, this means that $100\% - 99.9999993\% = 7 \times 10^{-7}\%$ are in the $E_1$ state.

\[
\frac{N_0}{N_1} = \frac{99.9999993\%}{7 \times 10^{-7}\%} = 1.429 \times 10^8
\]

(b) The ratio of hydrogen atoms in the $E_0$ state versus the $E_1$ state should be indicative of their probabilities:

\[
\frac{N_0}{N_1} = \frac{P(E_0)}{P(E_1)}
\]

To find this ratio, we divide the Boltzmann factors:

\[
\frac{P(E_0)}{P(E_1)} = \frac{e^{-E_0/kT}}{4e^{-E_1/kT}} = \frac{1}{4} e^{-(E_0 - E_1)/kT}
\]

This should be equal to the ratio we found above:

\[
\frac{1}{4} e^{-(E_0 - E_1)/kT} = 1.429 \times 10^8
\]

\[
\frac{-(E_0 - E_1)}{kT} = \ln\left(4(1.429 \times 10^8)\right)
\]

\[
T = \frac{E_1 - E_0}{k\ln(4(1.429 \times 10^8))} = 5872 \text{ K}
\]

0.5 Thermodynamic and Reversible Processes

10. What is the relationship between volume and pressure during isothermal and adiabatic processes for an ideal gas, respectively?

In an isothermal process, the temperature is constant and we can apply the ideal gas law: $pV = Nk_bT = \text{constant}$ (The ideal gas law is also used for isobaric and isochoric processes).

In an adiabatic process, $pV^\gamma = \text{constant}$ where $\gamma = \frac{C_p}{C_v} = (D + 2)/D$. Where $C_v$ is the constant volume heat capacity, $C_p$ is the constant pressure heat capacity, and $D$ is the number of degrees of freedom. Since no heat is added (adiabatic) but work can still be done, $\Delta S = 0$. As the gas compresses, the temperature increases, and as the gas expands, the temperature decreases.

11. The following two questions refer to the setup described below.

A piston of volume 0.05 m$^3$ contains 5 moles of a monatomic ideal gas at 300 K. If it undergoes an isothermal process and expands until the internal pressure matches the external pressure, $P_E = 1$ atm.
(i) How much work is done by the gas on the environment?
   a) $7.42 \times 10^3$
   b) $1.13 \times 10^4$
   c) $-1.13 \times 10^4$
   d) $1.83 \times 10^4$
   e) $-1.83 \times 10^4$

(ii) Suppose that the piston undergoes an adiabatic expansion instead, what is the final volume of the piston, $V_f$? (Values have units of cubic meters)
   a) 0.086
   b) 0.095
   c) 0.123

(i) The answer is (b). The work done ON the gas is $dW_{on} = -p dV$ while work done BY the gas is $dW_{by} = p dV$. They are different by a minus sign, so make sure to check your signs. To find the total work, we integrate both sides of the equation, writing $p$ in terms of $V$ using the ideal gas law

$$W_{by} = \int_{V_i}^{V_f} p \ dV = \int_{V_i}^{V_f} \frac{Nk_b T}{V} \ dV = Nk_b T \ln \left( \frac{V_f}{V_i} \right)$$

The final volume can be found using the ideal gas law (make sure pressure is in Pascals)

$$V_f = \frac{Nk_b T}{P_E} = 0.123$$

(ii) The answer is (a). The $P-V$ relation in an adiabatic process follows $pV^\gamma = \text{constant}$. Since the ideal gas is monatomic, $\gamma = \frac{5}{3}$. We can find the initial pressure using ideal gas law

$$P_i = \frac{Nk_b T}{V_i} = 249420 \ \text{Pa}$$

which allows us to calculate the constant $PV^\gamma = 249420 \times 0.05\frac{5}{3} = 1693$

After the expansion, we can use the $p-V$ relation again and plug in $P_f = P_E$ to get

$$P_E V_f^\gamma = 1693 \rightarrow V_f = 0.086$$

12. When a system is colder than the temperature of the environment (i.e. $T_{sys} < T_{env}$) its free energy is:

a) Smaller than its value when $T_{sys} = T_{env}$

b) Larger than its value when $T_{sys} = T_{env}$

c) The same as its value when $T_{sys} = T_{env}$
The answer is (b). When the system reaches thermal equilibrium ($T_{sys} = T_{env}$), the free energy is minimized. So when the system has a different temperature from the environment, the free energy will always be larger than the minimum at equilibrium.

