The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Saturday, November 12, from 3-5 pm Jonah, Wesley, and Jay

Can’t make it to a session? Here’s our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:
1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/844
2. Click “New Question”
3. Add your NetID and Name
4. Press “Add to Queue”

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!
0.1 Entropy, Energy, and Heat

1. Consider 5 coins, each initially starting on heads.
(a) What is the entropy, $S$, of this system in its current configuration?
(b) List all the macrostates available to this system.
(c) Identify the most probable macrostates. Hint: there are two.
(d) How many microstates would lead to the macrostates identified above?
(e) Calculate the change in entropy, $\Delta S$, if the system changed to either of its most probable macrostates.

2. Let’s investigate the classic “cook a whole chicken by slapping it“ experiment. The average whole raw chicken has a mass of 1.4 kg with a specific heat capacity of 3350 J/kg K. Your hand (along with a heat-insulating glove you’re wearing) weighs 0.7 kg. Let’s assume the chicken is in an insulated environment and is held in place, so the chicken cannot transfer heat to its surroundings and cannot move. The chicken starts out at room temperature, $T_0 = 298.15$ K.
(a) Determine how much energy must be added to the chicken to fully cook it, i.e. bring it to $T_f = 350$ K.
(b) If you wanted to cook the chicken in one slap, determine how fast your hand must be moving during the slap.
(c) If you wanted to cook the chicken with multiple normal slaps ($\approx 7$ m/s), determine how many slaps you would need.

0.2 Heat Capacity

3. Two blocks, A and B, come in to contact. Block A starts out at $T_A = 150$ K, while block B starts at $T_B = 400$ K. The heat capacity of block A is 15 J/K, and that of block B is 5 J/K.
(a) Suppose Block A has a mass of 5 kg, for Block B, 1 kg. What would be the specific heat capacity for each?
(b) Determine the equilibrium temperature, $T_f$.
(c) Determine the net change in entropy. Which block lowered in entropy, and which block rose in entropy? Hint:

\[
\frac{1}{T} = SU, \quad C = UT
\]
4. Timmy buys an ice cream cone on a hot summer day, but he gets distracted and leaves it on a park bench. The specific latent heat of fusion of ice cream is $2.34 \times 10^5$ J/kg, and his scoop has a mass of 75 g.

(a) If the sun is adding energy to his ice cream at a rate of 5 W, estimate how long it will take for his ice cream to completely melt, assuming it’s already at its melting point.

(b) Now, let’s say the specific heat capacity of melted ice cream is 2400 J/kg K. Assuming that all of the ice cream has to melt before the liquid ice cream starts to increase in temperature, and that the melting point of ice cream is about 273.15 K (which is also its initial temperature), determine the total time for the ice cream to go from solid to a room temperature liquid (room temp. = 298.15 K).

0.3 Equipartition and Ideal Gases

5. Determine the specific heat capacity of solid aluminum via equipartition. Use the value of molar mass in the equation sheet.

6. We have helium gas at temperature of 400 K. The molar mass of helium is 4.003 g/mol.

(a) Determine the RMS velocity of the helium particles.

(b) How does the RMS velocity of these particles compare to the RMS velocity of neon gas at the same temperature? The molar mass of neon is 20.180 g/mol.

0.4 Boltzmann Factors

7. We have a particle with two possible energy states $\pm \varepsilon$, where $\varepsilon = 6 \times 10^{-22}$ J. It is in contact with a thermal reservoir at temperature $T$.

(a) Determine the temperature $T$ such that $P(-\varepsilon)/P(+\varepsilon) = 20$.

(b) What is the average energy at this temperature?
(c) What is the probability of measuring $+\varepsilon$ as $T$ becomes extremely large? What is the average energy at extremely large temperatures?

8. We have a 10 km-tall cylinder filled with helium particles. The cylinder is heated to $T = 500$ K. The mass of a helium particle is $m_{\text{He}} = 6.643 \times 10^{-24}$ kg.

(a) Write the potential energy of a single Helium particle as a function of height $h$.

(b) Determine the ratio of probabilities between a particle at height $h_1$ and a particle at height $h_2$ as a function of $h_1$ and $h_2$.

(c) Locations of high probability in the cylinder correspond to higher-pressure areas, i.e.

\[ p \propto P(h) \]

Determine the ratio of pressures between the top and bottom of the cylinder.

