The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Monday, October 24th from 6-8 pm Matthew, Jonah, and Wesley

Can’t make it to a session? Here’s our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/844
2. Click “New Question”
3. Add your NetID and Name
4. Press “Add to Queue”

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!
1. Write out the first two laws of thermodynamics.

(a) The first law of thermodynamics, which is also known as the Law of Conservation of Energy, states that energy can neither be created nor be destroyed, it can only be transferred from one form to another. \( U = W + Q \)

(b) The second law of thermodynamics says that the entropy of any isolated system always increases. \( \Delta S \geq 0 \)

2. Consider 5 coins, each initially starting on heads.

(a) What is the entropy, \( S \), of this system in its current configuration?

(b) List all the macrostates available to this system.

(c) Identify the most probable macrostates. Hint: there are two.

(d) How many microstates would lead to the macrostates identified above?

(e) Calculate the change in entropy, \( \Delta S \), if the system changed to either of its most probable macrostates.

(a) The first thing to note here is that this system is in a 5H 0T macrostate (in this scenario, macrostates are the number of heads up (or tails) in a given configuration). There’s only one way to attain 5 heads — all of them are heads! So there is one microstate corresponding to this macrostate. We can now use the statistical definition of entropy, \( S = k_b \ln(\Omega) \), with the microstates in this configuration, \( \Omega \), equalling one. Thus,

\[
S = k_b \ln(1) = 0 \tag{1}
\]

Which is a reasonable answer because this system is highly uniform.

(b) The macrostates of this system are all the combinations of heads (H) and tails (T) available to the 5 coins.

- 5H 0T
- 4H 1T
- 3H 2T
- 2H 3T
- 1H 4T
- 0H 5T

(c) The most probable macrostates have the highest number of associated microstates.

- 3H 2T
- 2H 3T

(d) To calculate the number of microstates in either of these macrostates, we must use the combination formula, \( _nC_r \), where \( n \) is the total number of items in the set (5), and \( r \) is the number of items we’re choosing (hence \( n \ choose \ r \)).

\[
_5C_3 = _5C_2 = 10 \text{ microstates}
\]
(e) The change in entropy is found by taking the difference of the final state entropy, $S_{\text{final}}$, and initial state entropy, $S_{\text{initial}}$.

$$\Delta S = S_{\text{final}} - S_{\text{initial}} = k_b \ln(10) - k_b \ln(1) = k_b \ln(10)$$

Note that the entropy increased, as we would expect by the second law of thermodynamics.

3. Consider two containers A and B filled with the same amount of the same ideal gas. Heat is added to each one but A is held at constant volume while B is held at constant pressure. Both are heated through a temperature increase of 10 K.

(a) Which of the following is true of their added heats?

A) $Q_A > Q_B$
B) $Q_A = Q_B$
C) $Q_A < Q_B$

(b) Which of the following is true of their change in internal energy?

A) $\Delta U_A > \Delta U_B$
B) $\Delta U_A = \Delta U_B$
C) $\Delta U_A < \Delta U_B$

(c) Assuming the situation described above is carried out, and both containers increase their temperatures by 10 Kelvin, how much more heat must be added to container B?

(a) The answer is (C). Since container A is held at constant volume, the gas cannot expand. On the contrary, container B is held at constant pressure, so it does expand. Therefore, more heat must be added to container B to reach the same temperature because some of the added heat goes into the work done by the gas expansion. This is equivalent to saying that the environment does negative work on the gas.

(b) The answer is (B). The internal energy of a gas only depends on the number of particles, the degrees of freedom per particle, and the temperature, $U = \frac{D}{2} N k_b T$, where $D$ is the degrees of freedom. Each container has the same amount of the same gas and undergo the same temperature change. Therefore, their internal energies must be equal.

(c) From part (b), we know that after the heating process, the gases have equal internal energies. However, from part (a) we know more energy was added to container B as a result of expansion. Using the first law of thermodynamics, $dU = dQ - pdV$, we can say

$$Q_B - Q_A = (U_B - U_A) + p(\Delta V_B - \Delta V_A)$$
$$= p\Delta V_B = W_{\text{byB}}$$

Where this equals exactly the additional work done by gas B during expansion.

