Phys 212 – University Physics: Electricity and Magnetism

Midterm 3 Worksheet Solutions

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Thurs, Dec 6:00-7:30 pm Jung and Conor

Session 2: Fr, Dec 2, 3:30-5:00 pm Conor and Ray

Can’t make it to a session? Here’s our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/848
2. Click “New Question”
3. Add your NetID and Name
4. Press “Add to Queue”

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!
1. A circuit is composed of a battery with voltage $V = 10$ V, one resistor $R = 75$ Ω, a capacitor $C = 20$ pF, an inductor $L = 20$ mH and a switch $S$. The switch has been open for a long time; at $t = 0$, it is closed.

(i) What is the voltage across the capacitor after the switch has been closed for a long time?
   a) 10 V  
   b) 0 V  
   c) 15 V  
   d) 7.5 V

(ii) What is the charge across the capacitor at this time?
   a) 20 pC  
   b) 0 C  
   c) 0.2 nC  
   d) 2 nC

(iii) What is the voltage across $R$ after the switch has been closed for a long time?
   a) 7.5 V  
   b) 10 V  
   c) 0 V  
   d) 5 V

(iv) If the switch is now reopened after a long period of time, what will the initial current going through $R$ be?
   a) 0.133 A  
   b) 0 A  
   c) 0.5 A  
   d) 0.167 A

(i) The voltage source and capacitor are in parallel, and after a long time, the capacitor charge has approached its maximum. So voltage across capacitor equals the battery voltage $V$. The answer is (a).
(ii) Using $Q = CV$

$C = 20$ and $V = 10$

$Q = (20) \times (10) = \frac{200 \text{ pC}}{}$ or $0.2 \text{ nC}$

(iii) After a long time, the inductor acts as a normal wire, so there is no voltage drop across it. Essentially, this means that the resistor is in parallel with both the battery and capacitor, both of which have a voltage drop of $10V$.

(iv) When the switch is opened, the inductor prevents an instantaneous change in current. Therefore, the current right before and right after must be the same. Using Ohm’s law for the current before the open switch:

$$I = \frac{V}{R}$$

$I = 0.133 \text{ A}$

2. The electric field for a plane electromagnetic wave in a vacuum is given by

$$\vec{E}(y, t) = 2100 \sin(\omega t + 0.8y) \hat{x}$$

(i) What is the frequency of the wave?

(ii) What is the magnitude of the magnetic field?

(iii) What is the direction of the poynting vector?

   a) $\hat{y}$
   b) $-\hat{y}$
   c) $-\hat{z}$
   d) $\hat{z}$

(iv) What is the direction of $\vec{B}$?

   a) $\hat{y}$
   b) $-\hat{y}$
   c) $-\hat{z}$
   d) $\hat{z}$

(i) In a vacuum the speed of light is $c$. We can use the following dispersion relation to find $\omega$.

$$c = \frac{\omega}{k}$$

where $k = 0.8$ from the equation given. Rearrange the equation to solve for $\omega$. This can then be related to the normal frequency by $\omega = 2\pi f$. Solving for $f$:

$[f = 38 \text{ MHz}]$
(ii) The relationship between the magnitude of the electric and magnetic fields is

\[ E = cB \]

We can solve for \( B \) using \( E = 2100 \) from the equation

\[ |B| = \frac{2100}{c} = 7 \times 10^{-6} \text{T} \]

(iii) The Poynting vector points in the direction of the wave’s motion. In this case, that’s along the \( y \)-axis because \( E \) is a function of \( y \). To determine the sign, we must examine the signs of the function’s argument. Since both \( \omega t \) and \( ky \) have the same sign, the wave moves along \(-\hat{y}\).

(iv) The equation for the Poynting vector is

\[ \vec{S} \propto \vec{E} \times \vec{B} \]

Since we know the direction of \( \vec{S} \) (\(-\hat{y}\)) and the direction of \( \vec{E} \) we can determine the direction of \( \vec{B} \) using the right hand rule. We must determine the unit vector \( \hat{A} \) that makes \(-\hat{y} = \hat{x} \times \hat{A} \) true.

From the diagram, we can see that the only unit vector that works is \( +\hat{z} \).