9. An astrophysics major is trying to measure the temperature of a distant star. The star is primarily comprised of hydrogen. For simplicity, we’ll describe the energy states of hydrogen as the following:

\[
\begin{align*}
E_1 & \quad \quad \quad \quad \quad \\
E_0 & \\
\end{align*}
\]

Here, $E_0 = 0$ J and $E_1 = 1.634 \times 10^{-18}$ J. Through science-y magic, the student determines that 99.9999993% of the hydrogen in the star is in the $E_0$ energy state.

(a) Calculate the ratio of hydrogen atoms in the $E_0$ state to those in the $E_1$ state.

(b) Determine the temperature. Remember that the energy $E_1$ has 4 available microstates.
0.5 Thermodynamic and Reversible Processes

10. What is the relationship between volume and pressure during isothermal and adiabatic processes for an ideal gas, respectively?

11. The following two questions refer to the setup described below.

A piston of volume 0.05 m$^3$ contains 5 moles of a monatomic ideal gas at 300 K. If it undergoes an isothermal process and expands until the internal pressure matches the external pressure, $P_E = 1$ atm.

(i) How much work is done by the gas on the environment?

 a) $7.42 \times 10^3$
 b) $1.13 \times 10^4$
 c) $-1.13 \times 10^4$
 d) $1.83 \times 10^4$
 e) $-1.83 \times 10^4$

(ii) Suppose that the piston undergoes an adiabatic expansion instead, what is the final volume of the piston, $V_f$? (Values have units of cubic meters)

 a) 0.086
 b) 0.095
 c) 0.123

12. When a system is colder than the temperature of the environment (i.e. $T_{sys} < T_{env}$) its free energy is:

 a) Smaller than its value when $T_{sys} = T_{env}$
 b) Larger than its value when $T_{sys} = T_{env}$
 c) The same as its value when $T_{sys} = T_{env}$
13. Using the second law of thermodynamics, show that it is impossible for a heat engine to operate at $\epsilon = 1$.

**0.6 Quantum Harmonic Oscillators**

14. We are given a quantum system with 3 quantum harmonic oscillators and 7 Quanta of energy to distribute into them. How many different ways can we distribute the energy into the system?

**0.7 Helmholtz Free Energy and Chemical Potential**

15. A hot brick with heat capacity 100 J/K is used as a hot reservoir in a 270 K environment. The bricks starts at 400 K. Determine the maximum amount of work we could extract from this system.

16. Given an internal energy function $U(N) = \sin(N^2) + \ln(\alpha N)$ and an entropy function $S(N) = N e^N$ determine the chemical potential as a function of $N$ ($T$ is constant).

17. Show that $-T \left( \frac{dS}{dN} \right)_{U,V} = \left( \frac{dF}{dN} \right)_{T,V}$ using the fundamental relation.

18. Why does adding salt on the sidewalks in winter prevent ice from forming?
0.8 Gibbs Free Energy

19. What is the difference between Helmholtz free energy and Gibbs free energy? In which situations would you use one over the other?

20. For this problem, we’ll be playing around with differentials.

(a) Recall that the expression for Gibbs Free Energy is

\[ G = U - TS + pV \]

Without making any substitutions, write out the full differential form of \( G \), \( dG \).

(b) Determine the derivative of \( G \) with respect to volume \( V \) at fixed pressure, temperature, and number of particles. That is, find

\[ (GV)_{T, p, N} \]

and simplify as much as you can.

(c) Should \( G \) be minimized or maximized at equilibrium? What does this mean \( dG/dV \) should be equal to at equilibrium?

(d) Determine a formula for the equilibrium volume \( V_{eq} \) given that this gas is not ideal, i.e.

\[ S = Nk \ln(V - bN) \]

(e) Determine the equilibrium volume if \( T = 340 \) K, \( p = 50 \) kPa, \( n = 4 \) mol, and \( b = 8.1 \times 10^{-27} \) m³.

0.9 Phase Changes

21. How does the phase diagram for water differ from other pure substances?

22. A sample of Substance A is at its melting point \( T = 313.3 \) K. Given that it takes 1566.5 J of energy to melt the sample at this temperature, what is the change in entropy per particle of the sample, given that the sample is made up of 1.5 moles of substance A?
(a) $5.5 \times 10^{-24}$ J/K/particle
(b) 3.33 J/K/particle
(c) 5 J/K/particle
(d) $7.624 \times 10^{-24}$ J/K/particle