4. What is the difference between heat capacity, specific heat capacity, and molar heat capacity? Physically, these three quantities are measuring the same thing: an object’s tendency to increase in temperature as heat is added to it. However, the difference between these three is the units that they are expressed in.
Heat Capacity
\[ C \equiv \frac{dQ}{dT} \]

Specific Heat
\[ c_{sp} \equiv \frac{C}{m} \]

Molar Heat
\[ c_{mol} \equiv \frac{C}{n} \]

Where \( m \) is the mass and \( n \) is the number of moles of the substance.

Therefore, the units on each of these quantities is

Heat Capacity: \( \text{J/K} \)
Specific Heat: \( \text{J/kg K} \)
Molar Heat: \( \text{J/mol K} \)

5. Substance A has a heat capacity of 3 J/K while substance B has a heat capacity of 5 J/K. Starting from the same temperature, which one cools at a faster rate.

The units for heat capacity are J/K. Interpreting this, a heat capacity tells us the amount of energy required to raise or lower an object’s temperature by 1 Kelvin. So you may be tricked into thinking at first that a higher heat capacity means a greater tendency to heat up, but that’s not true. A lower heat capacity means less energy is required for temperature changes (heating and cooling), so objects with lower heat capacities will change temperature at faster rates.

Substance A

6. True or False: For an ideal gas, the molar heat capacity at constant pressure will always be greater than the molar heat capacity at constant volume.

From the definition of molar heat capacity at constant pressure
\[ c_{p,mol} = \frac{1}{n} \frac{dU}{dT} + \frac{p}{n} \frac{dV}{dT} \]

and the definition of molar heat capacity at constant volume
\[ c_{v,mol} = \frac{1}{n} \frac{dU}{dT} \]

we can relate the two by substitution, ending up with
\[ c_{p,mol} = c_{v,mol} + \frac{p}{n} \frac{dV}{dT} \]

So we already see that \( c_{v,mol} \) is less than \( c_{p,mol} \) as long as \( \frac{p}{n} \frac{dV}{dT} > 0 \). Now let’s confirm that, for an ideal gas, \( \frac{p}{n} \frac{dV}{dT} \) is greater than zero.

Starting from the ideal gas law \( pV = Nk_bT \), we can differentiate \( V \) with respect to \( T \) to get \( \frac{p}{n} \frac{dV}{dT} = Nk_b \). Substituting this into the equation relating the two molar heat capacities and simplifying, we get
\[
c_{p,mol} = c_{v,mol} + \frac{p}{n} \frac{dV}{dT}
\]
\[
c_{p,mol} = c_{v,mol} + \frac{N}{n} k_b
\]
\[
c_{p,mol} = c_{v,mol} + N_A k_b
\]
\[
c_{p,mol} = c_{v,mol} + R
\]

Note that \( \frac{N}{n} \) is Avogadro’s Number, \( N_A \), and \( R \) is the universal gas constant. Since \( R > 0 \), we have confirmed that \( c_{p,mol} > c_{v,mol} \) for an ideal gas.

\[\text{True}\]

7. Explain why the heat capacity at constant volume of an ideal solid is twice that of the same amount of a monatomic ideal gas.

At constant volume, heat capacity is the derivative of internal energy

\[C_v = \frac{dU}{dT}\]

Where \( U = \frac{D}{2} N k_b T \) where \( D \) is the number of degrees of freedom.

For a ideal solid, there are six degrees of freedom; 3 translational and 3 vibrational (\( D = 6 \)). But for a monatomic ideal gas, there are only 3 degrees of freedom; 3 translational (\( D = 3 \)). When the derivative of internal energy is taken to obtain heat capacity, these constants remain.

8. A sealed container with a mass of 2.7 kg is filled with 4 moles of helium gas. Initially, the helium gas is at a temperature of 140°C and the container is at 38°C. The helium-container system is thermally isolated.

Note that the specific heat of the material making the container is 386 J/(kg·K) and the molar specific heat of helium is 12.5 J/(mol·K).

Find the equilibrium temperature of the system in Celsius and Kelvin.