13. Using the second law of thermodynamics, show that it is impossible for a heat engine to operate at $\epsilon = 1$.

Start with the fact that $\Delta S_{total} = \Delta S_H + \Delta S_C$ (the total change in entropy is the sum of the change in entropy of the hot and cold reservoir).

We know from the definition of temperature that $\frac{1}{T} = \frac{dS}{dU}$. This let’s us rewrite the above as

$$\Delta S_{total} = \frac{\Delta U_H}{T_H} + \frac{\Delta U_C}{T_C} = -\frac{Q_H}{T_H} + \frac{Q_C}{T_C}$$

Where the last step is a result of $Q = \Delta U$ since the reservoirs do no work. Now using the second law, $\Delta S_{total} \geq 0$, we can see $\frac{Q_C}{Q_H} \geq \frac{T_C}{T_H}$

Combining this with the definition of engine efficiency: $\epsilon = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H}$

We can see the that there is a limit on $\epsilon$

$$\epsilon \leq 1 - \frac{T_C}{T_H}$$

0.6 Quantum Harmonic Oscillators

14. We are given a quantum system with 3 quantum harmonic oscillators and 7 Quanta of energy to distribute into them. How many different ways can we distribute the energy into the system?

From our formula sheet, we can use the binomial coefficient expansion formula.

$$\binom{N-1+q}{q} = \binom{3-1+7}{7} = \frac{9!}{7!2!} = 36$$

So there are 36 different ways to distribute the energy into the oscillators.

0.7 Helmholtz Free Energy and Chemical Potential

15. A hot brick with heat capacity 100 J/K is used as a hot reservoir in a 270 K environment. The bricks starts at 400 K. Determine the maximum amount of work we could extract from this system.

First we find the change in internal energy of the brick.

$$\Delta U = C\Delta T = -13000 \text{ J}$$
Next, we find the change in entropy of the brick.

\[
\Delta S = \int_{T_i}^{T_f} \frac{C dT}{T} = C \ln \frac{T_f}{T_i} = -39.3 \text{ J}
\]

Next, we know \( \Delta F = \Delta U - T_{env} \Delta S \), so \( \Delta F = -2389 \text{ J} \). This is equal to the negative maximum work, so \( W_{max} = 2389 \text{ J} \).

16. Given an internal energy function \( U(N) = \sin(N^2) + \ln(\alpha N) \) and an entropy function \( S(N) = Ne^N \) determine the chemical potential as a function of \( N \) (\( T \) is constant).

Remember that the definition of chemical potential is the derivative of \( F \) with respect to \( N \)

\[
\mu = \frac{dF}{dN} = \frac{d}{dN} (U - TS) = \frac{d}{dN} \left[ \sin(N^2) + \ln(\alpha N) - TNe^N \right] = 2N \cos(N^2) + \frac{1}{N} - T(e^N + Ne^N)
\]

17. Show that \( -T \left( \frac{dS}{dN} \right)_{U,V} = \left( \frac{d\mu}{dN} \right)_{T,V} \) using the fundamental relation.

The fundamental relation of thermodynamics is

\[
dS = \frac{1}{T} dU + \frac{p}{T} dV - \mu \frac{dN}{T}
\]

Taking the first derivative, \( \frac{dS}{dN} \), assuming constant \( U \) and \( V \) (meaning \( dU = dV = 0 \)) and multiplying by \( -T \), we obtain \( \mu \). Likewise, the second derivative, \( \frac{dF}{dN} \), is the definition of chemical potential, which is \( \mu \). Thus the two expressions are equal.

18. Why does adding salt on the sidewalks in winter prevent ice from forming?

When we add salt to a water-based solution, the entropy of the solution increases because there are more microstates associated with it. This increased entropy makes it harder for the substance to freeze, since nature favors higher entropy systems. That is, the temperature would have to go even lower than it currently is for nature to favor the solid form of this substance over its liquid form. As a result, the freezing point decreases.

0.8 Gibbs Free Energy

19. What is the difference between Helmholtz free energy and Gibbs free energy? In which situations would you use one over the other?
Helmholtz free energy is used when a constant volume system is in contact with a constant temperature environment. This is why Helmholtz free energy is used to calculate the available work done by a heated or cooled brick. The brick remains at constant volume as energy is transferred to or from a constant temperature environment.