3. An ideal transformer has \( N_1 = 100 \) turns in the primary coil and \( N_2 = 10 \) turns in the secondary coil. An RMS voltage of \( V = 120 \) V and 60Hz AC voltage is connected to the primary coil. A resistor with resistance \( R = 20 \) \( \Omega \) is connected to the secondary coil as shown in the figure.

(i) What is the average voltage across the resistor?

a) 120 V  
b) 1200 V  
c) 12 V  
d) 24 V

(ii) What is the average power in the resistor?
a) 0 W  
b) 14.4 W  
c) 0.6 W  
d) 7.2 W  

(i)  
Using \[ \frac{V_1}{N_1} = \frac{V_2}{N_2} \]  
where \( N_1 = 100, N_2 = 10, \) and \( V_1 = 120 \text{ V} \) to get  
\[ V_2 = 12 \text{ V} \]  

(ii) Using the average voltage found in the previous question  
\[ P = \frac{V^2}{R} = \frac{12^2}{20} = 7.2 \text{ W} \]  

4. A beam of unpolarized light of intensity \( I_0 \) passes through a series of ideal polarizing filters with their transmission axis turned to various angles, as shown in the figure (\( \theta_1 = 75^\circ \) and \( \theta_2 = 90^\circ \), both relative to the vertical)

(i) What is the light intensity (in terms of \( I_0 \)) in regions A, B and C?  
(ii) If we remove the middle filter, what will be the intensity at point C?  
(iii) If the second filter was rotated 15° clockwise, what would the light intensity be at point C?  
(iv) How far should the first filter be rotated in order to maximize the intensity of light at point C?  

(i) Unpolarized light through a linear polarizer cuts intensity in half. Then use  
\[ I = I_0 \cos^2(\theta) \]  
to solve for intensities going through the other two filters  
\[ I_A = \frac{I_0}{2} \]  
\[ I_B = I_A \cos^2(\theta_1) = 0.0335I_0 \]  
\[ I_C = I_B \cos^2(\theta_2 - \theta_1) = 0.03125I_0 \]
(ii) If the middle filter is removed, the angle between the first and third filters is $90^\circ$. Vertically polarized light cannot enter a horizontal polarizer.

\[ I_C = 0 \]

(iii) If the second filter is rotated $15^\circ$ clockwise, \( \theta \) between the first and second filter would be $90^\circ$, so \( I_B = 0 \) and \( I_C = 0 \)

\[ I_C = 0 \]

(iv) To maximize the intensity of light at point C, rotate the first filter $75^\circ$ clockwise to make \( \theta \) between the first and second filter $0$. This allows \( I_1 = I_2 \) so there’s no decrease in intensity through the second filter.

5. A light ray is incident from the air \( (n_1 = 1) \) into a glass with index of refraction \( n_2 = 1.4 \) at angle \( \theta_1 \). The angle between the normal and the refracted ray is \( \theta_2 = 45^\circ \).

\[ n_2 = 1.4 \]

(i) What is the value of \( \theta_1 \)?

(ii) What is the value of \( \theta_3 \)?

(iii) If \( n_2 \) were to decrease but stay above the index of refraction of air and we fix \( \theta_2 \) to be $45^\circ$, how would the values of \( \theta_1 \) and \( \theta_3 \) be affected?

   a) Decrease \( \theta_1 \), increase \( \theta_3 \)
   b) Decrease \( \theta_1 \), decrease \( \theta_3 \)
   c) Increase \( \theta_1 \), increase \( \theta_3 \)
   d) Increase \( \theta_1 \), decrease \( \theta_3 \)
(i) Use Snell’s Law \( n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \)
\[
\sin(\theta_1) = 1.4 \sin(45°) \\
\theta_1 = \sin^{-1}(1.4 \sin(45°)) = 81.9°
\]

(ii) Use Snell’s Law \( n_2 \sin(\theta_2) = n_1 \sin(\theta_3) \)
\[
\sin(90° - \theta_3) = 1.4 \sin(45°) \\
\theta_3 = 90° - \sin^{-1}(1.4 \sin(45°)) = 8.1°
\]
Notice that \( \theta_3 \) is taken with respect to the horizontal, and that Snell’s Law involves angles with respect to the normal.

(iii) If the index of refraction decreases, \( n_2 \sin(45°) \) also decreases which decreases \( \theta_1 \). If \( \theta_1 \) decreases, \( \theta_3 = 90° - \theta_1 \) must increase. The answer is therefore (a).