The helium-container system is isolated from any heat conducting environment. So, by conservation of energy, we know that \( Q_{He} = -Q_C \). One thing to see is that the heat capacities in the question have different units — one is specific heat, the other is molar heat. Using the fact that \( Q = C \Delta T \) where \( C \) is regular heat capacity, we can do the following

\[
Q_{He} = -Q_C
\]
\[
n c_{mol}(T_f - T_{He,i}) = -m c_{sp}(T_f - T_{C,i})
\]

were \( n \) is the moles of Helium, \( m \) is the mass of the container, and \( T_f \) is the equilibrium temperature (which is the same for both the Helium and the container by the definition of thermal equilibrium). Solving for \( T_f \)
\[ T_f = \frac{n_c \text{mol} T_{H, i} + m_{sp} T_{C, i}}{n_c \text{mol} + m_{sp}} \]

And when the values given above are put into this equation, we get

\[ T_f = 315.8 \text{ K} = 42.7^\circ \text{C} \]

**Remember to convert the temperatures to Kelvin in the equations, then back to Celsius for the final answer.**

9. A 10 kg block of aluminum initially at 90°C is placed in contact with a 12 kg block of copper initially at 10°C. The molar heat capacities of aluminum and copper are 24.2 J/(K-mol) and 24.5 J/(K-mol), respectively. Molar masses are on the equation sheet.

(a) When the two blocks come into thermal equilibrium, what temperature will they reach?

A) 75.4°C  
B) 62.9°C  
C) 49.8°C  
D) 33.5°C  
E) 24.5°C

(b) How does the entropy of the aluminum change during this process?

A) 0  
B) +695 J/K  
C) -695 J/K  
D) +3.211 \times 10^3 \text{ J/K}  
E) -3.211 \times 10^3 \text{ J/K} 

(a) The answer is (B). It is assumed in these problems that when two blocks come into contact, the only energy (heat) flow is between them, and not the surrounding environment. That is, \( Q_{Al} = -Q_{Cu} \) (remember that negative sign).

We know from the definition of heat capacity at constant volume that

\[ dQ = C_v dT = \frac{m}{m_{mol}} c_{mol} dT \]

where \( Q \) is the heat exchanged, \( C_v \) is the heat capacity at constant volume, \( m \) is the mass, \( m_{mol} \) is the molar mass, \( c_{mol} \) is the molar heat capacity, and \( T \) is the temperature. This is written in terms of moles because the problem gives us the relevant quantities in these units. We also know that the final temperature of the two blocks will be equal after the heat is exchanged because they will be in thermal equilibrium, \( T_f \).

Using this information, we can do the following
\[ Q_{\text{Al}} = -Q_{\text{Cu}} \]
\[ C_{v,\text{Al}}(T_f - T_{i,\text{Al}}) = -C_{v,\text{Cu}}(T_f - T_{i,\text{Cu}}) \]

Notice that regular heat capacity \( (C_v = \frac{m}{m_{\text{mol}}}c_{\text{mol}}) \) is being used here; this makes the resulting expression for \( T_f \), the final temperature of both blocks, look much friendlier.

\[ T_f = \frac{C_{v,\text{Al}}T_{i,\text{Al}} + C_{v,\text{Cu}}T_{i,\text{Cu}}}{C_{v,\text{Al}} + C_{v,\text{Cu}}} \]

And just for reference here’s what the equation looks like if you don’t use heat capacity, instead opting for all the given quantities in the question.

\[ T_f = \frac{m_{\text{Al}}c_{\text{mol,Al}}T_{i,\text{Al}} + m_{\text{Cu}}c_{\text{mol,Cu}}T_{i,\text{Cu}}}{m_{\text{Al}}c_{\text{mol,Al}} + m_{\text{Cu}}c_{\text{mol,Cu}}} \]

When you use either of these with the given values, you end up with the boxed answer choice. **Remember to convert the temperatures to Kelvin in the equations, then back to Celsius for the final answer.**

(b) The answer is (C). From the fundamental relation for entropy, we know that

\[ \Delta S = \int_{U_i}^{U_f} \frac{1}{T}dU = \int_{T_i}^{T_f} \frac{C_v}{T}dT \]

Where we use the fact that \( dU = C_vdT \) at constant volume, which is applicable here because the aluminum block does not change volume during the heat exchange.

We can now substitute the given values in part (a) into the second integral (remember to convert the temperatures to Kelvin!)

\[ \Delta S = \int_{90+273}^{62.9+273} \frac{8963}{T}dT = 8963 \ln \left( \frac{335.9}{363} \right) \approx -695.4 \text{ J/K} \]

Don’t get confused about this. The negative entropy here is totally valid *as long as* the entropy increase in the copper block exceeds the magnitude of this entropy change. You can calculate the copper block’s entropy to verify that the sum is greater than zero.

10. Consider a sealed container with a volume of 1 m\(^3\) filled with \(10^{23}\) helium atoms and \(2 \times 10^{23}\) molecules of nitrogen. Recall that helium is a monatomic gas and nitrogen is a diatomic gas. Initially the temperature of the gas mixture is 1000 K.

