Graph Partitioning, Cake-cutting, and Applications to Political Redistricting

Graph partitioning problems appear in a variety of practical applications, including VLSI circuit layout, communication network design, and political redistricting. In the political redistricting problem, the graph defined by indivisible geographic units of voters must be partitioned into a fixed number of contiguous districts. Algorithmic approaches to political redistricting are often motivated by the salient issue of *gerrymandering*, the intentional manipulation of district shapes to favor one party. Several states in the U.S., notably Maryland, North Carolina, Wisconsin, and Pennsylvania, have recently been involved in gerrymandering court cases [1]. While algorithmic districting does not intrinsically prevent gerrymandering, the algorithm can be made publicly available to make the process more transparent. The proposed research will **provide innovative districting methods and promote transparent elections.**

Since finding an optimal districting plan is NP-hard, recent endeavors have focused on local-search meta-heuristics to efficiently find acceptable solutions [2]. To support these methods, King et al. developed a *geo-graph* model for enforcing contiguity constraints [3, 4, 5]. This model reduces the computational complexity of checking contiguity during a local-search transition in which a unit moves from one district to another. To verify contiguity, the geo-graph approach only examines the transferred unit's augmented neighborhood, defined as the units with which it shares at least one boundary point. In computational experiments, this vastly improves upon previous contiguity verification algorithms [3]. Approaching the political districting problem from a game-theoretic perspective, Pegden et al. proposed an alternating *I-cut-you-freeze* protocol for drawing districts, in which one party draws a districting plan, then the other chooses one district to freeze and redraws the rest, then the first party freezes one district and redraws, and so on [6]. The authors proved optimal strategies for the *continuous* (fractional voters allowed) *non-geometric* (contiguity not enforced) setting and provided evidence that this protocol can produce fairer outcomes than the prevailing "one party decides" method.

Research Plan: I will extend current approaches to political districting in three ways:

1. <u>Generalize the I-cut-you-freeze protocol to a more realistic setting.</u>

The equilibrium strategies in [6] sometimes require the weaker party to draw districts in which they have no vote share, which is impossible in practice. Therefore, I have added a constraint that the vote share in each district must fall within δ of an even ½- ½ split for some $0 < \delta < ½$. I proved that the same equilibrium strategies hold with this constraint, but the vote share thresholds for achieving specific seat shares change (manuscript in progress). I am further generalizing to the *discrete* non-geometric case, in which indivisible units with integral vote shares for each party are assigned to nearly-equipopulous districts. I plan to fully characterize the optimal strategies for each party, determine the complexity of finding these strategies, and evaluate the generated district plans by four fairness metrics: partisan symmetry, efficiency gap, responsiveness, and proportionality. This analysis will give **upper bounds on the performance of the I-cut-you-freeze protocol**, as it does not account for geographic constraints.

2. <u>Design and evaluate an I-cut-you-freeze variant in which an algorithm draws the districts.</u>

A potential drawback of the I-cut-you-freeze protocol is that each party could gerrymander on its turns, leading to a final districting plan with bizarre district shapes. I propose to remedy this issue by drawing districts algorithmically in each round with a compactness objective, where the "drawing" party can control secondary objective parameters. If the districting algorithm is made publicly available, then the two parties and the citizenry have equal access, **promoting a fair and transparent districting process**. Election simulations will illumine the practicality of this hybrid approach.

3. Extend the geo-graph model to higher dimensions.

The three-dimensional geographic partitioning problem is formulated as follows: Given a three-dimensional volume divided into cells (convex polyhedra glued together along faces), partition the cells into *k* zones such that every zone is contiguous and the chosen objectives (e.g. population balance, compactness) are optimized. Two units are *adjacent* if their polyhedra share a two-dimensional face. Basak defines this cell structure as a *combinatorial cell complex* (c.c.c.) with a partial order diagram representation [7]. The dual of a c.c.c.'s partial order diagram encodes adjacency relationships between cells, and I have used this representation to prove several 3-D analogs of the intermediate results in [3] (manuscript in progress). However, the necessary and sufficient conditions for contiguity that only examine the augmented neighborhood do not extend to 3-D, as I proved by a simple counterexample. The conditions remain necessary, so I now seek: (a) restrictions on the district topologies in the 3-D problem which make these conditions sufficient, and (b) stronger conditions which are both necessary and sufficient for the 3-D problem in full generality. I will then extend these results to the *n*-dimensional setting, which the partial order diagram representation naturally supports.

Qualifications/Resources: My undergraduate research project in discrete optimization, strong coursework history in CS and math, and research progress in the first year of my PhD program have prepared me to tackle the proposed projects. As a researcher in the Simulation and Optimization Laboratory at the University of Illinois, I enjoy access to extensive resources across the Computer Science (CS) and Industrial and Enterprise Systems Engineering (ISE) departments. I meet regularly with Douglas King and Sheldon Jacobson (my advisor), two of the innovators behind the geo-graph model. Thanks to the diversity of our CS and ISE departments, I also have access to faculty in computational geometry and algorithmic game theory.

Intellectual Merit: My proposed research of the I-cut-you-freeze protocol will prove new results in game theory and provide actionable insights about the performance of the protocol. By extending the geo-graph model, I will contribute theoretical results in computational geometry/topology and discrete optimization.

Broader Impacts: My research will extend and unify existing approaches to the political districting problem, resulting in robust, efficient methods with the power to transform districting practices and mitigate gerrymandering. Furthermore, my results for the 3-D geo-graph model have a wide range of applications beyond political districting, including office space allocation, finite element analysis, and 3-D image segmentation for medical scans. In these applications, the geo-graph model can lead to increased business productivity, better mechanical design processes, and faster medical diagnoses.

With the support of the NSF, my work will improve the political districting process and impact several other practical partitioning problems.

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