On the other hand, Gibbs free energy is used when a constant pressure system is in contact with a constant temperature environment. This is more useful in chemical reactions where some sort of gaseous expansion causes the pressure of the system to remain constant.

20. For this problem, we’ll be playing around with differentials.

(a) Recall that the expression for Gibbs Free Energy is

\[ G = U - TS + pV \]

Without making any substitutions, write out the full differential form of \( G \), \( dG \).

(b) Determine the derivative of \( G \) with respect to volume \( V \) at fixed pressure, temperature, and number of particles. That is, find

\[ (GV)_{T, p, N} \]

and simplify as much as you can.

(c) Should \( G \) be minimized or maximized at equilibrium? What does this mean \( dG/dV \) should be equal to at equilibrium?

(d) Determine a formula for the equilibrium volume \( V_{eq} \) given that this gas is not ideal, i.e.

\[ S = Nk \ln(V - bN) \]

(e) Determine the equilibrium volume if \( T = 340 \text{ K} \), \( p = 50 \text{ kPa} \), \( n = 4 \text{ mol} \), and \( b = 8.1 \times 10^{-27} \text{ m}^3 \).

(a) To find the differential, we must apply chain rule to the other variables:

\[ G = U = TS + pV \]

\[ dG = dU = -SdT - TdS + Vdp + pdV \]

(b) First, we divide the expression we found in (a) by the differential \( dV \):

\[ GV = UV - STV - TSV + VpV + pVV \]

\[ = UV - STV - TSV + VpV + p \]

Then, we fix the variables to be fixed: \( T \), \( p \), and \( N \). To deal with them, we set any derivative of these variable to 0:

\[ (GV)_{T, p, N} = UV - S(0) - TSV + V(0) + p \]

\[ = UV - TSV + p \]

Finally, recall that \( U = \frac{N_{DOF}}{2} NkT \). In other words, \( U \) depends on \( T \) and \( N \), but since these variables are fixed, \( U \) is also fixed, so derivatives of \( U \) are equal to 0.

\[ (GV)_{T, p, N} = -TSV + p \]
(c) $G$ should be minimized at equilibrium. Thus, its derivative with respect to volume should be 0 at equilibrium.

(d) We can plug this $S$ equation into the expression we found in (b):

$$\frac{(GV)_{T, p, N}}{T, p, N} = -TV (Nk \ln(V - bN)) + p$$

$$0 = -T \left( \frac{Nk}{V - bN} \right) + p$$

$G$ should be minimized, so we set $dG/dV$ equal to 0:

$$0 = -T \left( \frac{Nk}{V - bN} \right) + p$$

$$p = \frac{NkT}{V - bN}$$

$$pV - pbN = NkT$$

$$V = \frac{NkT + pbN}{p}$$

(e) We simply plug the given quantities into the equation we found in (d). Note, however, that we’re given the number of moles which must be converted into the number of molecules ($N = nN_A$). We get $V = 0.246 \text{ m}^3$.

### 0.9 Phase Changes

21. How does the phase diagram for water differ from other pure substances?

The phase diagram of water is unique in that as we increase pressure, the substance will become a liquid as opposed to remaining a solid. This indicates that the density of the solid will form of water (ice) is less than the density of the liquid form, which is why ice floats in water.
22. A sample of Substance A is at its melting point $T = 313.3$ K. Given that it takes 1566.5 J of energy to melt the sample at this temperature, what is the change in entropy per particle of the sample, given that the sample is made up of 1.5 moles of substance A?

(a) $5.5 \times 10^{-24}$ J/K/particle  
(b) 3.33 J/K/particle  
(c) 5 J/K/particle  
(d) $7.624 \times 10^{-24}$ J/K/particle

The latent heat of a substance is:

$$L = T \Delta S$$  \hfill (3)$$

We are given that it takes 1566.5 J of energy to melt the substance, this must be $L$ since it represents the total amount of energy necessary to complete a phase change for our sample. Dividing by $T$, we get:

$$L/T = 1566.5/313.3 = 5$$ J/K

We are asked for the entropy per particle however. We convert the moles of substance A to particles by multiplying by Avogadro’s Number, and then divide our total entropy change by this the total number of particles to get:

$$\frac{5}{1.5 \times N_A} = 5.5 \times 10^{-24}$$ J/K/particle $= \Delta S$