(a) Find the pressure inside the container (Answer in Pa).
   
   A) 8980
   
   B) 8290
(b) Find the ratio of the total molecular rotational energy to the total translational energy (molecular and atomic) inside the container. Assume that equipartition applies.

A) 0.333
B) 0.444
C) 0.666
D) 0.777

(a) The answer is (C). This container holds a mixture of two gases, so the ideal gas law has to be used on each separately, and then sum of the pressures will be the total pressure from the gas mixture.

Helium
\[ p_{He} = \frac{N_{He}k_bT}{V} \]

Nitrogen
\[ p_{N} = \frac{N_{N}k_bT}{V} \]

Given these relations, the total pressure is the sum of each partial pressure. Note that the volume and temperature is the same for these two gases.

\[ p = p_{He} + p_{N} = \frac{(N_{He} + N_{N})k_bT}{V} \]

(b) The answer is (B). First recall that the formula for the internal energy of a gas as a result of \( D \) degrees of a freedom is \( U = \frac{D}{2} N k_b T \).

The total translational energy is the sum of the energy from the He atoms and the N atoms, both of which have 3 translational degrees of freedom per particle. Let \( T \) be the total translational kinetic energy, then

\[ T = T_{He} + T_{N} = \frac{3}{2} N_{He} k_b T + \frac{3}{2} N_{N} k_b T \]

The total rotational energy, unlike the translational energy, comes entirely from the N atoms since they’re diatomic. There are only 2 degrees of freedom here. Let \( R \) be the rotational kinetic energy, then

\[ R = N_{N} k_b T \]

Taking the ratio of \( R \) to \( T \), \( \frac{R}{T} \), gives

\[ \frac{R}{T} = \frac{\frac{N_{N} k_b T}{\frac{3}{2} N_{He} k_b T + \frac{3}{2} N_{N} k_b T}}{\frac{2}{3} \frac{N_{N}}{N_{He} + N_{N}}} \]

Note that this took into account that Helium is monatomic \( (D = 3) \), and Nitrogen is diatomic \( (D = 5, \text{split among translation and rotation}) \).
11. The heat capacity of a solid is linear with temperature. How does its entropy change with temperature?

The entropy will also increase linearly. To show this, let \( C = \alpha T \)

\[
\Delta S = \int \frac{C}{T} \, dT = \int \alpha \, dT = \alpha T
\]

Essentially, the \( T \) in the denominator reduces the heat capacity by a factor of \( T \), but that factor is restored by the integral.

12. A monatomic, ideal gas is contained at fixed volume under pressure \( p \). Now suppose the pressure is tripled. What is the ratio of the initial \( v_{rms} \) to the final \( v'_{rms} \) ? \( (v_{rms}/v'_{rms}) \)

In order to calculate \( v_{rms} \), we must equate kinetic energy with the internal energy (of a single particle of the gas).

\[
\frac{1}{2} m v_{rms}^2 = \frac{3}{2} k_b T = \frac{3}{2} \frac{pV}{N}
\]

Where the ideal gas law was used to make the last equation. Solving for \( v_{rms} \), we get \( \sqrt{\frac{3pV}{Nm}} \)

If we then triple the pressure, we get \( v'_{rms} = \sqrt{9pV} \)

Taking the ratio, \( \frac{v_{rms}}{v'_{rms}} \), we get

\[
\frac{v_{rms}}{v'_{rms}} = \sqrt{\frac{3pV}{Nm}} \cdot \sqrt{\frac{9pV}{Nm}} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}
\]

13. Given the following expression for entropy:

\[
S = \ln \left( \frac{T}{a} \right) + \cos \left( \frac{PV}{k} \right) \tan(\sin(N))
\]

with \( a = 20 \) Kelvin, minimize the entropy with respect to

A) \( T \), Temperature
B) \( P \), Pressure
C) \( N \), Number of molecules (You can use Wolfram Alpha for this one :))

To minimize the entropy, we need to take its partial derivative with respect to the variable that is changing in the question, while holding all other variables constant.

(a)

\[
\frac{\partial S}{\partial T} = \frac{1}{T \tan(\sin(N))}
\]

Setting the derivative equal to zero, we find that entropy has no minimum for a varying temperature.
(b) \[
\frac{\partial S}{\partial P} = -\frac{V \sin \left( \frac{PV}{b} \right)}{b \tan(\sin(N))}
\]

Setting the derivative equal to zero and solving for \( P \), we get

\[0 = \sin \left( \frac{PV}{b} \right) \implies n\pi = \frac{PV}{b} \implies P = \frac{nb\pi}{V}\]

for \( n = 0, 1, 2, 3, \ldots \) And plugging this back into \( S \) will give you the minimized entropy.

(c) \[
\frac{\partial S}{\partial N} = -\frac{\ln \left( \frac{T}{a} \right) + \cos \left( \frac{PV}{b} \right)}{\tan^2(\sin(N)) \sec^2(\sin(N)) \cos(N)}
\]

Setting this derivative equal to zero (we can ignore the denominator since only the numerator can equal zero):

\[0 = -\ln \left( \frac{T}{a} \right) + \cos \left( \frac{PV}{b} \right) \cos(N)\]

(We brought the secant squared term to the denominator as cosine squared). Solving for \( N \), we get:

\[-\ln \left( \frac{T}{a} \right) = \cos(N) \implies N = \arccos \left( -\frac{\ln \left( \frac{T}{a} \right)}{\cos \left( \frac{PV}{b} \right)} \right)\]

There is no good way to simplify the above answer, but I hope you got some good practice finding minima! Putting \( N \) back into the equation for \( S \) will give you the minimized entropy.

14. A meteor is heading towards Earth! To simplify this situation, let’s say Earth \((m_E = 5.972 \times 10^{24} \text{ kg})\) is moving in the positive \( x \) direction at 30,000 m/s, and the meteor \((m_m = 8 \times 10^{22} \text{ kg})\) is heading on a collision course in the negative \( x \) direction at 20,000 m/s.

(a) Calculate the initial kinetic energy of this system, before the collision.

(b) When the meteor collides with Earth, the two stick together and continue floating through space. Find the resulting velocity of the Earth-meteor object.

(c) Calculate the kinetic energy of the Earth-meteor object.

(d) Determine the difference between the initial and final kinetic energies, and explain where this “lost energy” goes.

(a) \[
KE_{\text{before}} = \frac{1}{2} m_E v_E^2 + \frac{1}{2} m_m v_m^2
= 2.708 \times 10^{33} \text{ J}
\]

(b) The initial momentum before the collision is

\[
P_{\text{before}} = p_E + p_m
= m_E v_E - m_m v_m
\]
This momentum must be conserved after the collision.

\[
p_{\text{after}} = (m_E + m_m)v_{E+m}
\]

but \( p_{\text{after}} = p_{\text{before}} \)

\[
(m_E + m_m)v_{E+m} = m_Ev_E - m_mv_m
\]

\[
v_{E+m} = \frac{m_Ev_E - m_mv_m}{(m_E + m_m)v_{E+m}}
\]

= 29.339 m/s

(c) 

\[
KE_{\text{after}} = \frac{1}{2}(m_E + m_m)v_{E+m}^2
\]

= 2.605 \times 10^{33} \text{ J}

(d) \( KE_{\text{after}} - KE_{\text{before}} = -9.869 \times 10^{31} \text{ J} \). A portion of the kinetic energy was converted into internal energy through the collision/sticking process.

15. We’re playing a game with three 6-sided dice. After rolling, you get a point for every even number you roll.

(a) Write the total number of possible microstates.

(b) How many microstates are associated with a perfect score of 3? What is the probability of this score?

(c) The number of microstates that are associated with a score of 1 is 81 (Try it!). What is the probability of this score?

(d) Use \( S = k\ln(\Omega) \) to find the entropies of scores 3 and 1 (\( \Omega = \) the number of microstates). Which has the greater entropy?

(a) Total microstates = \( 6^3 = 216 \).

(b) To earn a point, a roll must be a 2, 4, or 6 (3 available options). Thus the number of microstates for a score of 3 is \( 3^3 = 27 \). The probability of getting this score is \( 27/216 = 0.125 \) (which could also be found by realizing each die has a 0.5 probability to score, resulting in \( 0.5^3 = 0.125 \)).

(c) Given the number of microstates is 81, we simply divide by the total number of microstates, which is 216, to get 0.325.

(d)

\[
S_3 = k\ln(27) = 4.548 \times 10^{-23} \text{ J/K}
\]

\[
S_1 = k\ln(81) = 6.054 \times 10^{-23} \text{ J/K}
\]

A score of one has greater entropy, meaning that it has more microstates associated with it (this makes sense intuitively, since a perfect score should be the least likely, therefore smallest entropy).